Exercises in GCSE Mathematics Higher Level

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Preface

This book covers the GCSE syllabi examined for the first time in 2003 and the higher part of the new two tier examination system beginning in September 2006.

All graphs can be accommodated on A4 size graph paper used in ‘portrait’ mode.

I would like to thank my wife Jenny and my two daughters Abigail and Hannah for all the help and encouragement they have given me in writing this.

R Joinson

September 2006
Chester

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1. Estimations and Calculations

In questions 1 to 36 use your calculator to work out the value of the problem. In each case
a) write down the calculator display correct to four significant figures
b) write down an estimate you can do to check your answer to part a.
c) write down your answer to part b.

1) $\dfrac{1.631 \times 5.482}{4.8143}$ 2) $\dfrac{12.62 \times 3.851}{2.415}$ 3) $\dfrac{9.143 \times 8.639}{4.384}$

4) $\dfrac{0.4831 \times 6.8159}{3.286}$ 5) $\dfrac{2.841 \times 0.356}{9.416}$ 6) $\dfrac{8.143 \times 0.3142}{7.8154}$

7) $\dfrac{0.7154 \times 0.832}{5.416}$ 8) $\dfrac{0.1843 \times 0.6315}{6.4153}$ 9) $\dfrac{0.5714 \times 0.8354}{7.831}$

10) $\dfrac{2.453 \times 0.154}{0.3651}$ 11) $\dfrac{8.415 \times 0.3142}{0.2164}$ 12) $\dfrac{15.815 \times 0.2231}{0.8145}$

13) $\dfrac{14.264 \times 27.742}{0.6421}$ 14) $\dfrac{27.98 \times 96.41}{0.415}$ 15) $\dfrac{184 \times 32.415}{0.283}$

16) $\dfrac{16.854 \times 0.0182}{0.3145}$ 17) $\dfrac{27.684 \times 0.8142}{0.0156}$ 18) $\dfrac{67.48 \times 29.684}{0.00142}$

19) $\dfrac{(27.63)^2 \times 14.32}{27.61}$ 20) $\dfrac{4.831 \times (0.821)^2}{0.614}$ 21) $\dfrac{23.41 \times (17.83)^2}{126.4}$

22) $\dfrac{(15.84)^2 \times 1.982}{0.614 \times 3.28}$ 23) $\dfrac{105 \times (0.325)^2}{0.761 \times 26.45}$ 24) $\dfrac{(0.312)^2 \times 27.4}{0.51 \times 6.72}$

25) $\dfrac{43.1 \times 6.74}{8.931 + 12.82}$ 26) $\dfrac{14.71 \times 23.42}{16.84 + 33.21}$ 27) $\dfrac{27.41 \times 0.8541}{4.312 + 6.842}$

28) $\dfrac{(26.3)^2 \times 0.416}{2.84(3.14 + 2.86)}$ 29) $\dfrac{41.6(8.42 + 3.86)}{0.415}$ 30) $\dfrac{(27.64)^2 \times 19.68}{2.54(2.84 + 6.41)}$

31) $\dfrac{27.63 \times (18.41)^2}{0.31(8.62 - 4.31)}$ 32) $\dfrac{(28.31)^2 \times (0.843)^2}{0.415}$ 33) $\dfrac{36.38 \times (4.321)^2}{0.31(2.865 + 7.416)}$

34) $\dfrac{14.841 \times 26.832}{(5.483)^2 + 16.943}$ 35) $\dfrac{(41.821)^2 \times 0.0154}{0.831 \times 0.0132}$ 36) $\dfrac{(0.531)^2 \times 0.00614}{0.8312 \times 0.0154}$

37) If $v = \dfrac{(8.64)^2 + 29.83}{0.0154}$ a) Use your calculator to find the value of v, correct to 3 significant figures. b) What figures would you use to check the value of v? c) Write down the answer to part b.

38) If $D = \dfrac{(27.61)^3 \times 0.00814}{3.61(2.48 + 5.61)}$ a) Use your calculator to find the value of D correct to 4 significant figures. b) What figures would you use to check the value of D? c) Write down the answer to part b.

2. Multiplication and Division

Do not use a calculator

Exercise 1
Short division with or without remainders

1) 57 ÷ 7	2) 83 ÷ 6	3) 94 ÷ 8	4) 106 ÷ 4
5) 183 ÷ 9	6) 401 ÷ 6	7) 372 ÷ 3	8) 861 ÷ 7
9) 974 ÷ 5	10) 462 ÷ 8	11) 341 ÷ 9	12) 576 ÷ 6

Exercise 2
Long division with or without remainders

1) 87 ÷ 17	2) 96 ÷ 23	3) 84 ÷ 11	4) 143 ÷ 34
5) 176 ÷ 26	6) 541 ÷ 67	7) 341 ÷ 44	8) 183 ÷ 14
9) 196 ÷ 16	10) 215 ÷ 18	11) 326 ÷ 24	12) 184 ÷ 17
13) 285 ÷ 22	14) 497 ÷ 31	15) 567 ÷ 34	16) 674 ÷ 23
17) 841 ÷ 21	18) 456 ÷ 27	19) 845 ÷ 42	20) 956 ÷ 51

Exercise 3
Division without remainders (answer in decimal form)

1) 15.0 ÷ 2	2) 25.0 ÷ 4	3) 58 ÷ 8	4) 34 ÷ 5
5) 30 ÷ 4	6) 93 ÷ 6	7) 188 ÷ 8	8) 90 ÷ 8
9) 81 ÷ 4	10) 273 ÷ 6	11) 27.6 ÷ 5	12) 210 ÷ 8
13) 145 ÷ 4	14) 238 ÷ 8	15) 214 ÷ 4	16) 156 ÷ 8
17) 14.7 ÷ 5	18) 50.4 ÷ 5	19) 58.8 ÷ 7	20) 583 ÷ 4

Exercise 4
Long multiplication

1) 27×32	2) 84×19	3) 26×47	4) 33×34	5) 86×54
6) 121×17	7) 216×27	8) 143×34	9) 256×47	10) 354×3
11) 374×63	12) 542×73	13) 431×86	14) 853×64	15) 427×27
16) 862×73	17) 491×93	18) 354×76	19) 529×69	20) 592×74

3. Negative Numbers

Do not use a calculator

Exercise 1

Calculate the final temperature.

1) 5ºC increases by 9ºC
2) 5ºC falls by 3ºC
3) 12ºC falls by 15ºC
4) –2ºC increases by 4ºC
5) –5ºC falls by 8ºC
6) 9ºC – 4ºC
7) –8ºC – 12ºC
8) –4ºC + 2ºC
9) 8ºC – 12ºC
10) –6ºC – 5ºC
11) –17ºC + 3ºC
12) –1ºC + 15ºC
13) 0ºC – 6ºC
14) 12ºC – 12ºC
15) –6ºC + 6ºC
16) –17ºC – 6ºC
17) –43ºC + 26ºC
18) –17ºC + 26ºC
19) –7ºC – 19ºC
20) –31ºC + 27ºC

Exercise 2

What is the change in temperature between each of the following?

1) 3ºC and 7ºC
2) 17ºC and 23ºC
3) –5ºC and 4ºC
4) –7ºC and 2ºC
5) –6ºC and –3ºC
6) –7ºC and 0ºC
7) 5ºC and 2ºC
8) 7ºC and –2ºC
9) 5ºC and –3ºC
10) –2ºC and –7ºC
11) –8ºC and –4ºC
12) 0ºC and –12ºC
13) –17ºC and –12ºC
14) 8ºC and –16ºC
15) –9ºC and –15ºC
16) –12ºC and 22ºC
17) –12ºC and 34ºC
18) –16ºC and –8ºC
19) –16ºC and 0ºC
20) 12ºC and –20ºC

Exercise 3

In each of the following, write down the number represented by the '?'

1) 5 – ? = 1
2) 3 – ? = 3
3) 4 – ? = –2
4) 7 – ? = –9
5) –2 + ? = 3
6) –5 + ? = 1
7) –4 – ? = –7
8) –3 + ? = 4
9) ? + 3 = 5
10) ? – 4 = 3
11) 5 – ? = –2
12) 5 – ? = –3
13) ? + 2 = –7
14) 4 + ? = –9
15) ? – 2 = –2
16) 7 + ? = 0
17) 8 + ? = 1
18) 10 – ? = –6
19) 4 + ? = –6
20) ? – 14 = –4

Exercise 4

1) Two numbers are multiplied together to make –30.
   One of the numbers is 6. What is the other?
2) Two numbers are multiplied together to make –18.
   One of the numbers is –6. What is the other?
3) Two numbers are multiplied together to make –60.
   The sum of the two numbers is 4. What are they?
4) Two numbers are multiplied together to make –144.
   The sum of the numbers is 0. What are the numbers?

4. Use of the Calculator

Exercise 1

Calculate each of the following pairs of problems. Predict the answers before you do them.

1)	$4 + 8 \div 4$	and	$(4 + 8) \div 4$
2)	$3 + 5 \times 4$	and	$(3 + 5) \times 4$
3)	$18 - 2 \times 3$	and	$(18 - 2) \times 3$
4)	$30 - 6 \div 2$	and	$(30 - 6) \div 2$
5)	$16 \div 4 + 4$	and	$16 \div (4 + 4)$
6)	$40 \div 8 + 2$	and	$40 \div (8 + 2)$
7)	$6 \times 4 + 2$	and	$6 \times (4 + 2)$

Exercise 2 (give your answer correct to 4 significant figures wherever necessary)

1) $\dfrac{16.59 - 8.25}{3.8}$

2) $\dfrac{12.7 - 2.4}{3.6 - 1.4}$

3) $5.7 + 3.6 \div 2.4$

4) $4.3 - 2.4 \div 3.8$

5) $\dfrac{6.3 - 2.8}{1.7 + 3.6 \times 1.4}$

6) $\dfrac{3.2}{5.7 + 3.6 \times 1.4}$

7) $5.3 - (2.6 - 1.4)$

8) $(4.3 + 3.6) \times (2.7 - 1.63)$

9) $\dfrac{9.7 - 7}{4.2 - 3.5 \div 5}$

10) $\dfrac{2.63 \times 3.8}{11.4 - 6.3}$

11) $\dfrac{5.8 \times (7 + 3)}{8 \div 5}$

12) $7.83 - (12.41 - 6.32)$

13) $\dfrac{7.2}{9.8 + 12.7}$

14) $\dfrac{7.2 + 12.7}{9.8}$

15) $\dfrac{9.48 \times 2.54 - 1.48}{6.42}$

16) $\dfrac{9.48 \times 2.54 - 1.48}{2.67 + 3.14}$

17) $\dfrac{18.31 - (2.48 + 3.65)}{6.51 - (2.87 + 2.61)}$

18) $\dfrac{26.14 \div 15.41}{3.87 \div 7.63}$

19) $\dfrac{(16.14 - 3.65) \div 2.16}{4.27 - 3.18}$

20) $\dfrac{19.42 - 3.15 \times 4.26}{3.17 \div (4.16 + 3.67)}$

21) $5 - 6\cos 20°$

22) $5.2^2 - 2.3\cos 30°$

23) $\sqrt{(6 + 3\tan 25)}$

24) $6 + 5\sin 40°$

25) $4\tan 35° + 3\sin 45°$

26) $\sqrt{(3.12 + 3\tan 40°)} + (2.56)^2$

27) $\dfrac{4.63 + 3.12^2 - 3\sin 55°}{\sqrt{5.84 + 0.31}}$

28) $\dfrac{8.91 \times 3.14 + 4\tan 27°}{4.35^2 - 3.86}$

5. Standard Form

Exercise 1

Write down these numbers in standard form

1) 457
2) 1427
3) 9431
4) 156,321
5) 17 million
6) 0.2813
7) 0.08142
8) 0.000486
9) 0.0000097

Exercise 2

Change these numbers from standard form

1) 2.8×10^5
2) 6.4×10^7
3) 9.3×10^4
4) 4.315×10^6
5) 8.614×10^9
6) 4.31×10^{-2}
7) 3.2×10^{-6}
8) 6.84×10^{-7}
9) 4.38×10^{-9}

Exercise 3

Calculate each of the following leaving your answers in standard form. Round off to four significant figures wherever necessary.

1) $(6.4 \times 10^2) \times (3.8 \times 10^4)$
2) $(5.4 \times 10^6) \times (8.3 \times 10^2)$
3) $(4.6 \times 10^{-2}) \times (3.4 \times 10^{-3})$
4) $(4.3 \times 10^{-7}) \times (8.8 \times 10^5)$
5) $(8.3 \times 10^{-2}) \times (6.4 \times 10^8)$
6) $(5.3 \times 10^{-7}) \times (4.6 \times 10^3)$
7) $(5.7 \times 10^{-2}) \div (3.4 \times 10^2)$
8) $(8.3 \times 10^{-6}) \div (5.4 \times 10^3)$
9) $(8.4 \times 10^5) \div (2.4 \times 10^{-3})$
10) $(5.4 \times 10^{-7}) \div (4.3 \times 10^{-2})$
11) $137,000 \times 10^5$
12) 0.08123×10^6
13) $27.31 \times 4.82 \times 10^6$
14) $571.31 \times 4.2 \times 10^{-7}$
15) $\dfrac{3.841 \times 10^6}{3.182 \times 10^2}$
16) $\dfrac{7.41 \times 10^{-4}}{3.54 \times 10^{-6}}$
17) $\dfrac{(27.41 \times 10^3) \times (2.684 \times 10^7)}{7.41 \times 10^5}$
18) $\dfrac{(2.641 \times 10^{-3}) \times (2.84 \times 10^{-6})}{3.82 \times 10^5}$
19) $\sqrt{4 \times 10^4}$
20) $\sqrt{9 \times 10^{-6}}$

Exercise 4

1) The distance from the earth to a star is 8×10^{13} kilometres. Light travels at a speed of approximately 3.0×10^5 kilometres per second.
   a) How far will light travel in one year?
   b) How long will light take to travel from the star to the earth? Give your answer correct to 3 significant figures.
2) The mass of the earth is 5.976×10^{24} kilograms and the mass of the moon is 7.35×10^{22} kilograms.
   a) Write down these masses in tonnes.
   b) How many times is the mass of the earth greater than the mass of the moon?
3) A neutron has a mass of 1.675×10^{-27} kilograms and an electron 9.109×10^{-31} kilograms.
   a) How many electrons are needed for their mass to be equal to that of a neutron?
   b) How many electrons are required to have a mass of 1 kg?

6. Rational and Irrational Numbers

Exercise 1

Convert the following numbers into fractions

1) $0.2\dot{5}$ 2) $0.3\dot{1}$ 3) $0.5\dot{4}$ 4) $0.6\dot{2}$
5) $0.0\dot{4}$ 6) $0.7\dot{3}$ 7) $0.00\dot{7}$ 8) $0.01\dot{7}$
9) $0.\dot{1}\dot{5}$ 10) $0.\dot{3}\dot{2}$ 11) $0.\dot{5}\dot{3}$ 12) $0.0\dot{7}\dot{2}$

Exercise 2

Which of the following numbers are rational?

1) $\frac{1}{5}$ 2) $\frac{7}{8}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{\sqrt{7}}{\sqrt{8}}$
5) π 6) π^2 7) $0.2\dot{3}$ 8) $\sqrt{3}$
9) $\frac{\sqrt{4}}{\sqrt{25}}$ 10) $\sqrt{\frac{9}{49}}$ 11) $\frac{\sqrt{5}}{5^2}$ 12) $\left(\frac{2}{3}\right)^2$
13) $\sqrt{6.25}$ 14) $\sqrt{2.5}$ 15) $\sqrt{4}$ 16) $\sqrt{4.1}$

Exercise 3

Which of the following are irrational?

1) $1+\frac{1}{3}$ 2) $1+\frac{1}{\sqrt{3}}$ 3) $1+\frac{\sqrt{3}}{2}$
4) $3+\frac{1}{\sqrt{3}}$ 5) $2-\frac{2}{\sqrt{2}}$ 6) $\frac{18}{\sqrt{3}}-\frac{12}{\sqrt{3}}$
7) $2^0+2^{-1}+2^{-2}$ 8) $(1+\sqrt{2})\times(1+\sqrt{2})$ 9) $2\sqrt{2}$
10) 2.58 11) $0.3\dot{7}$ 12) $1.\dot{5}\dot{1}$
13) $\sqrt{8}\times\sqrt{2}$ 14) $\sqrt{12}\times\sqrt{2}$ 15) $\sqrt{2}\times\sqrt{3}$
16) $6^{\frac{1}{2}}+3^0$ 17) $4^{\frac{1}{2}}\times 9^{-2}$ 18) $4^{\frac{1}{2}}\times 9^{-\frac{1}{2}}$

Exercise 4

Which of the following equations have rational answers?

1) $2x = 5$ 2) $3x = \sqrt{3}$ 3) $4x = 2^2$
4) $\sqrt{2x} = \frac{\sqrt{2}}{4}$ 5) $5x = \frac{7}{8}$ 6) $\sqrt{3x} = \frac{4}{\sqrt{3}}$
7) $5x^2 = 7$ 8) $9x^2 = 4$ 9) $4x^2 = 17$
10) $3x^2 = \frac{1}{2}$ 11) $\frac{3}{4}x^2 = 12$ 12) $7x^2 = 5$

Exercise 5

Which two of the following are descriptions of irrational numbers?

a) A number which, in its decimal form, recurs.
b) A number written in its decimal form has a finite number of decimal places.
c) A number whose exact value cannot be found.
d) A number which can be represented by the ratio of two integers.
e) An infinite decimal which does not repeat itself.

7. Surds 1

Do not use a calculator

Exercise 1

Simplify each of the following by writing as products of whole numbers and surds. The first one has been done for you.

1) $\sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$

2) $\sqrt{24}$ 3) $\sqrt{27}$ 4) $\sqrt{32}$ 5) $\sqrt{45}$

6) $\sqrt{63}$ 7) $\sqrt{48}$ 8) $\sqrt{50}$ 9) $\sqrt{72}$

10) $\sqrt{75}$ 11) $\sqrt{80}$ 12) $\sqrt{125}$ 13) $\sqrt{147}$

Exercise 2

Simplify each of the following by rationalising the denominator. The first one has been done for you.

1) $\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{\sqrt{5\times 5}} = \frac{10\sqrt{5}}{\sqrt{25}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{7}}$ 5) $\frac{1}{\sqrt{11}}$

6) $\frac{2}{\sqrt{2}}$ 7) $\frac{3}{\sqrt{3}}$ 8) $\frac{6}{\sqrt{3}}$ 9) $\frac{6}{\sqrt{2}}$

10) $\frac{9}{\sqrt{3}}$ 11) $\frac{7}{\sqrt{5}}$ 12) $\frac{10}{\sqrt{2}}$ 13) $\frac{21}{\sqrt{7}}$

14) $\frac{14}{\sqrt{7}}$ 15) $\frac{22}{\sqrt{3}}$ 16) $\frac{21}{\sqrt{3}}$ 17) $\frac{30}{\sqrt{2}}$

Exercise 3

Simplify each of the following, the first one has been done for you.

1) $\sqrt{3}\times\sqrt{8} = \sqrt{3}\times\sqrt{4\times 2} = \sqrt{3}\times\sqrt{4}\times\sqrt{2} = \sqrt{3}\times 2\times\sqrt{2} = 2\sqrt{3\times 2} = 2\sqrt{6}$

2) $\sqrt{5}\times\sqrt{20}$ 3) $\sqrt{2}\times\sqrt{6}$ 4) $\sqrt{3}\times\sqrt{12}$

5) $\sqrt{3}\times\sqrt{21}$ 6) $\sqrt{5}\times\sqrt{10}$ 7) $\sqrt{2}\times\sqrt{12}$

8) $\sqrt{6}\times\sqrt{3}$ 9) $\sqrt{3}\times\sqrt{18}$ 10) $\sqrt{10}\times\sqrt{2}$

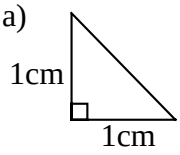
11) $\sqrt{5}\times\sqrt{30}$ 12) $\sqrt{2}\times\sqrt{14}$ 13) $\sqrt{10}\times 2\sqrt{2}$

14) $\sqrt{2}\times 3\sqrt{12}$ 15) $4\sqrt{8}\times\sqrt{12}$ 16) $\sqrt{10}\times 2\sqrt{2}$

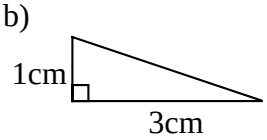
Exercise 4

In each of the following right angled triangles, write down the length of the unknown side as a surd.

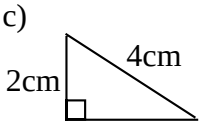
a)



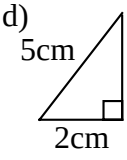
b)



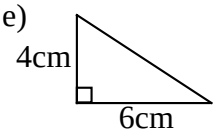
c)



d)



e)



8. Surds 2

Exercise 1

Simplify each of the following by writing as products of whole numbers and surds

1) $\sqrt{8}$ 2) $\sqrt{12}$ 3) $\sqrt{24}$ 4) $\sqrt{28}$
5) $\sqrt{108}$ 6) $\sqrt{40}$ 7) $\sqrt{50}$ 8) $\sqrt{18}$
9) $\sqrt{48}$ 10) $\sqrt{32}$ 11) $\sqrt{20}$ 12) $\sqrt{125}$
13) $\sqrt{200}$ 14) $\sqrt{216}$ 15) $\sqrt{192}$ 16) $\sqrt{320}$

Exercise 2

Simplify

1) $\frac{2}{\sqrt{2}}$ 2) $\frac{3}{\sqrt{3}}$ 3) $\frac{4}{\sqrt{4}}$ 4) $\frac{6}{\sqrt{2}}$
5) $\frac{14}{\sqrt{7}}$ 6) $\frac{8}{\sqrt{2}}$ 7) $\frac{9}{\sqrt{3}}$ 8) $\frac{12}{\sqrt{3}}$
9) $\frac{14}{\sqrt{2}}$ 10) $\frac{20}{\sqrt{2}}$ 11) $\frac{30}{\sqrt{3}}$ 12) $\frac{50}{\sqrt{5}}$
13) $\frac{70}{\sqrt{5}}$ 14) $\frac{39}{\sqrt{3}}$ 15) $\frac{49}{\sqrt{7}}$ 16) $\frac{63}{\sqrt{21}}$

Exercise 3

Simplify

1) $\sqrt{2} + 2\sqrt{2}$ 2) $\sqrt{3} + 3\sqrt{3}$ 3) $2\sqrt{2} + 3\sqrt{2}$
4) $\sqrt{8} + \sqrt{2}$ 5) $\sqrt{8} - \sqrt{2}$ 6) $\sqrt{12} - \sqrt{3}$
7) $2\sqrt{5} - \sqrt{5}$ 8) $\sqrt{32} - 2\sqrt{2}$ 9) $2\sqrt{5} - \sqrt{5}$
10) $3\sqrt{5} - 2\sqrt{5}$ 11) $4\sqrt{7} - \sqrt{28}$ 12) $\sqrt{500} - 3\sqrt{5}$

Exercise 4

Simplify

1) $2\sqrt{2} - \frac{2}{\sqrt{2}}$ 2) $2\sqrt{2} - \frac{3}{\sqrt{2}}$ 3) $2\sqrt{3} + \frac{3}{\sqrt{3}}$
4) $\frac{12}{\sqrt{2}} + 2\sqrt{2}$ 5) $\frac{18}{\sqrt{3}} + 2\sqrt{3}$ 6) $\frac{28}{\sqrt{7}} - 3\sqrt{7}$
7) $\frac{30}{\sqrt{5}} + \sqrt{5}$ 8) $\frac{144}{2\sqrt{3}} - 23\sqrt{3}$ 9) $\frac{49}{\sqrt{7}} - 3\sqrt{7}$
10) $\frac{45}{3\sqrt{5}} - \sqrt{5}$ 11) $\frac{60}{\sqrt{20}} + 3\sqrt{5}$ 12) $\frac{80}{\sqrt{8}} - 2\sqrt{2}$

Exercise 5

Simplify

1) $\sqrt{6} \times \sqrt{3}$ 2) $\sqrt{5} \times \sqrt{10}$ 3) $\sqrt{3} \times \sqrt{12}$
4) $\sqrt{6} \times \sqrt{12}$ 5) $\sqrt{7} \times 2\sqrt{7}$ 6) $3\sqrt{2} \times 2\sqrt{8}$
7) $2\sqrt{2} \times 4\sqrt{12}$ 8) $5\sqrt{6} \times 4\sqrt{3}$ 9) $4\sqrt{2} \times 3\sqrt{8}$
10) $5\sqrt{10} \times 2\sqrt{2}$ 11) $7\sqrt{2} \times 3\sqrt{12}$ 12) $4\sqrt{8} \times 7\sqrt{12}$

9. Prime Factors

Exercise 1

Express the following numbers as products of their prime factors.

1) 300	2) 900	3) 630	4) 700	5) 792
6) 945	7) 1960	8) 1815	9) 1512	10) 8580
11) 2640	12) 5460	13) 3744	14) 6336	15) 9240

Exercise 2

Express each of the following numbers as products of their prime factors. In each case state the smallest whole number it has to be multiplied by to produce a perfect square.

1) 660	2) 300	3) 450	4) 700	5) 1575
6) 2205	7) 600	8) 396	9) 1350	10) 1872
11) 4950	12) 3840	13) 8820	14) 11,760	15) 11,340

Exercise 3

Calculate the largest odd number that is a factor of each of the following.

1) 120	2) 210	3) 432	4) 416	5) 440
6) 704	7) 1144	8) 1200	9) 1840	10) 1848
11) 2464	12) 2112	13) 5880	14) 4725	15) 9240

Exercise 4

1) a) What is the highest common factor of 735 and 756?

An area of land measures 73.5 metres by 75.6 metres. It is to be divided up into square plots of equal size.

b) What is the size of the largest squares that will fit on it?

c) How many squares will fit on it?

2) a) What is the highest common factor of 60 and 75?

b) Emily's mum organises a party for her. She makes 60 cakes and 75 sandwiches. Everyone at the party is allowed the same amount of food to eat. She invites as many children as possible. How many does she invite?

3) a) What is the highest common factor of 990 and 756?

b) A shop is moving its stock. It has 9900 type A items and 7560 type B items. They have to be packed into boxes, each box containing both item A and item B. The same number of type A items and the same number of type B items are in each box. What is the maximum number of boxes needed and c) how many items will be in each box?

10. Fractions

Exercise 1
Simplify into single fractions

1) $\frac{1}{5}+\frac{2}{5}$ 2) $\frac{1}{3}+\frac{2}{3}$ 3) $\frac{7}{10}+\frac{7}{10}$ 4) $\frac{1}{5}+\frac{1}{3}$
5) $\frac{1}{8}+\frac{1}{7}$ 6) $\frac{1}{3}-\frac{1}{4}$ 7) $\frac{3}{5}+\frac{2}{3}$ 8) $\frac{4}{5}-\frac{2}{7}$
9) $\frac{7}{9}+\frac{7}{12}$ 10) $1\frac{3}{4}-\frac{3}{5}$ 11) $2\frac{7}{8}-\frac{11}{12}$ 12) $4\frac{3}{7}-\frac{4}{5}$
13) $\frac{a}{7}+\frac{2a}{7}$ 14) $\frac{4a}{5}+\frac{2a}{5}$ 15) $\frac{a}{4}+\frac{3a}{4}$ 16) $\frac{x}{5}+\frac{x}{3}$
17) $\frac{x}{5}+\frac{x}{7}$ 18) $\frac{b}{2}-\frac{b}{5}$ 19) $\frac{3x}{8}-\frac{x}{4}$ 20) $\frac{7x}{10}-\frac{2x}{7}$
21) $\frac{3x}{10}+\frac{4x}{9}$ 22) $\frac{3a}{2}+\frac{3a}{4}$ 23) $\frac{15x}{7}+\frac{x}{2}$ 24) $\frac{11x}{4}-\frac{x}{11}$

Exercise 2
Simplify

1) $\frac{2}{x}+\frac{3}{x}$ 2) $\frac{7}{a}-\frac{6}{a}$ 3) $\frac{6}{x}+\frac{9}{x}$ 4) $\frac{1}{x}+\frac{3}{2x}$
5) $\frac{5}{2x}+\frac{3}{2x}$ 6) $\frac{3}{4x}-\frac{1}{5x}$ 7) $\frac{2}{3x}+\frac{3}{2x}$ 8) $\frac{5}{4a}-\frac{4}{5a}$
9) $\frac{1}{a}+\frac{1}{b}$ 10) $\frac{2}{x}-\frac{1}{y}$ 11) $\frac{3}{x}+\frac{5}{y}$ 12) $\frac{1}{5}+\frac{1}{b}$
13) $\frac{2}{5}+\frac{3}{b}$ 14) $\frac{4}{5}+\frac{x}{5}$ 15) $\frac{4}{x}-\frac{x}{4}$ 16) $\frac{x}{4}-\frac{3}{x}$

Exercise 3
Simplify

1) $\frac{3}{4}+\frac{2}{a+1}$ 2) $\frac{5}{8}-\frac{1}{x+1}$ 3) $\frac{3}{10}-\frac{2}{x-1}$
4) $\frac{x}{3}+\frac{x-1}{4}$ 5) $\frac{a}{4}-\frac{x+1}{3}$ 6) $\frac{2b}{5}+\frac{3a}{4}$
7) $\frac{4x+3}{9}+\frac{5x}{2}$ 8) $\frac{5a}{4}+\frac{2a+3}{5}$ 9) $\frac{7x+4}{3}-\frac{5x}{8}$
10) $\frac{x}{2}-\frac{2x-1}{3}$ 11) $\frac{4y+3}{6}-\frac{y}{5}$ 12) $\frac{7a}{4}+\frac{3a+5}{7}$
13) $\frac{6}{x-1}+\frac{3}{x-4}$ 14) $\frac{7}{x+2}+\frac{4}{2x-1}$ 15) $\frac{5}{3x+2}-\frac{2}{4x+1}$
16) $\frac{5}{2x}+\frac{y}{3}$ 17) $\frac{3x}{a}+\frac{4x}{5a}$ 18) $\frac{4x}{a}-\frac{5x}{3a}$
19) $\frac{1}{5}(x+1)-\frac{1}{6}(x+3)$ 20) $\frac{5}{8}(x+1)-\frac{2}{5}(x-3)$ 21) $\frac{3}{5}(a-2)+\frac{3}{7}(a-1)$
22) $\frac{3(x+3)}{4}+x$ 23) $\frac{5(a+6)}{4}-a$ 24) $\frac{11(x+3)}{4}-2x$
25) $x+\frac{2}{x}$ 26) $3a+\frac{4a+1}{3}$ 27) $5x-\frac{3x+3}{4}$

11. Fractions, Decimals and Percentages 1

Exercise 1
Change into decimals (correct to 4 decimal places where necessary)

1) $\frac{2}{5}$	2) $\frac{3}{8}$	3) $\frac{4}{5}$	4) $\frac{7}{8}$	5) $\frac{3}{4}$	6) $\frac{3}{20}$
7) $\frac{9}{20}$	8) $\frac{7}{20}$	9) $\frac{4}{15}$	10) $\frac{7}{30}$	11) $\frac{17}{35}$	12) $\frac{7}{28}$
13) $\frac{13}{15}$	14) $\frac{4}{17}$	15) $\frac{4}{9}$	16) $\frac{9}{14}$	17) $\frac{12}{35}$	18) $\frac{7}{13}$

Exercise 2
Change these decimals into percentages

1) 0.15	2) 0.42	3) 0.31	4) 0.94	5) 0.38	6) 0.56
7) 0.72	8) 0.387	9) 0.552	10) 0.673	11) 0.841	12) 0.529
13) 0.781	14) 0.7	15) 0.1	16) 4.5	17) 2.78	18) 5.23

Exercise 3
Change into percentages correct to 4 significant figures

1) $\frac{3}{8}$	2) $\frac{7}{15}$	3) $\frac{9}{20}$	4) $\frac{6}{15}$	5) $\frac{4}{21}$	6) $\frac{7}{12}$
7) $\frac{7}{25}$	8) $\frac{17}{30}$	9) $\frac{14}{35}$	10) $\frac{7}{25}$	11) $\frac{23}{35}$	12) $\frac{12}{33}$
13) $\frac{34}{53}$	14) $\frac{31}{42}$	15) $\frac{53}{88}$	16) $\frac{32}{45}$	17) $\frac{17}{123}$	18) $\frac{72}{95}$

Exercise 4
Compare each of the following sets of numbers by first changing them into percentages and then writing them down in order of size, smallest to largest.

1) $\frac{1}{2}$	0.4	48%	2) $\frac{5}{8}$	0.61	67%
3) $\frac{7}{12}$	0.6	67%	4) $\frac{7}{32}$	0.2	23%
5) $\frac{3}{10}$	0.32	34%	6) $\frac{5}{16}$	0.35	33.5%
7) $\frac{7}{25}$	0.27	30%	8) $\frac{6}{17}$	0.37	29.4%
9) $\frac{6}{24}$	0.24	27%	10) $\frac{9}{30}$	0.34	27.4%

Exercise 5 Calculate

1) $\frac{1}{4}$ of 30	2) $\frac{3}{4}$ of 192	3) $\frac{3}{8}$ of 88metres	4) $\frac{3}{7}$ of 168m
5) $\frac{7}{8}$ of £44	6) $\frac{1}{4}$ of $32\frac{1}{2}$ kg	7) $\frac{7}{12}$ of 15.6kg	8) $\frac{5}{8}$ of £12.80
9) $\frac{7}{40}$ of 8m	10) $\frac{7}{32}$ of £1.60	11) $\frac{7}{16}$ of 160m	12) $\frac{7}{20}$ of 8.40m

12. Fractions, Decimals and Percentages 2

Exercise 1

1) 37% of 600	2) 24% of 50	3) 36% of 950	4) 41% of500
5) 15% of £6	6) 40% of £1.50	7) 60% of £19	8) 17% of 8 metres
9) 24% of £9	10) 72% of £4.50	11) 52% of £16.50	12) 93% of 1200

Exercise 2

Change these marks into percentages. (Give your answer correct to the nearest whole number)

1) 24 out of 50	2) 38 out of 60	3) 27 out of 40	4) 37 out of 80
5) 56 out of 90	6) 97 out of 150	7) 43 out of 200	8) 63 out of 70
9) 84 out of 120	10) 156 out of 250	11) 17 out of 20	12) 76 out of 110
13) 43 out of 76	14) 58 out of 95	15) 62 out of 68	16) 27 out of 45

Exercise 3

Find the percentage profit on each of the following, correct to the nearest whole number.

	Buying Price	Selling Price
1)	£100	£120
2)	£50	£80
3)	£60	£80
4)	£1.50	£1.80
5)	£2.80	£3.10
6)	£1,500	£1,700
7)	£45,000	£47,000
8)	£42.50	£45.00
9)	£900	£950
10)	£2010	£2500

Exercise 4

Find the selling price for each of these.

	Buying Price	Profit
1)	£100	17%
2)	£200	21%
3)	£150	20%
4)	£2000	15%
5)	£4200	32%
6)	£200	7.5%
7)	£70	25%
8)	£49,000	15%
9)	£80	27%
10)	£450	22%

13. Percentages

1. By selling a car for £2,500, Ben made a profit of 25%. How much did he pay for it?
2. A company makes a profit for the year of £75,000 before tax is paid. What percentage tax does it pay if its tax bill amounts to £11,250?
3. a) What is the total cost of a television set if it is priced at £240 plus VAT of $17\frac{1}{2}$%?

   b) A radio costs £21.60 in a sale. If it had a reduction of 10%, what was its original price?

   c) A computer costs £998.75 including VAT at $17\frac{1}{2}$%. What is its price before VAT is added?
4. Rachel invests £1500 in a bank account which pays interest of $6\frac{1}{2}$% per annum.

   a) How much interest has she earned at the end of 1 year?

   b) She has to pay tax on this interest at 22%. How much tax does she pay?
5. William earns £23,000 per year as a shop manager.

   a) If he is offered a pay rise of $7\frac{1}{2}$%, what will his new wages be?

   b) Instead he is offered a new job by a different firm and the rate of pay is £25,400 per annum. What percentage increase does this represent on his old wages?
6. The population of a certain town was 50,000 at the beginning of 1998. It is expected to rise by 7% each year until the end of the year 2000. What is the expected population at the end of this period?
7. A shopkeeper buys 35 radios for £435.75. If she sells them at £15 each, what is her percentage profit?
8. A car was bought at the beginning of 1994. During the first year it depreciated in value by 23% and then by 9% each subsequent year. If its original price was £9,000, what was its value at the end of 1997, to the nearest pound?
9. A can of cola has a label on it saying ‘20% extra free’.

   a) If the can holds 960ml, what did the original can hold?

   b) The original can cost 45p. If the company increase the price of the new can to 60p, does this represent an increase in price? Explain your answer.
10. In the general election, Maureen Johnson got 22,016 votes, which was 43% of all the votes cast.

    a) If Anthony Jones got 19,968 votes, what percentage of the people voted for him?

    b) If John Parry got 8% of the vote, how many people voted for him?
11. The cost of building a bridge in 1995 was estimated as £24 million. When it was finally completed in 1998 its total cost amounted to £37.7 million. What was the percentage increase?
12. It is estimated that a certain rainforest gets smaller by 8% each year. Approximately how many years will it take to be 39% smaller?
13. A firms profits were £500,000 in 1991. In 1992 they were 15% higher. However in 1993 they were 5% lower than in the previous year. What were the profits in 1993?
14. £1000 is invested at 6% compound interest. Interest is added to the investment at the end of each year. For how many years must the money be invested to in order to get at least £400 interest?
15. A lady wants a room to be built onto her house. Builder A quotes £11,400 which includes VAT of $17\frac{1}{2}$%. However he will reduce this by 10% if it is accepted within one week.

    Builder B quotes £9000 excluding VAT. Which is the cheapest quotation and by how much?
16. VAT of $17\frac{1}{2}$% is added to the cost of a computer. If the VAT is £166.25, what is the total cost of the computer?

14. Interest

Exercise 1

Find the simple and compound interest (without using the compound interest formula) on each of the following. Wherever necessary give your answer correct to the nearest penny.

1) £100 invested for 2 years at 2% interest per annum.

2) £150 invested for 2 years at 12% interest per annum.

3) £500 invested for 3 years at 9% interest per annum.

4) £1000 invested for 4 years at 10% interest per annum.

5) £1500 invested for 3 years at 7% interest per annum.

6) £2000 invested for 3 years at 4% interest per annum.

7) £5200 invested for 4 years at 5% interest per annum.

8) £120 invested for 2 years at 7% interest per annum.

9) £550 invested for 3 years at 8% interest per annum.

10) £2100 invested for 4 years at 6% interest per annum.

Exercise 2

The Compound Interest Formula is

$$x = P\left(1 + \frac{R}{100}\right)^n$$

Where x represents the amount in the bank after n years with a rate of R% on a principle of P.

1) Use the compound interest formula to calculate the amount of money in a bank account when

   a) 200 Euros is invested for 5 years at a rate of 4.5%

   b) 500 Euros is invested for 7 years at a rate of 3.7%

   c) 1,200 Euros is invested for 12 years at a rate of 5.6%

2) 6,000 Euros is invested in an account that pays interest at a compound rate of 4.7%

   a) Calculate the value of $x = 1 + \frac{R}{100}$

   b) By using the x^y key on your calculator, make a list of the amounts of money in the account at the end of each of the 10 years the money is left in the account.

3) Calculate the interest gained when 10,000 Euros is invested for 15 years in a bond which pays an interest of 3.74% per annum.

4) What is the difference between the simple and compound interest earned on an investment of 5,000 Euros over a period of 12 years at a rate of 4.86%?

15. Scale Drawings and Ratio

Do not use a calculator

Exercise1

Fill in the missing values for each of the following

Scale	Dimensions on Drawings	Actual Dimensions
1:4	10cm	
1:5		40cm
1:10	6.2cm	
1:20		140cm
1:40	10cm	
1:8		1.28cm
	20cm	6 metres
	15cm	3 metres
1:50		2.5 metres
1:100	2.5cm	
	2.5cm	5 metres
	7cm	17.5cm
1:500		27.5m
1:75	6cm	
	15cm	4.5m
1:12		138cm
1:250	3.6cm	
	4.5cm	22.5cm
1:75		600cm
1:40	2.6cm	

Exercise 2

Divide each of the following into the ratios given.

1) £900 into the ratio 4:5
2) £1000 into the ratio 3:7
3) £200 into the ratio 3:5
4) £600 into the ratio 7:8
5) £800 into the ratio 5:11
6) £700 into the ratio 5:9
7) £630 into the ratio 7:11
8) £1265 into the ratio 9:14
9) £2205 into the ratio 8:13
10) £1200 into the ratio 3:4:5
11) £450 into the ratio 5:6:7
12) £315 into the ratio 2:3:4
13) £1008 into the ratio 7:8:9
14) £1215 into the ratio 7:9:11
15) £550 into the ratio 5:8:9
16) £78.40 into the ratio 3:4:7
17) £150 into the ratio 6:8:11
18) £13.86 into the ratio 3:7:11
19) £864 into the ratio 4:7:13
20) £343 into the ratio 3:4:7

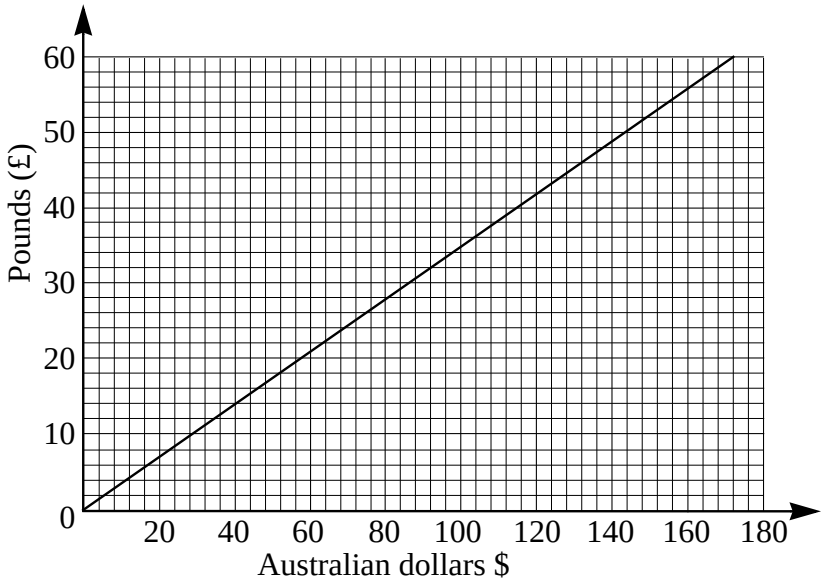
Exercise 3

Three people, A, B and C, share an amount of money in the ratios shown below.
In each case calculate the total amount of money shared out and the amount C gets.

1) Ratio 2:3:4. A gets £8
2) Ratio 3:4:5. B gets £12
3) Ratio 3:8:10. B gets £24
4) Ratio 3:5:7. A gets £33
5) Ratio 7:11:14. B gets £99
6) Ratio 3:5:11. A gets £1.65
7) Ratio 2:5:8. B gets £3.35
8) Ratio 3:6:13. B gets £6.72
9) Ratio 5:7:9. A gets £11.55
10) Ratio 4:11:13. B gets £56.10

16. Conversion Graphs 1

1. This graph shows the relationship between the Australian dollars and the pound sterling



From the graph determine:
a) The rate of exchange (i.e. the number of dollars to the pound)
b) The number of dollars that can be obtained for £22.
c) The amount of £ sterling that can be exchanged for $140.

2. Christmas trees are priced according to their height. The table below shows some of the prices charged last Christmas.

Height (metres)	1.0	1.5	2.0	2.5	3.0
Price (£)	8.50	11.75	15.00	18.25	21.50

Draw a conversion graph using 8cm to represent 1 metre on the vertical axis and 1cm to represent £2 on the horizontal axis.
a) From your graph, calculate the cost of a tree measuring 2m 35cm.
b) What size tree can be purchased for £12.50?

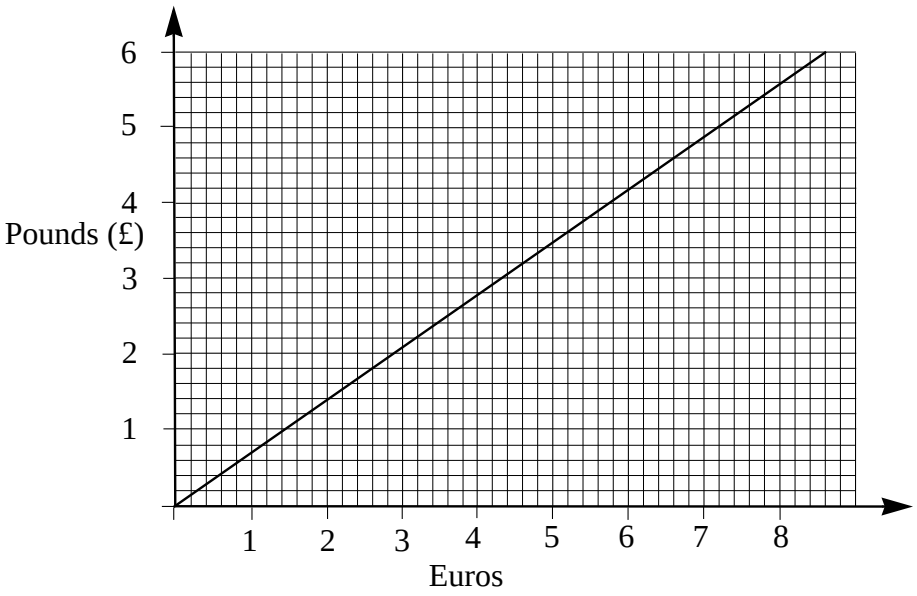
3. Bill is a craftsman who makes wooden bowls on his lathe. He advertises that he can make any size bowl between 20cm and 60cm diameter. In his shop he gives the price of five different bowls as an example.

Diameter of bowl (cm)	20	30	40	50	60
Price	£8.80	£20.00	£42.40	£79.00	£133.60

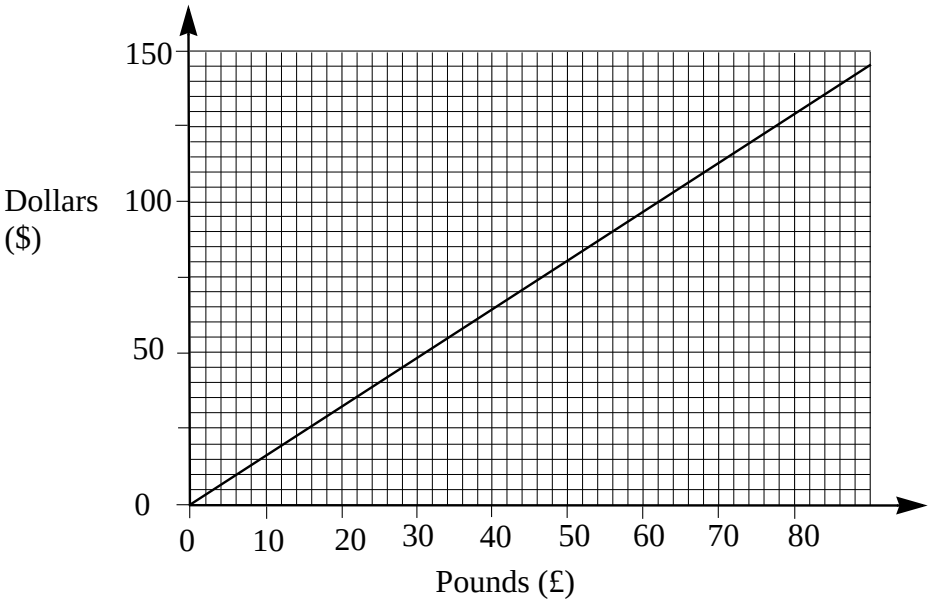
a) Use these figures to draw a conversion graph. Use a scale of 2cm to represent a diameter of 10cm on the horizontal axis and 2cm to represent £10 on the vertical axis.
b) Jane has £50 to spend. From your diagram, estimate the size of bowl she can buy.
c) What is the cost of a bowl of 34cm diameter?

17. Conversion Graphs 2

1) The graph can be used to convert pounds (£) into Euros. Use it to convert;
   a) £4.50 into Euros
   b) 5.00 Euros into pounds and pence.



2) The graph can be used to convert pounds (£) into US dollars ($). Use it to convert;
   a) £70 into dollars
   b) $60 into pounds.



18. Conversion Graphs 3

1) 1kg is approximately 2.2lbs. Calculate what 40 kg is in pounds. From this information draw a conversion graph to convert kg into pounds.
Use a horizontal scale of 4cm to 10kg and a vertical scale of 4cm to 20lbs.
From your graph convert;
a) 23kg into pounds
b) 75 pounds into kg.

2) It is known that 1 gallon is approximately equal to 4.5 litres. Use this information to change 10 gallons into litres. Plot a graph to convert gallons into litres using a scale of 2cm to represent 2 gallons on the horizontal axis and 2cm to represent 5 litres on the vertical axis.
From your graph; a) convert 11 gallons into litres
b) convert 32 litres into gallons
In each case give your answer correct to 1 decimal place.

3) The table below shows the cost of gas. There is a fixed charge of £10.00.

Cost	£10.00	£25.00	£85.00	£160.00
Units Used	0	1,000	5,000	10,000

Use this information to plot a conversion graph with a scale of 2cm to represent 2000 units on the horizontal axis and 2cm to represent £20 on the vertical axis.
From your graph find;
a) the cost of 5,200 units
b) the number of units that can be bought for £145.00.

4) Water is run from a tap into a container which has a large base and narrower neck. The height of the water in the container is measured every 30 seconds. The following table gives the results;

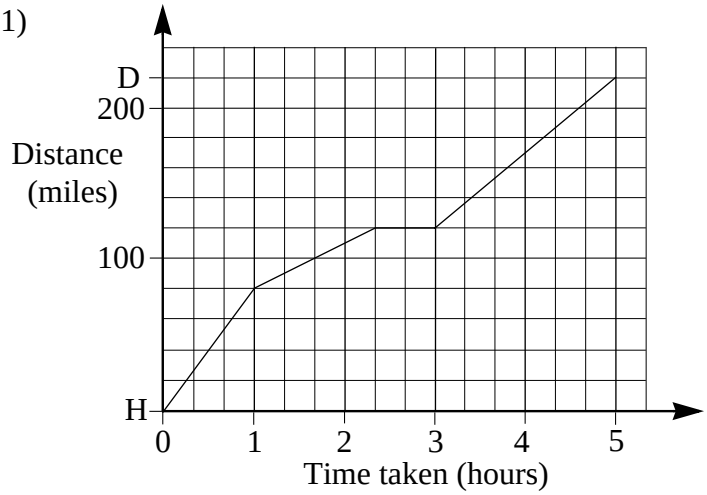
Height of water (cm)	0	2	8	18	32	50
Time (secs)	0	30	60	90	120	150

Using a vertical scale of 2cm to represent 10cm for the height of the water and a horizontal scale of 2cm to represent 20 secs for the time, plot the above information to produce a conversion chart. From your graph find;
a) the time it takes to reach a height of 25cm
b) the height of water after the tap has been running for $1\frac{1}{4}$ minutes.

5) David has to make pastry but his scales measure in ounces and the recipe uses grammes. He has a tin of beans which say on the label that $15\frac{1}{2}$ ounces is equivalent to 440 grammes. Using a scale of 2cm to represent 2oz on the horizontal axis and 2cm to represent 50 grammes on the vertical axis, draw a line to show the relationship between ounces and grammes.
From the graph convert the following to the nearest half ounce, so that David can use his scales;
a) 85g of butter
b) 200 g of flour
When he has mixed all the ingredients together he weighs out $13\frac{1}{2}$ ounces of pastry.
c) What is this weight in grammes?

19. Distance Time Diagrams 1

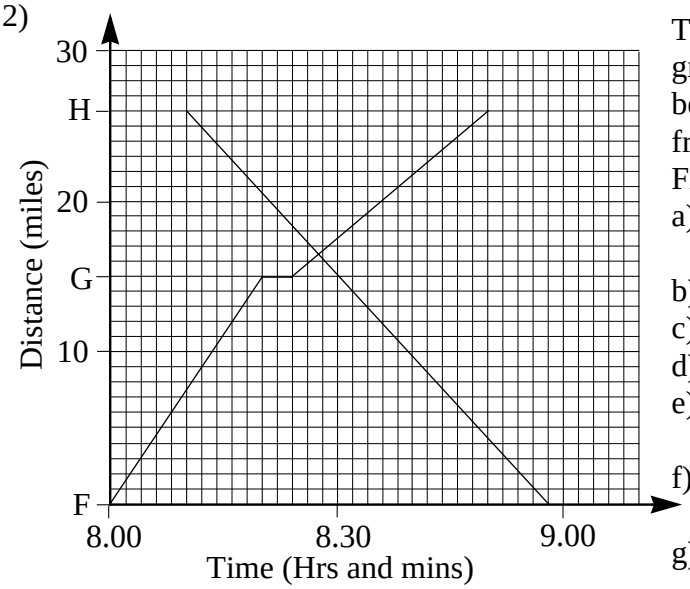
1)



The diagram shows the journey of a lorry from home H to destination D.

a) What is the distance between H and D?
b) For how long did the driver stop?
c) What was his average speed when travelling slowest?
d) What was the average speed for the whole journey?

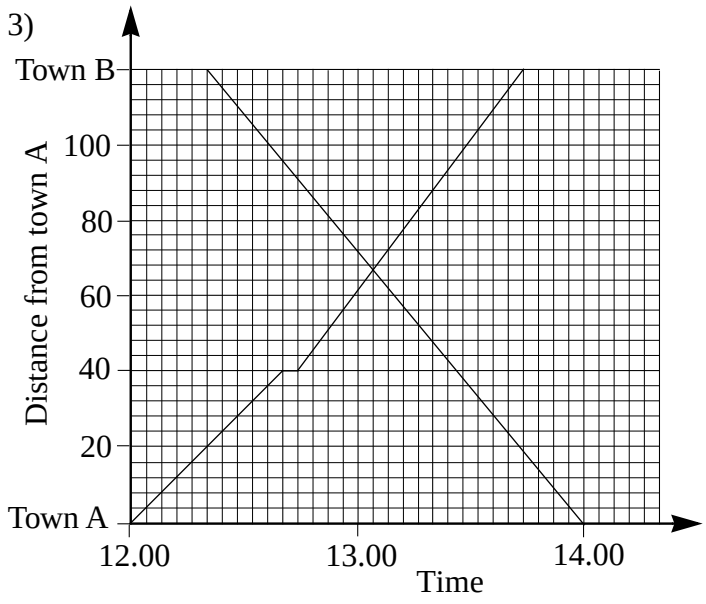
2)



The diagram shows a distance time graph for two buses A and B, travelling between towns F, G and H. Bus A travels from F to H and bus B from H to F.
Find

a) the average speed of bus A between F and G in miles per hour.
b) the length of time bus A stops at G
c) the time at which bus B leaves H
d) the average speed of bus B in m.p.h
e) the approximate time at which the buses pass each other
f) the approximate distance from G at which the buses pass
g) the time at which bus B arrives at F.

3)

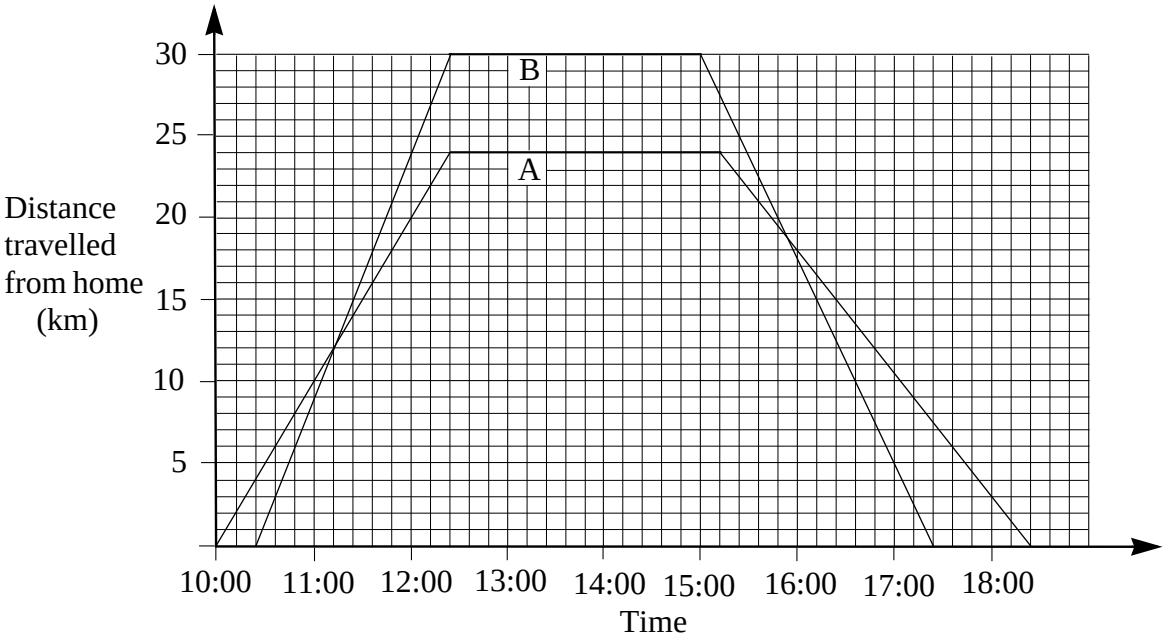


Two towns are 120 miles apart.
The graph shows the journeys of two trains.
The first goes from A to B.
The second goes from B to A.
From the graph find

a) the speed of the first train over the first part of its journey.
b) the time at which the first train stopped and for how long.
c) the speed of the first train during the second part of its journey.
d) the average speed of the second train.
e) the time and distance from town A of the two trains when they passed each other.

20. Distance Time Diagrams 2

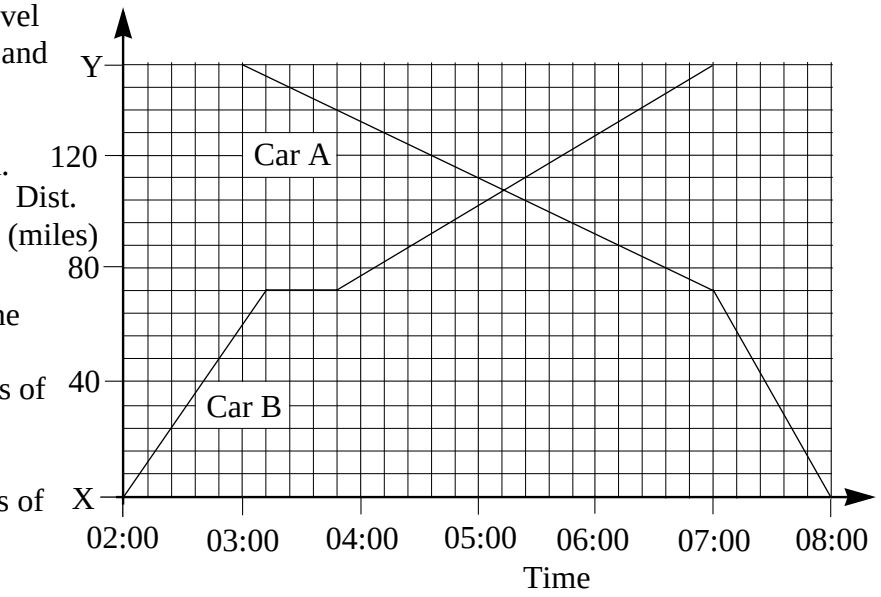
1)



The diagram shows a distance-time graph for two journeys.
One journey is by bicycle, the other is jogging.

a) Which journey do you think is by bicycle and why, A or B?
b) What is the average speed of the cyclist on her outward journey?
c) Who travelled furthest?
d) What is the average speed of the jogger on his homeward journey?
e) For how long did the jogger stop?
f) If both journeys were made along the same road, at what approximate times did they meet?
g) At what time did the cyclist arrive home?

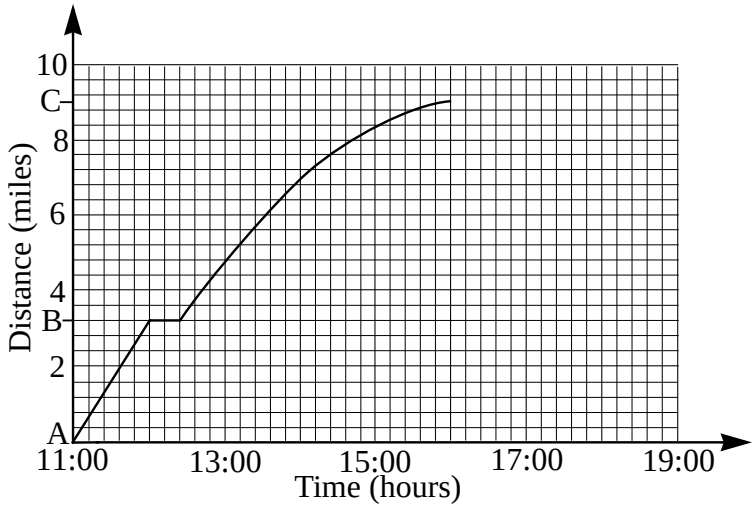
2) Two cars, A and B travel between two towns X and Y. The distance time graph shows the distance from town X. Half the journey is along a motorway and half is not.



a) How far apart are the two towns?
b) Calculate the speeds of car A over the two sections.
c) Calculate the speeds of car B over the two sections.
d) For how long did car B stop?
e) At what time, and how far from town X , are the two cars when they pass each other?
f) Approximately how far apart are the two cars at 06:00?
g) At what times will the cars be 50 miles apart?

21. Distance Time Diagrams 3

1. The graph shows a journey undertaken by a group of walkers. From the graph determine
   a) their average speed between points A and B.
   b) their average speed between points B and C.
   c) their approximate speed at 2pm.
   d) At C they rest for half an hour and then return to A at a constant speed. If they arrive home at 8.00pm, what is their average speed?

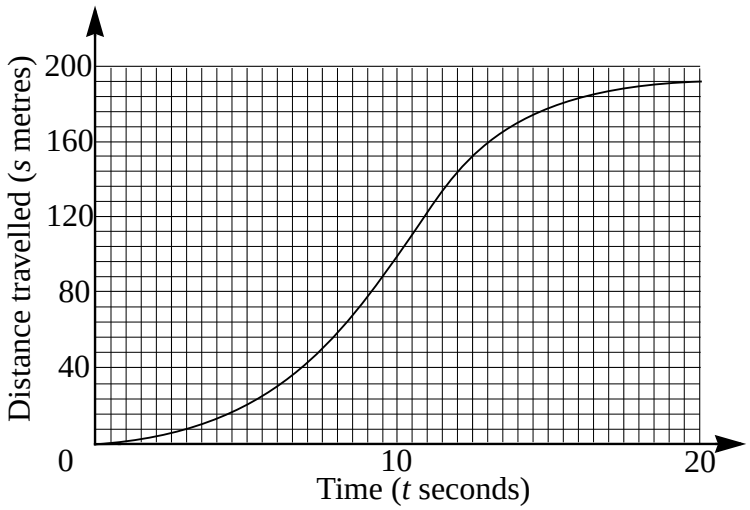


2. An object is projected vertically upwards so that its height above the ground *h* in time *t* is given in the following table.

Time *t* seconds	0	0.5	1	1.5	2	2.5	3	3.5	4
Height *h* metres	0	7	12	15	16	15	12	7	0

Draw a graph to show this information using a scale of 4cm to represent 1 second on the horizontal axis and 2cm to represent 2 metres on the vertical axis. From your graph find
a) the time, to the nearest 0.1 second, it takes for the object to reach 10 metres.
b) the velocity of the object when $t = 1.7$secs.

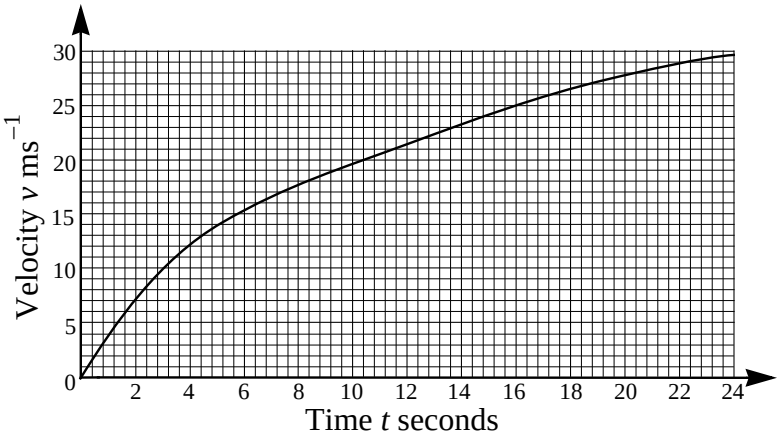
3. The curve shows the distance travelled (*s*) by a car in time (*t*).
   a) Find its approximate speed when $t = 14$ seconds.
   Explain what is happening to the car between
   b) $t = 0$ and $t = 8$ secs
   c) $t = 8$ secs and $t = 12$ secs
   d) $t = 12$ secs and $t = 20$ secs



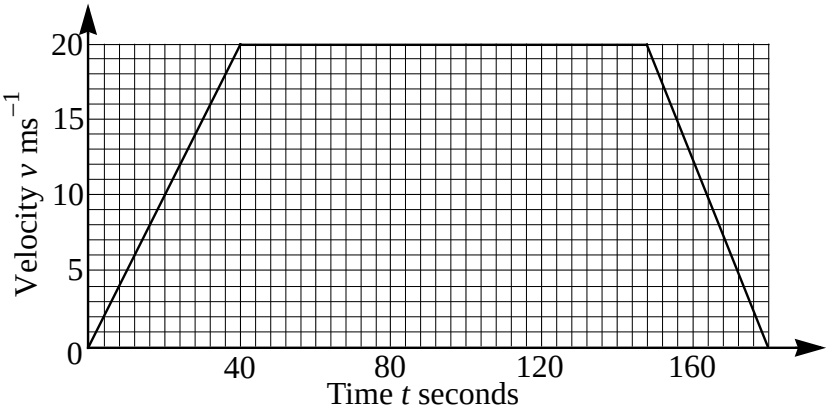
4. An object is projected vertically upwards. Its height *h* above the ground after time *t* is given by the formula $h = 30t - 6t^2$ where *h* is measured in metres and *t* is in seconds. Draw a graph to show this relationship for values of *t* from 0 to 5 seconds. From your graph find
   a) the height when $t = 1.4$ seconds.
   b) the approximate speed of the object when $t = 2$ seconds.
   c) the distance travelled in the fourth second.
   d) the maximum height gained by the object.

22. Velocity-Time Graphs

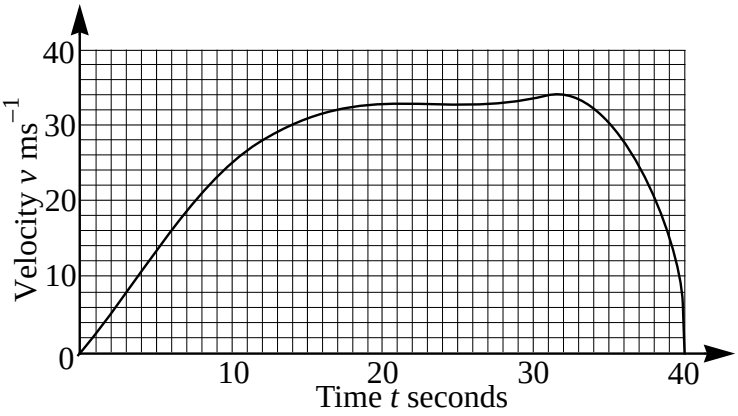
1. The velocity of a vehicle after time t seconds is given by the graph on the right.
   a) Find the area underneath the graph between $0 < t < 20$ seconds.
   b) What is the approximate distance travelled by the vehicle in this time?
   c) What is the acceleration of the vehicle when the velocity is $10\,\text{ms}^{-1}$?



2. The diagram below shows a velocity-time graph for the journey a car makes. Use it to calculate
   a) the total distance travelled in 180 seconds
   b) the average acceleration in the first 40 seconds.



3. The diagram on the right shows the journey a car makes between two sets of traffic lights. Use it to find the approximate distance between the traffic lights



4. The velocity v ms^{-1} of a particle over the first 5 seconds of its motion is represented by the equation $v = t(5 + 2t)$ where t is in seconds.
   a) Copy and complete the table below and from it draw the graph of $v = t(5 + 2t)$. Use a

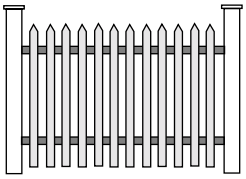
Time t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Velocity v											

   scale of 2cm to represent 1 second on the horizontal axis and 1cm to represent $5\,\text{ms}^{-1}$ on the vertical axis.
   b) Estimate the distance travelled by the particle in the first 3 seconds.
   c) What is the approximate acceleration of the particle when $t = 3$ seconds?

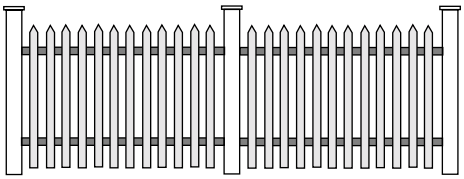
23. Number Patterns and Sequences

1) A fence consists of posts, horizontal bars and uprights, as shown below.

1 post
No horizontals
No uprights



2 posts
2 horizontals
12 uprights



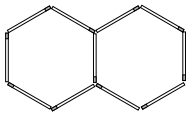
3 posts
4 horizontals
24 uprights

Write down the number of uprights and horizontals needed with:

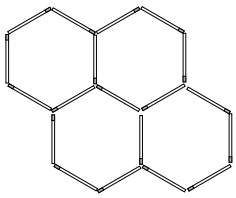
a) 4 posts b) 5 posts c) *n* posts d) 20 posts

e) A fence is made from 72 uprights. How many posts and horizontals are needed?

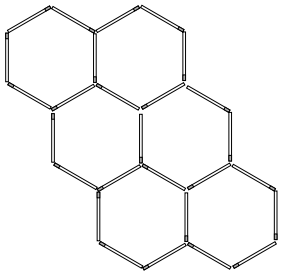
2) The shapes shown below are made from matchsticks.



1 Layer
11 Matchsticks



2 Layers
19 Matchsticks



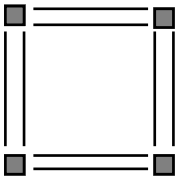
3 Layers
27 Matchsticks

Write down the number of matches needed for shapes with a) 4 layers b) 5 layers c) *n* layers

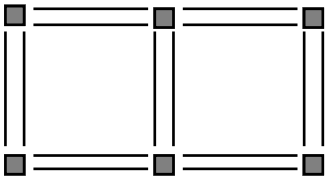
d) Calculate how many matches are needed for a shape having 20 layers.

e) A shape is made with 99 matchsticks. How many layers does it have?

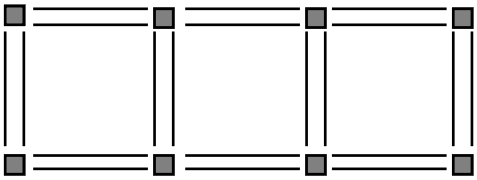
3) Pens, in which animals are kept are made from posts and cross bars. One pen requires 4 posts and 8 cross bars, 2 bars along each side.



1pen
8 cross bars
4 posts



2 pens
14 cross bars
6 posts



3 pens
20 cross bars
8 posts

If more pens are made in this way, write down the number of posts and cross bars needed for a) 4 pens b) 5 pens c) *n* pens.

d) Calculate the number of posts needed if there are 122 cross bars.

24. Number Sequences

Exercise 1

Write down the next two numbers in the following sequences

1) 2, 5, 8, 11, 14, 17 . . .
2) 1, 5, 9, 13, 17, 21 . . .
3) 4, 5, 7, 10, 14, 19 . . .
4) 2, 2, 3, 5, 8, 12 . . .
5) 17, 19, 22, 26, 31, 37 . . .
6) 20, 19, 17, 14, 10, 5 . . .
7) 10, 8, 6, 4, 2, 0 . . .
8) 15, 14, 11, 6, –1, –10 . . .
9) 6, 7, 7, 6, 4, 1 . . .
10) 2, 4, 8, 16, 32, 64 . . .
11) –1, –2, –4, –7, –11, –16 . . .
12) –6, –6, –7, –9, –12, –16 . . .
13) –7, –1, 5, 11, 17, 23 . . .
14) –3, –1, 0, 0, –1, –3 . . .
15) 1, 3, 7, 15, 31, 63 . . .
16) 1, 4, 9, 16, 25, 36 . . .
17) 7, 5, 3, 1, –1, –3 . . .
18) 1, 8, 27, 64, 125, 216 . . .
19) –7, 0, 19, 56, 117, 208 . . .
20) –2, 1, 6, 13, 22, 33 . . .
21) 1, 2, 4, 8, 16, 32 . . .
22) 2, 11, 26, 47, 74, 107 . . .
23) 0, 1, 1, 2, 3, 5 . . .
24) 6, 7, 13, 20, 33, 53 . . .
25) 1, 3, 6, 10, 15, 21 . . .
26) 4, 9, 15, 22, 30, 39 . . .

Exercise 2

Write down the next two numbers and find the rule, in terms of the n^{th} number, for each of the following sequences

1) 4, 7, 10, 13, 16, 19 . . .
2) –3, 2, 7, 12, 17, 22 . . .
3) –2, –6, –10, –14, –18, –22 . . .
4) 8, 3, –2, –7, –12, –17 . . .
5) 39, 31, 23, 15, 7, –1 . . .
6) 6, 11, 16, 21, 26, 31 . . .
7) –19, –12, –5, 2, 9, 16 . . .
8) 1, 4, 9, 16, 25, 36 . . .
9) 3, 6, 11, 18, 27, 38, . . .
10) –6, –3, 2, 9, 18, 29, . . .
11) 0, 1, 4, 9, 16, 25 . . .
12) 0, 2, 6, 12, 20, 30 . . .
13) $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \frac{10}{11}, \frac{12}{13}$
14) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$
15) $\frac{1}{4}, \frac{4}{5}, \frac{9}{6}, \frac{16}{7}, \frac{25}{8}, 4$
16) $3, \frac{6}{7}, \frac{9}{17}, \frac{12}{31}, \frac{15}{49}, \frac{18}{71}$

Exercise 3

1) a) Write down an expression for the n^{th} term of the sequence 2, 3, 4, 5, 6 . . .
   b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.
2) a) Write down an expression for the n^{th} term of the sequence 3, 5, 7, 9, 11 . . .
   b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.
3) a) Write down an expression for the n^{th} term of the sequence 1, 4, 9, 16, 25, 36 . . .
   b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.
4) a) Write down an expression for the n^{th} term of the sequence 5, 9, 13, 17, 21, 27 . . .
   b) Show algebraically that the product of any two terms in the sequence is itself a term in the sequence.

25. Indices 1

Exercise 1

Write down the values of the following.

1) 3^2 2) 3^3 3) 3^4 4) 3^5 5) 10^2 6) 10^3 7) 10^4 8) 10^5

Exercise 2

Use a calculator to write down the values of the following.

1) 6^5 2) 5^6 3) 4^7 4) 7^6 5) 9^5 6) 11^5 7) 13^6 8) 7^9

Exercise 3

Write down the answers to these both in index form and, where necessary, numerical form.

1) $2^3 \times 2^4$ 2) $3^4 \times 3^5$ 3) 4×4^5 4) $10^4 \times 10^3$ 5) $7^4 \times 7^4$

6) $8 \times 8^3 \times 8$ 7) $x^5 \times x^2$ 8) $a^3 \times a^{10}$ 9) $b^2 \times b^3 \times b^4$ 10) $y^{10} \times y^{15}$

Exercise 4

Write down the answers to each of the following in index form.

1) $4^8 \div 4^4$ 2) $5^9 \div 5^4$ 3) $7^7 \div 7^4$ 4) $10^{10} \div 10^7$ 5) $15^7 \div 15^4$ 6) $\frac{10^4}{10^2}$

7) $\frac{9^7}{9^4}$ 8) $\frac{12^6}{12^3}$ 9) $\frac{8^{10}}{8^4}$ 10) $\frac{20^7}{20^4}$ 11) $a^5 \div a^2$ 12) $y^{15} \div y^3$ 13) $\frac{x^7}{x^2}$

Exercise 5

Write down the answers to each of the following in index form.

1) $\left(2^2\right)^4$ 2) $\left(4^2\right)^5$ 3) $\left(7^3\right)^3$ 4) $\left(4^4\right)^3$ 5) $\left(5^2\right)^3$ 6) $\left(2^3\right)^5$

7) $\left(3^2\right)^8$ 8) $\left(7^2\right)^4$ 9) $\left(3^2\right)^5$ 10) $\left(5^2\right)^4$ 11) $\left(x^2\right)^5$ 12) $\left(y^3\right)^3$

Exercise 6

Calculate the answers to each of these in numerical form.

1) $(2 \times 3)^4$ 2) $(4 \times 3)^5$ 3) $(7 \times 2)^3$ 4) $(4 \times 2)^3$ 5) $(5 \times 3)^3$

6) $(2 \times 5)^5$ 7) $(3 \times 4)^6$ 8) $(7 \times 3)^4$ 9) $(3 \times 2)^5$ 10) $(5 \times 4)^4$

Exercise 7

Simplify each of the following

1) a) $x^2 \times x^3$ b) $x^5 \times x^6$ c) $a^4 \times a^8$ d) $y^2 \times y^{11}$

2) a) $a^4 \div a^2$ b) $a^2 \div a^2$ c) $x^5 \div x^3$ d) $2^{10} \div 2^4$

3) a) $\left(a^6\right)^4$ b) $\left(x^3\right)^6$ c) $\left(y^2\right)^4$ d) $\left(b^3\right)^6$

4) a) $(xy)^2 \times x^2$ b) $(ab)^3 \times a^2$ c) $(xy)^4 \times y^2$ d) $(ab)^3 \times b^3$

5) a) $(3x)^2$ b) $(2x)^3$ c) $(3x)^3$ d) $(5a)^2$

26. Indices 2

Exercise 1
Simplify each of the following

1) $3x^2 \times 4$ 2) $5a^3 \times a^4$ 3) $6y^2 \times 4y^3$ 4) $7x^4 \times 5x^7$

5) $(a^2)^3$ 6) $(c^3)^6$ 7) $(x^7)^4$ 8) $(x^3)^4$

9) $x^2y \times xy^2$ 10) $a^3b \times a^4b^5$ 11) $x^4y^5 \times y^3$ 12) $a^3b^3 \times a^4b^4$

13) $(4x)^2$ 14) $(3x^2)^2$ 15) $(4y^3)^2$ 16) $(2a^2)^4$

17) $12a^2 \div 4$ 18) $18x^5 \div x^2$ 19) $16y^5 \div 4y^2$ 20) $20a^5 \div 5a^2$

21) $\frac{24a^4}{6a^2}$ 22) $\frac{18b^7}{3b}$ 23) $\frac{12a^3b^2}{3ab}$ 24) $\frac{21x^3y^5}{7x^2y^4}$

25) $x^5 \times x^2 \div x^3$ 26 $3a^2 \times 2a \times a^5$ 27) $4y^2 \times 7y^3 \div 2y$ 28) $4x^7 \times 3x^4 \div 4x^3$

Exercise 2
Simplify

1) $x^9 \div x^9$ 2) $a^5 \div a^7$ 3) $20y^3 \div 10y^4$ 4) $4b^3 \div 8b^5$

5) $y^2 \times y^{-3}$ 6) $2x^2 \times 3x^{-5}$ 7) $4a^3 \times 3a^{-6}$ 8) $3a^2b \times 2ab^{-2}$

9) $x^3 \times x^0$ 10) $y^{-3} \times y^0$ 11) $3x^2 \times 2x^0$ 12) $6a^0 \times 4a^0$

13) $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ 14) $x^{-\frac{1}{3}} \times x^{\frac{1}{3}}$ 15) $b^0 \times b^{\frac{1}{2}}$ 16) $y^{\frac{1}{2}} \times y^{-\frac{3}{2}}$

17) $x^{\frac{1}{2}} \div x^{\frac{1}{2}}$ 18) $y^{\frac{1}{2}} \div y^{\frac{1}{4}}$ 19) $a^{\frac{1}{4}} \div a^{-\frac{1}{2}}$ 20) $b^{\frac{1}{2}} \div b^0$

21) $(x^{\frac{1}{2}})^3$ 22) $(a^{\frac{1}{4}})^2$ 23) $(2b^{\frac{1}{3}})^3$ 24) $(5y^3)^{\frac{1}{2}}$

Exercise 3
Simplify

1) $25^{\frac{1}{2}}$ 2) $25^{-\frac{1}{2}}$ 3) $8^{\frac{1}{3}}$ 4) $27^{-\frac{1}{3}}$

5) $8^{\frac{2}{3}}$ 6) $4^{\frac{3}{2}}$ 7) $(4^{\frac{1}{2}})^3$ 8) $9^{1\frac{1}{2}}$

9) $(81)^{\frac{3}{4}}$ 10) $(64)^{\frac{2}{3}}$ 11) $(32)^{\frac{3}{5}}$ 12) $25^{\frac{3}{2}}$

13) $(25)^{-\frac{3}{2}}$ 14) $(64)^{-\frac{5}{6}}$ 15) $(6\frac{1}{4})^{\frac{1}{2}}$ 16) $(\frac{125}{8})^{\frac{2}{3}}$

Exercise 4
Solve the following equations

1) $36^{\frac{1}{x}} = 6$ 2) $81^{\frac{1}{x}} = 9$ 3) $81^{\frac{1}{x}} = 3$ 4) $64^{\frac{1}{x}} = 4$

5) $32^x = 2$ 6) $25^x = \frac{1}{5}$ 7) $81^{\frac{1}{2}} = 3^x$ 8) $512^{\frac{1}{3}} = 2^x$

9) $32^{-\frac{1}{x}} = \frac{1}{2}$ 10) $x^{-\frac{1}{4}} = \frac{1}{3}$ 11) $x^{-\frac{1}{4}} = \frac{1}{2}$ 12) $16^x = \frac{1}{4}$

27. Substitution

Exercise 1

Evaluate each of the following, given that $a = 6$, $b = 4$ and $c = -5$.

1) $3a + 5b$ 2) $4a - 6b$ 3) $2a - 7b$ 4) $4c + 2a$

5) $5a - 4c$ 6) $3a^2 + 2b$ 7) $4b^2 - 2a$ 8) $5a + 3c^2$

9) $(4b - 6c)^2$ 10) $(2a)^2 + 3b$ 11) $4c - 5b^2$ 12) $6a - (5c)^2$

Exercise 2

1) If $y = 3x + 4$ a) Calculate the value of y when $x = 6$ b) What value of x is needed if $y = 19$?

2) If $a = 3b - 6$ a) Calculate the value of a when $b = -7$ b) What value of b is needed if $a = 54$?

3) If $y = 7x + 3$ a) Calculate the value of y when $x = -6$ b) What value of x is needed if $y = -81$?

4) Carol works for a garden centre and plants rose bushes in her nursery. She works out the length of each row of bushes using the formula $L = 50R + 200$, where L represents the length of the row in centimetres and R is the number of bushes she plants. Use the formula to calculate
   a) the length of a row containing 10 bushes
   b) the number of bushes in a row 15 metres long.

5) The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.
   a) Calculate the volume of a cone when $\pi = 3.142$, $r = 3$cm and $h = 2.5$cm.
   b) A cone has a volume of $183cm^3$. Calculate the value of its height h, if $r = 5$cm and $\pi = 3.142$. Give your answer correct to the nearest millimetre.

6) Simple interest can be calculated from the formula $I = \frac{PTR}{100}$.
   a) If the principal (P) = £250, the time (T) = 3 years and the rate (R) = 9.5%, calculate the interest.
   b) If the interest required is £200, what principal needs to be invested for 6 years at 7%?

7) The temperature F (degrees fahrenheit) is connected to the temperature C (degrees celsius) by the formula $C = \frac{5}{9}(F - 32)$.
   a) Calculate C if $F = -20°F$.
   b) Convert $-10°C$ into $°F$

8) A bus company uses the formula $T = \frac{D}{20} + \frac{S}{60} + \frac{1}{4}$ to calculate the time needed for their bus journeys. D is the distance in miles and S the number of stops on the journey. If T is measured in hours, calculate
   a) the time needed for a bus journey of 10 miles with 10 stops.
   b) the time needed for a bus journey of 20 miles with 4 stops.
   c) During the rush hour, more people get on the bus and the extra traffic slows the bus down. What would you do to the formula to take this into account?

28. Simplifying 1

Exercise 1 Simplify

1) $7+4$
2) $10-5$
3) $12-3$
4) $8-9$
5) $6-9$
6) $7-10$
7) $-4+8$
8) $-6+9$
9) $-4+10$
10) $-5-3$
11) $-7-4$
12) $-9-6$
13) $4-3+2$
14) $6-7+1$
15) $5-9+5$
16) $6-10-2$
17) $-4+6-3$
18) $-7+2+4$
19) $8-15+3$
20) $-5-4+9$
21) $-5+3-4+8$
22) $-6+4-9-4$
23) $-8-6-4+3$
24) $8-10-6+4$
25) $5-6-4+8$
26) $-9-6+3-4$
27) $-9-4+2-8$
28) $-7+2+3-9$
29) $8+6-5-4$
30) $-6-4+3-8$

Exercise 2 Simplify

1) $3y+8y$
2) $5y+3y$
3) $9y-6y$
4) $12x-4x$
5) $16y-18y$
6) $27x-19x$
7) $-12y+3y$
8) $-23x+17x$
9) $-16a-7a$
10) $-14w-5w$
11) $12b+3b+2a+3a$
12) $9x+7y+3x+6y$
13) $4b+5a+3b+3a$
14) $x+6y+y+x$
15) $6a-2a+3b+4b$
16) $12p-4p+3q+7q$
17) $12a+3b-4a-b$
18) $5x+7y-y-x$
19) $16x+8y-10x-9y$
20) $21a+3b-17a-2b$
21) $6x+3y-8x-6y$
22) $12a+9b-6a-12b$
23) $5xy+3y-6xy$
24) $4xy+4y+2xy$
25) $-7ab+6b-3ab-4b-3ab$
26) $-5xy+7x-2xy-3xy-2x$
27) $5ab+3bc-4ab+5bc-6ab-3bc$
28) $7xy+9yz-3xy-3yz+7xy-2yz$
29) $9xy-4x+2xy-5x+3xy$
30) $12ab-4a-3ab+5a+9ab$
31) x^2+3x^2
32) $7y^2+6y^2$
33) $x^2+2y^2+4x^2+5y^2$
34) $7x^2+4y^2-3x^2-4y^2$
35) $3xy+2x^2+3xy-x^2$
36) $9x^2-3x+5x-3x^2$
37) $-6x^2y+2xy^2+3xy^2+2x^2y$
38) $7x^2y-12xy^2-5x^2y+3xy^2$
39) $\frac{1}{4}x+\frac{1}{2}x$
40) $\frac{3}{4}y-\frac{1}{4}y$

29. Simplifying 2

Exercise 1

Simplify each of the following by expanding the brackets where necessary

1) $7x - 3x$
2) $5y - 7y$
3) $-8y + 3y$
4) $-6x - 7x$
5) $4x + 3y + 5x + 6y$
6) $9y + 7x - 11y - 4x$
7) $-6x + 3y - 4x + 2y$
8) $-7x - 6y + 3x - 4y$
9) $2ab + 3b - 4a - 6ab$
10) $12b - 4a + 3ab - 7a$
11) $4x^2 + 3x - 2x^2$
12) $6y^2 - 4y - 5y^2$
13) $4y^2 - 4xy + 4xy$
14) $-6x^2 + 3y^2 - 4x^2$
15) $3(x + y) + y$
16) $5(2x - y) + 2y$
17) $3x + 4(x + y)$
18) $7x - 3(x + 2y)$
19) $4y - 2(x - y)$
20) $6x - 4(2x - 2y)$
21) $3(x + y) + 2(x - y)$
22) $4(2x + 3y) - (x + 2y)$
23) $5(2x - 3y) - (2x - y)$
24) $3(2x - y) - 2(3x - y)$
25) $3(2x - 3) - 3(x - 4)$
26) $5(2x - 3y) - 3(5x - 2y)$
27) $4x(2x - 3) - 3x(2x + 4)$
28) $7x(y - 2) - 6y(2x - 3)$
29) $4y(2y - 3x) - 2x(x - 3y)$
30) $4x(4y + 3x) - 3y(4x - 3y)$
31) $\frac{1}{4}x + \frac{1}{3}x$
32) $\frac{1}{2}x - \frac{1}{3}x$
33) $\frac{2}{3}y - \frac{1}{2}y$
34) $\frac{1}{3}y^2 + \frac{1}{6}y^2$

Exercise 2

Expand and simplify

1) $(x + 1)(x - 3)$
2) $(x + 2)(x - 4)$
3) $(2x + 3)(x - 7)$
4) $(2x + 5)(3x - 2)$
5) $(2x - 7)(3x + 2)$
6) $(3x + 4)(2x - 5)$
7) $(5x + 2)(7x - 3)$
8) $(4x + 8)(3x - 2)$
9) $(4x - 4)(2x + 1)$
10) $(3x + 2y)(2x + 3y)$
11) $(4x + y)(x + 3y)$
12) $(x - 4)(2x - 6)$
13) $(x + 1)^2$
14) $(3x + 2)^2$
15) $(5x - 2)^2$
16) $(3x + 4)^2$
17) $(5x - 6)^2$
18) $(7x + 2)^2$
19) $(-3x + 2)^2$
20) $(-5x - 7)^2$
21) $(-4x - 6)^2$
22) $(x + y)^2$
23) $(2x + 3y)^2$
24) $(4x - 2y)^2$

30. Multiplying Brackets

Exercise 1
Calculate

1) 8×3
2) 5×7
3) $4\times(-6)$
4) $6\times(-4)$
5) -3×2
6) -8×5
7) $-5\times(-4)$
8) $-6\times(-5)$
9) $-7\times(-3)$
10) -6×5
11) $4\times(-3)$
12) $-8\times(-7)$

Exercise 2
Expand and simplify

1) $3(x+y)$
2) $6(3x+4)$
3) $-(2x-3)$
4) $-(3x+2)$
5) $-4(2x+5)$
6) $-7(3x-4)$
7) $4(-3x-3)$
8) $-5(-2x+3)$
9) $-3(-3x-2)$
10) $7x+8y+3(2x+4y)$
11) $12x-3y-2(4x+y)$
12) $14x+8y-6(6x-2y)$
13) $7x-3y-(5x+2y)$
14) $12x+3y-(4x-2y)$
15) $12x-4y+(4y-2x)$
16) $2(3x+2y)+3(3x+3y)$
17) $4(2x+4y)+5(6x-7y)$
18) $5(3x-2y)-4(3x+4y)$
19) $7(3x-5y)-4(4x-5y)$
20) $5x(2x+3)-2x(2x-1)$
21) $3x(3x-2)-4x(3x+4)$
22) $5x(3x+2)+3x(4x-5)$
23) $6x(2x+1)-x(5x+3)$
24) $4x(3x-2)-x(-3x-2)$
25) $5x(2x+3)-3x(-4-2x)$
26) $3x(4x-6)-3x(2x+5)$

Exercise 3
Expand and simplify

1) $(x+2)(x+3)$
2) $(2x+1)(x+2)$
3) $(3x+2)(x+4)$
4) $(5x+2)(6x+7)$
5) $(3x+4)(2x-3)$
6) $(4x+5)(3x-5)$
7) $(6x+3)(4x-6)$
8) $(3x+2)(5x-3)$
9) $(4x-3)(2x+1)$
10) $(3x-4)(x+2)$
11) $(6x-5)(4x+3)$
12) $(3x-7)(2x-8)$
13) $(6x-4)(7x-5)$
14) $(3x-6)(4x-5)$
15) $(8x-6)(9x-2)$
16) $(3x+7)(5x-2)$
17) $(5x+3)^2$
18) $(6x-2)^2$
19) $(4x-5)^2$
20) $(-4x-9)^2$

31. Factorising

Exercise 1

Factorise each of the following

1) $4x + 8$
2) $6y - 9$
3) $7b - 14a$
4) $xy - x$
5) $xy + 3x$
6) $4y + 10xy$
7) $6x^2 + 2$
8) $5x^2 - x$
9) $9x^2 - 3x$
10) $a^2b + ab^2$
11) $4ab - a^2b$
12) $8ab + 6ab^2$
13) $a + ab - a^2$
14) $3ab - ac + a^2$
15) $5x^2y - 4y^2 - 3xy$
16) $\frac{x^2}{2} - \frac{x^3}{4}$
17) $\frac{y^2}{3} - \frac{xy}{6}$
18) $\frac{5x^2}{6} - \frac{2x}{3}$

Exercise 2

Factorise

1) $m^2 - n^2$
2) $a^2 - 4$
3) $(xy)^2 - z^2$
4) $(ab)^2 - 9$
5) $x^2y^2 - 4$
6) $v^2w^2 - 25$
7) $a^2b^2 - 9c^2$
8) $25a^2 - 9b^2$
9) $b^2 - 1$
10) $2a^2 - 50$
11) $8a^2 - 50$
12) $12x^2 - 27y^2$
13) $xy^2 - 4x^3$
14) $2xy^2 - 8x^3$
15) $4x^2y^2 - 9x^2$
16) $x^4 - y^4$
17) $16x^4 - 81y^4$
18) $3a^4 - 12b^2$

Exercise 3

Factorise

1) $x^2 + 4x + 3$
2) $x^2 + 4x + 4$
3) $x^2 + 8x + 7$
4) $x^2 + 7x + 10$
5) $x^2 + 7x + 12$
6) $x^2 + 11x + 30$
7) $x^2 + 2x - 3$
8) $x^2 - 2x + 3$
9) $x^2 + 4x - 5$
10) $x^2 - 2x - 8$
11) $x^2 - 2x - 15$
12) $x^2 - x - 12$
13) $x^2 - 10x + 24$
14) $x^2 - 8x + 15$
15) $x^2 - 11x + 28$

Exercise 4

Factorise

1) $2x^2 + 3x + 1$
2) $2x^2 + 9x + 4$
3) $2x^2 + 7x + 3$
4) $2x^2 + 8x + 6$
5) $2x^2 + x - 6$
6) $3x^2 - 7x - 6$
7) $2x^2 - 9x + 4$
8) $3x^2 - 10x + 3$
9) $3x^2 - 14x + 8$
10) $3x^2 + x - 14$
11) $3x^2 + 19x + 20$
12) $3x^2 - 12x + 12$
13) $4x^2 + 10x + 6$
14) $4x^2 - 10x + 6$
15) $4x^2 + 13x + 3$
16) $4x^2 + 21x + 5$
17) $5x^2 + 13x - 6$
18) $6x^2 - 5x - 6$
19) $6x^2 + 5x + 1$
20) $9x^2 + 12x + 4$
21) $8x^2 + 11x + 3$
22) $4x^2 - 23x + 15$
23) $5x^2 - 13x - 6$
24) $12x^2 - 13x + 3$

32. Re-arranging Formulae

Exercise 1

In each of the following questions, re-arrange the equation to make the letter in the bracket the subject.

1) $v = u + at$ (u)
2) $v = u + at$ (a)
3) $d = 3b - c$ (b)
4) $c = pd + w$ (w)
5) $x = 7y - z$ (z)
6) $a = 3b + c$ (b)
7) $w = \frac{4v + u}{3}$ (v)
8) $x = \frac{5y + b}{4}$ (b)
9) $2x = x + b$ (b)
10) $6y = 3a - 2y$ (a)
11) $p = \frac{1}{2}a + 3b$ (a)
12) $w = 2v + \frac{1}{4}u$ (v)
13) $c = \frac{a + b}{d}$ (d)
14) $p = \frac{2r - q}{3s}$ (s)
15) $x = 2(y + z)$ (z)
16) $a = 3(3b + 4c)$ (c)
17) $x = \frac{1}{2}(y + z)$ (y)
18) $3a = \frac{1}{3}(2b + c)$ (c)
19) $3x = \frac{1}{4}(y - z)$ (z)
20) $5w = \frac{1}{3}(3v - 2u)$ (u)
21) $7x - 4y = \frac{1}{2}(3x + 6y)$ (x)
22) $5a + 3b = \frac{2}{3}(3b - 2a)$ (a)

Exercise 2

In each of the following questions, re-arrange the equation to make the letter in the bracket the subject.

1) $a = \frac{b^2}{c}$ (b)
2) $x = \frac{y}{z^2}$ (z)
3) $c = \frac{4a^2}{b}$ (a)
4) $3v = \frac{9}{u^2}$ (u)
5) $\frac{1}{2}x = \frac{2}{3}x + y$ (y)
6) $\frac{3}{4}y - 2x = y$ (y)
7) $\frac{7}{2}x = \frac{1}{2}(x + y)$ (x)
8) $\frac{4}{9}b = \frac{1}{4}(b - 3c)$ (b)
9) $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ (x)
10) $\frac{2}{3x} = \frac{2}{y} + \frac{3}{z}$ (x)
11) $\frac{1}{\sqrt{x}} = \frac{1}{2a} + \frac{1}{3b}$ (x)
12) $\frac{2}{\sqrt{x}} = \frac{3}{2y} + \frac{b}{2}$ (x)
13) $\frac{2}{3x} = \frac{y}{2} - \frac{1}{z}$ (z)
14) $\frac{3}{x} = \frac{6}{y} - \frac{1}{z}$ (z)
15) $x = \frac{1}{a^2} + \frac{1}{b}$ (a)
16) $4y = \frac{2}{3a^2} + 3b$ (a)
17) $\frac{3}{b} = \frac{1}{b} + \frac{1}{c}$ (b)
18) $\frac{3}{x} - \frac{6}{y} = \frac{1}{x}$ (x)
19) $3x = \frac{2y + z}{y}$ (y)
20) $4a = \frac{b - 3c}{c}$ (c)
21) $\frac{1}{x} = \frac{x + 3y}{x}$ (x)
22) $\frac{2}{y} = \frac{x - 3y}{2y}$ (y)

33. Equations

Find the value of the letter in each of the following equations

Exercise 1

1) $x+4=6$
2) $x+7=17$
3) $7+y=19$
4) $x-2=4$
5) $y-7=11$
6) $a-9=18$
7) $6-y=4$
8) $12-x=2$
9) $19-x=5$
10) $12a=36$
11) $6x=42$
12) $8y=36$
13) $7b=-35$
14) $4y=-24$
15) $4b=-10$
16) $4a+2=10$
17) $9a+6=33$
18) $12x+6=30$
19) $7x-3=18$
20) $12x-7=17$
21) $6x-7=35$
22) $4y+4=14$
23) $3b+2=-4$
24) $6y-5=-35$

Exercise 2

1) $x+3=2x$
2) $6x-5=5x$
3) $7x-6=6x$
4) $3x+5=4x$
5) $2x+3=3x$
6) $4x+2=5x$
7) $4x-12=2x$
8) $5x-6=2x$
9) $4x-7=2x$
10) $3x+6=5x$
11) $8x+5=10x$
12) $7x+7=9x$
13) $4x+2=2x$
14) $4x+4=-12$
15) $3x-2=x+6$
16) $x+7=2x-2$
17) $6x-12=3x+12$
18) $5x-2=2x+4$
19) $4x+9=2x+15$
20) $3x+7=2x-1$
21) $4x+3=2x-3$

Exercise 3

1) $2(x+1)=8$
2) $3(x-1)=9$
3) $5(x+2)=15$
4) $4(x+2)=36$
5) $7(x-2)=21$
6) $2(2x+1)=26$
7) $3(2x-1)=27$
8) $2(5x+4)=28$
9) $3(3x-7)=15$
10) $2(x+1)=3x$
11) $4(x-2)=3x$
12) $5(x+6)=15x$
13) $2(2x+3)=10x$
14) $3(2x-5)=3x$
15) $6(2x+7)=33x$
16) $3(2x+1)=8x-5$
17) $6(x-6)=4x+4$
18) $4(3x+2)=11x+18$
19) $4(x+3)=5(3x-2)$
20) $2(x+3)=4(2x-9)$
21) $3(2x-1)=5(3x-15)$
22) $2(x+1)+x=11$
23) $3(2x-2)+x=29$
24) $5(3x+2)-4x=87$

34. Solving Equations 1

Exercise 1
Solve the following equations

1) $7x - 3 = 60$
2) $12x - 14 = 130$
3) $24 - 3x = 6$
4) $7 - 2x = -1$
5) $\frac{x}{4} = 30$
6) $\frac{x}{6} = 25$
7) $\frac{2}{3}x = 8$
8) $\frac{1}{2}x = 4$
9) $\frac{3}{x+2} = 1$
10) $\frac{2}{x-1} = 5$
11) $2x + 3 = 3x + 2$
12) $4x + 3 = 3x + 5$
13) $4x + 5 = 3x + 3$
14) $7x + 3 = 10x - 6$
15) $5(x + 3) = 7x + 5$
16) $3(2x - 1) = 5(3x - 15)$
17) $3(x + 2) - 2(3x - 5) = 10$
18) $4(2x - 3) - 3(3x - 10) = 11$
19) $\frac{x}{3} + \frac{x}{4} = 14$
20) $\frac{x-2}{3} + \frac{x-2}{2} = 25$
21) $\frac{2x-1}{3} - \frac{x+1}{4} = 4$
22) $\frac{3}{x} = \frac{4}{x+2}$
23) $\frac{2}{x-1} = \frac{3}{2x+4}$
24) $\frac{2x-1}{x} = \frac{3}{4}$
25) $\frac{1}{x} = \frac{3}{2x} + \frac{1}{x+1}$
26) $\frac{2}{3x} = \frac{1}{x} - \frac{1}{x+1}$

Exercise 2
Solve the following equations

1) $x^2 - 25 = 0$
2) $x^2 - 81 = 0$
3) $2x^2 - 72 = 0$
4) $3x^2 - 27 = 0$
5) $(x + 2)(x - 3) = 0$
6) $(x - 6)(x + 5) = 0$
7) $(3x + 4)(2x - 1) = 0$
8) $x(3x - 2) = 0$
9) $x(4x + 3) = 0$
10) $2x(x - 4) = 0$
11) $2x^2 + 3x = 0$
12) $6x^2 - 4x = 0$
13) $4x^2 - x = 0$
14) $x^2 - x - 6 = 0$
15) $2x^2 - x - 3 = 0$
16) $6x^2 + 4x - 2 = 0$
17) $x^2 - 3x - 10 = 0$
18) $3x^2 + 6x = 24$
19) $8x^2 = 2x + 15$
20) $4x^2 - 25 = 0$
21) $\frac{4}{x} + \frac{1}{x+1} = \frac{6}{x}$
22) $\frac{1}{x} = \frac{3}{2x} + \frac{1}{x+1}$

35. Solving Equations 2

Exercise 1

Solve the following equations correct to 2 decimal places each time.

1) $x^2 + 4x - 6 = 0$

2) $x^2 - 3x + 1 = 0$

3) $x^2 - 4x - 3 = 0$

4) $x^2 + 6x - 4 = 0$

5) $x^2 + x - 1 = 0$

6) $4x^2 - 7x - 4 = 0$

7) $6x^2 + 3x - 1 = 0$

8) $10x^2 + 2x - 7 = 0$

9) $4x^2 + 5x - 10 = 0$

10) $3x^2 - 4x - 2 = 0$

11) $3x^2 + 4x = 5$

12) $2x^2 + 5x = 1$

13) $4x^2 + 3 = 7x$

14) $3x(2x + 1) = 1$

15) $\frac{1}{x} + \frac{3}{x+2} = 7$

16) $\frac{1}{x-3} - \frac{1}{2x} = 4$

17) $\frac{5}{x} + \frac{1}{x+1} = 7$

18) $\frac{7}{2x} + \frac{2}{x-1} = 4$

19) $x^2 + 7x + 2 = 2x + 4$

20) $x^2 + 13x = 4x + 5$

21) $5x^2 - 3x = x^2 + 4$

22) $4x^2 - 3x = 2x^2 + 7$

23) $x(x + 4) = 3x^2 + 2x - 6$

24) $4(x + 2) = 3x(x + 1)$

25) $\frac{4}{x} = \frac{2x-3}{x-1}$

26) $\frac{2x+3}{x-1} = \frac{x}{4}$

Exercise 2

In each of the following calculate the value of a and the corresponding value of k.

1) $4x^2 - 20x + k = (2x - a)^2$

2) $(3x + a)^2 = 9x^2 + 12x + k$

3) $(5x - a)^2 = 25x^2 - 30x + k$

4) $x^2 - 14x + k = (x - a)^2$

5) $(3x + a)^2 = 9x(x + 1) + k$

6) $16x(x + 1) + k = (4x + a)^2$

7) $9x^2 - 30x + k = (3x - a)^2$

8) $(4x - a)^2 = 16x(x - 4) + k$

9) $(2x + a)^2 = 4x^2 + 24x + k$

10) $36x^2 + 48x + k = (6x + a)^2$

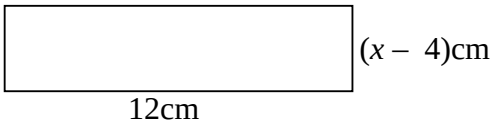
36. Using Simple Equations

1) A bus costs £200 to hire for a day. A social club charges £10 for each non member (n) and £6 for each member (m) to go on an outing.
   a) Write down an equation linking m and n and the cost of hiring the bus if the club is not to lose money.
   b) If twenty members go on the outing, how many non-members need to go?

2) Olivia has two bank accounts, both containing the same amount of money. She transfers £300 from the first account to the second. She now has twice as much money in the second account.
   a) If she originally had £x in each account, how much does she have in each after the transfer?
   b) Write down an equation linking the money in her two accounts after the money has been moved.
   c) How much money has she altogether?

3) Ella buys 400 tiles for her bathroom. Patterned tiles cost 34p each and plain white tiles cost 18p each. She spends exactly £100 on x patterned tiles and white tiles.
   a) Write down, in terms of x, the number of white tiles she buys.
   b) Write down an equation for the total cost of the tiles. Calculate the value of x.
   c) How many white tiles did she buy?.

4) The length of a rectangle is 12cm and its width is $(x - 4)$cm. If its perimeter is numerically the same as its area, calculate the value of x and hence its area

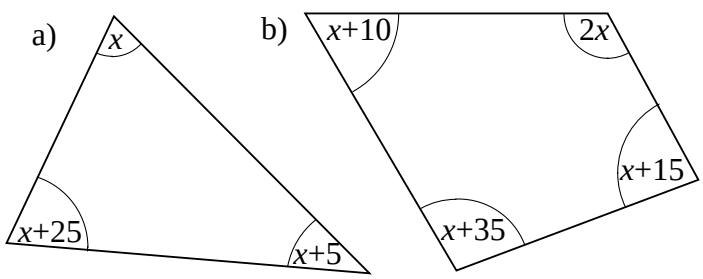


5) Three consecutive numbers are added together and their sum is 69.
   a) If the first number is x, write down expressions for the 2nd and 3rd numbers.
   b) Use these expressions to calculate the value of x and hence the three numbers.

6) The distance between two towns, A and B is 300 miles. A car travels between the two towns on motorways and ordinary roads. Its average speed on the motorways is 60mph and 40mph on the ordinary roads.
   a) If x is the distance travelled on the motorways, write down, in terms of x the distance travelled on ordinary roads.
   b) Write down, in terms of x, the time taken to travel the two parts of the journey.
   c) If the total time taken was 6 hours, write down an equation in terms of x and solve it. What distance was travelled on ordinary roads?

7) Sarah drives her car from her home to the railway station, a distance of x kilometres. She then gets the train and travels to London, 8 times the distance she travelled in her car. If her total journey is 36 kilometres, calculate the length of the car journey.

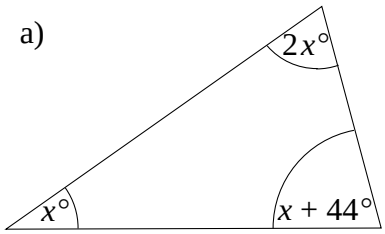
8) Calculate the sizes of the angles in each of these diagrams



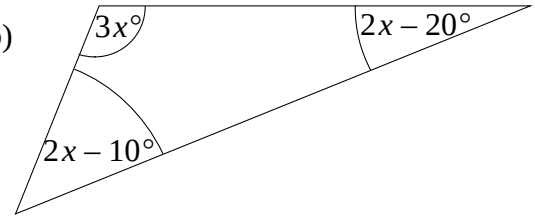
37. Problems Involving Equations

1. In each of the triangles below, calculate the value of x and hence the sizes of the angles of the triangles.

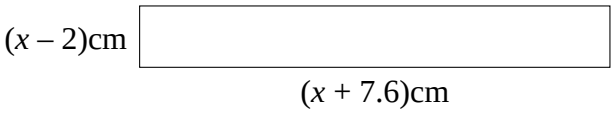
a)



b)



2. It takes an aeroplane $6\frac{1}{2}$ hours to travel from London to the USA, a distance of 3,500 miles.
   a) What was the average speed?
   b) If the same aeroplane travels from London to Spain, a distance of x miles, write down in terms of x the time taken.
   c) If the same aeroplane travels from London to Italy in y minutes, write down an expression in terms of y for the distance travelled.
3. The dimensions of a square and a rectangle are given in the two diagrams. If their areas are equal,
   a) calculate the value of x
   b) calculate their areas.



4. Three consecutive numbers add up to 156.
   a) If the middle number is x, what are the values of the other two numbers, in terms of x.
   b) Write down an equation in terms of x and solve it to find x.
5. A wine merchant has x bottles of wine in her shop and y bottles in her cellar. She transfers a quarter of the bottles from the cellar to the shop.
   a) How many bottles does she now have (i) in the shop (ii) in the cellar?
   She now finds that she has twice as many bottles in the cellar as she has in the shop.
   b) Write down an equation linking x and y and simplify it.
   c) If she originally had 2,000 bottles in the shop, how many has she altogether?
6. For the annual village fete, the vicar orders 250 bottles of drinks. He orders x bottles of lemonade and the remainder of cola.
   Bottles of lemonade cost 35p each and bottles of cola cost 38p each. He spends £91.70 altogether.
   a) Write down in terms of x the number of bottles of cola he bought.
   b) Write down an equation for the total cost of the bottles and from it calculate the value of x.
   c) How many bottles of cola did he buy?
7. Three numbers are added together. The second number is 6 more than the first number and the third number is 15 less than the first number.
   a) If the second number is x, write down in terms of x, the value of each of the other two numbers.
   b) The three numbers are added together. Write down a simplified expression for the total of the three numbers.
   c) If the sum of the numbers is 93, what are the three numbers?

38. Simultaneous Equations

1) $2x + 2y = 10$
$x + 2y = 6$

2) $3x + y = 18$
$2x + y = 13$

3) $4x + 2y = 2$
$2x + 2y = 0$

4) $5x + 3y = 18$
$5x + y = 16$

5) $x + y = 1$
$x - y = 5$

6) $3x + 4y = 29$
$x - 4y = -17$

7) $3x - 2y = 10$
$-3x + y = -11$

8) $3x + 4y = 18$
$3x - 4y = -6$

9) $4x + 3y = 11$
$2x + y = 7$

10) $5x + 2y = 33$
$2x + y = 14$

11) $6x + 2y = 10$
$4x + y = 7$

12) $3x - 2y = 13$
$x - y = 5$

13) $2x + 3y = 28$
$3x - y = 9$

14) $2x + 3y = 15$
$5x - y = 46$

15) $4x + 3y = 13$
$6x - 2y = 13$

16) $5x + 3y = 14$
$2x + 2y = 4$

17) A family of 2 adults and 2 children go to the cinema. Their tickets cost a total of £14.00. Another family of 1 adult and 4 children go to the same cinema and their bill is £13.60.
a) Letting x represent the cost of an adults ticket and y the cost of a child's ticket, write down two equations connecting x and y
b) Solve for x and y.

18) The sum of two numbers is 39 and their difference is 9.
a) Letting x and y be the two numbers, write down two equations connecting x and y.
b) Solve the equations.

19) A rectangle has a perimeter of 42cm. Another rectangle has a length double that of the first and a width one third of that of the first. The perimeter of the second is 57cm.
Letting x and y represent the dimensions of the first rectangle, write down two equations containing x and y. Solve the equations and write down the dimensions of the second rectangle.

20) 4 oranges and 3 apples weigh 720 grams. 3 oranges and 4 apples weigh 750 grams. Let x and y represent their weights. Write down two equations containing x and y. Calculate the weights of each piece of fruit.

21) Three mugs and two plates cost £7.20, but four mugs and one plate cost £7.90. Let x represent the cost of a mug and y the cost of a plate. Write down two equations involving x and y. Solve these equations and calculate the cost of seven mugs and six plates.

22) Sandra withdrew £400 from the bank. She was given £20 and £10 notes, a total of 23 altogether. Let x represent the number of £20 notes and y the number of £10 notes. Write down two equations and solve them.

23) A quiz game has two types of question, hard (h) and easy (e). Team A answers 7 hard questions and 13 easy questions. Team B answers 13 hard questions and 3 easy questions. If they both score 74 points, find how many points were given for each of the two types of question.

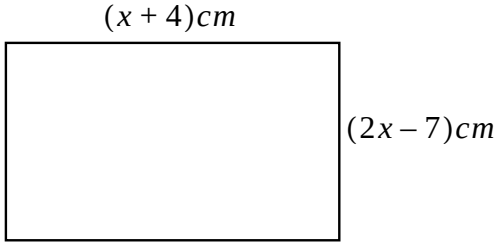
24) A man stays at a hotel. He has bed and breakfast (b) for three nights and two dinners (d). A second man has four nights bed and breakfast and three dinners. If the first man's bill is £90 and the second man's bill is £124, calculate the cost of a dinner.

25) Four large buckets and two small buckets hold 58 litres. Three large buckets and five small buckets hold 68 litres. How much does each bucket hold?

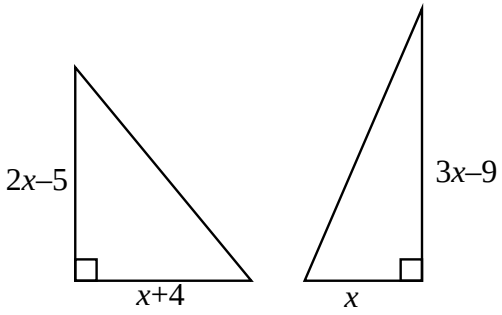
26) Caroline buys three first class stamps and five second class stamps for £1.94. Jeremy buys five first class stamps and three second class stamps for £2.06. Calculate the cost of each type of stamp.

39. Problems Involving Quadratic Equations

1. A rectangle has a length of $(x + 4)$ centimetres and a width of $(2x - 7)$ centimetres.
   a) If the perimeter is 36cm, what is the value of x?
   b) If the area of a similar rectangle is $63cm^2$, show that $2x^2 + x - 91 = 0$ and calculate the value of x.



2. The areas of the two triangles on the right are equal.
   a) Write down an equation in x and simplify it.
   b) Solve this equation and calculate the area of one of the triangles.



3. Bill enters a 30 kilometre race. He runs the first 20km at a speed of x kilometres per hour and the last 10km at $(x - 5)$ kph. His total time for the race was 4 hours.
   a) Write down an equation in terms of x and solve it.
   b) What are his speeds for the two parts of the run?

4. A piece of wire is cut into 2 parts. The first part is bent into the shape of a square. The second part is bent into the shape of a rectangle with one side 4cm long and the other side twice the length of the square's side. Let x represent one side of the square.
   a) Write down two expressions in x for the areas of the two shapes.
   b) If the sum of the two areas is $105cm^2$, show that $x^2 + 8x - 105 = 0$.
   c) Calculate the length of the original wire.

5. A small rectangular lawn is twice as long as it is wide. It has a path around it which is 2 metres wide. The area of the path is twice the area of the lawn
   a) If the small side of the lawn is x metres, write down the dimensions of the outside edge of the path.
   b) By writing down the area of the lawn in terms of x and using the answer to part a), form an equation in x.
   c) Simplify this equation so that it can be written as $4(x^2 - 3x - 4) = 0$ and solve it.
   d) Write down the dimensions of the lawn.

6. William and Daniel take part in a fun run. William's average speed is $(x + 4)$ kph and Daniel's is $(3x)$ kph. William completes his run in $(x - 1)$ hours and Daniel in $(x - 2)$ hour.
   a) Write down two expressions representing the distance travelled by both runners.
   b) Combine these expressions to find the value of x.
   c) What was William's speed?

40. Completing the Square

1) In each of the following expressions determine the number which must be added (or subtracted) to make a perfect square. Some have been done for you.

a) $x^2 + 6x$ b) $x^2 + 4x$ c) $x^2 - 3x$

$x^2 + 6x + c^2 = (x + c)^2$

$x^2 + 6x + c^2 = x^2 + 2cx + c^2$

So $6x = 2cx$

Therefore $c = 3$ and $c^2 = 9$

i.e. 9 must be added to make a perfect square

d) $x^2 - 7x$ e) $x^2 - x$ f) $x^2 + 9x$

g) $3x^2 + 9x$ h) $4x^2 - 3x$ i) $7x^2 + 4x$

$3x^2 + 9x + c^2 = (\sqrt{3}x + c)^2$

$3x^2 + 9x + c^2 = 3x^2 + 2\sqrt{3}cx + c^2$

So $9x = 2\sqrt{3}cx$

Therefore $c = \frac{9}{2\sqrt{3}}$ and $c^2 = \frac{81}{12} = \frac{27}{4}$

i.e. $\frac{27}{4}$ must be added to make a perfect square

j) $6x^2 + 2x$ k) $9x^2 - 3x$ l) $16x^2 + 5x$

2) Find the solution to these equations by first completing the square.

a) $x^2 - 8x - 20 = 0$ b) $x^2 - 4x - 21 = 0$ c) $x^2 + 11x + 18 = 0$

d) $x^2 + 5x + 6 = 0$ e) $x^2 - x - 2 = 0$ f) $x^2 + 3x - 4 = 0$

g) $x^2 - 4x - 5 = 0$ h) $x^2 + 3x - 28 = 0$ i) $x^2 + 4x + 3 = 0$

j) $2x^2 + 5x + 2 = 0$ k) $3x^2 - 4x - 4 = 0$ l) $2x^2 + 5x - 3 = 0$

m) $8x^2 + 2x - 1 = 0$ n) $6x^2 + x - 2 = 0$ o) $15x^2 + 4x - 3 = 0$

41. Iteration

1) Starting with $x = 4$ and using the iteration $x_{n+1} = 4 - \frac{1}{x_n}$ find a solution of $x = 4 - \frac{1}{x}$ correct to two decimal places.

2) Starting with $x = 5$ and using the iteration $x_{n+1} = 7 - \frac{9}{x_n}$ find a solution of $x = 7 - \frac{9}{x}$ correct to two decimal places.

3) Starting with $x = 4$ and using the iteration $x_{n+1} = 3 + \frac{3}{x_n}$ find a solution of $x = 3 + \frac{3}{x}$ correct to two decimal places.

4) Show that $x = 9 - \frac{2}{x}$ can be re-arranged into the equation $x^2 - 9x + 2 = 0$.

Use the iterative formula $x_{n+1} = 9 - \frac{2}{x_n}$ together with a starting value of $x_1 = 9$ to obtain a root of the equation $x^2 - 9x + 2 = 0$.

5) Show that $x = \frac{1}{x+1}$ can be re-arranged into the equation $x^2 + x - 1 = 0$.

Use the iterative formula $x_{n+1} = \frac{1}{x_n + 1}$ together with a starting value of $x_1 = 0.5$ to obtain a root of the equation $x^2 + x - 1 = 0$ correct to 2 decimal places.

6) Show that $x = \frac{12}{x+2} - 4$ can be re-arranged into the equation $x^2 + 6x - 4 = 0$.

Use the iterative formula $x_{n+1} = \frac{12}{x_n + 2} - 4$ together with a starting value of $x_1 = -6$ to obtain a root of the equation $x^2 + 6x - 4 = 0$ correct to 1 decimal place.

7) A sequence is given by the iteration $x_{n+1} = \frac{21}{x_n - 4}$

a) (i) The first term, x_1, of the sequence is –2. Find the next 4 terms, correct to two decimal places.

(ii) What do you think the value of x is as n approaches infinity?

b) Show that the equation this solves is $x^2 - 4x - 21 = 0$.

8) A sequence is given by the iteration $x_{n+1} = 6 + \frac{7}{x_n}$

a) (i) The first term, x_1, of the sequence is 5. Find the next 3 terms.

(ii) What do you think the value of x is as n approaches infinity?

b) Show that the equation this solves is $x^2 - 6x - 7 = 0$.

42. Direct and Inverse Proportion

1) y varies directly as x and $y = 4$ when $x = 6$.

a) Find an expression for y in terms of x. b) Calculate the value of y when $x = 8$.

c) Calculate the value of x when $y = 15$.

2) If v varies directly as r^3 and $v = 24$ when $r = 2$ find the value of v when $r = 3$.

3) y is directly proportional to $\sqrt{x}$ and $x = 9$ when $y = 2$.

a) Find an expression for y in terms of x. b) Find the value of y when $x = 4$.

c) Calculate the value of x when $y = 6$.

4) a varies directly as b^3. If $a = 32$ when $b = 2$ find a when $b = 4$.

5) Given that y is inversely proportional to the square of x, and y is equal to 0.75 when $x = 2$.

a) Find an expression for y in terms of x.

b) Calculate y when $x = 3$. c) Calculate x when $y = \frac{1}{12}$.

6) y varies inversely as x. If $y = 1.5$ when $x = 2$ find y when $x = 4$.

7) y is proportional to the cube root of x . $y = 10$ when $x = 8$.

a) Find an expression for y in terms of x.

b) Calculate (i) y when $x = 64$. (ii) x when $y = 25$.

8) y varies inversely as x^2. Copy and complete the following table.

x	1	2	3	4
y		0.75		

9) a varies directly as the square root of b. If $a = 1$ when $b = 9$ find a when $b = 4$.

10) y varies directly as the cube root of x. Copy and complete the following table.

x	0.125	1	8	64
y			1	

11) The table below shows values of y for values of the variable x, which are linked by the equation $y = 8x^n$.

x	0.5	1	2
y	1	8	64

Find (a) The value of n. (b) y when $x = 0.25$.

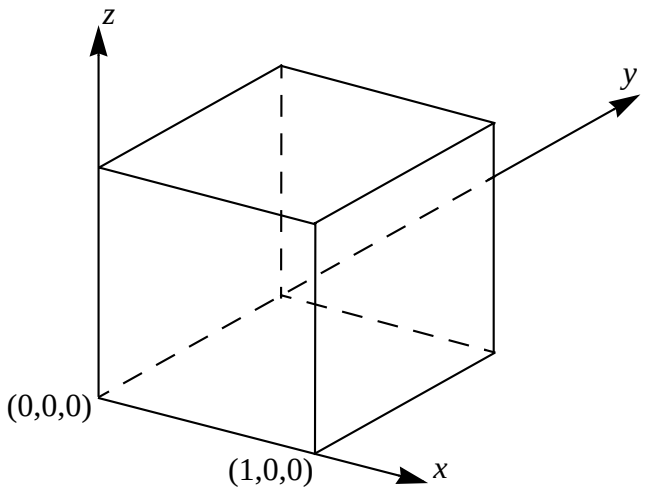
12) y varies inversely as the square of x. Copy and complete the table below.

x	0.5	0.75	1	1.5
y	6			

43. 3 Dimensional Co-ordinates 1

1) Point A has co-ordinates of (7, 9, 11).
Point B has co-ordinates of (1, 3, 5).
What are the co-ordinates of point P, the mid point of line AB?

2) a) The cube shown below has two corners of (0,0,0) and (1,0,0). What are the co-ordinates of the other corners?

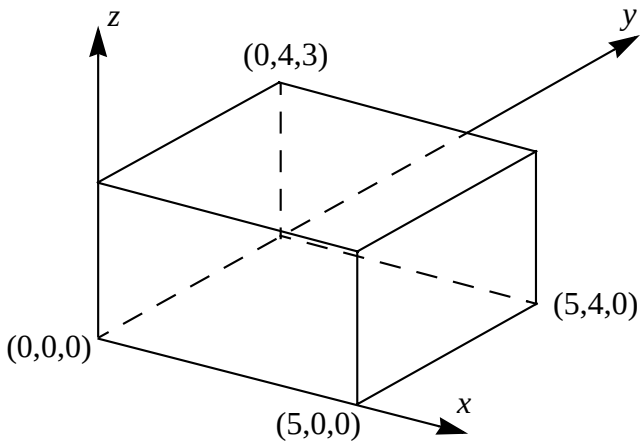


b) The cube is moved +2 units in each direction.
(i) What are the new co-ordinates of the corners?
(ii) What are the co-ordinates of the centre of the cube?

3) The co-ordinates of four of the corners of a cube are (0,0,0), (0,0,2), (0,2,2) and (2,2,2).
What are the co-ordinates of the other corners?

4) A cube has co-ordinates of (1,1,2), (5,1,2), (5,5,2), (1,5,2), (1,1,6), (5,1,6), (5,5,6) and (1,5,6).
a) Write down the co-ordinates of the centres of each of it's six faces.
b) What are the co-ordinates of the centre of the cube?

5) a) The cuboid shown below has four corners of (0,0,0), (5,0,0), (5,4,0), and (0,4,3). What are the co-ordinates of the other corners?

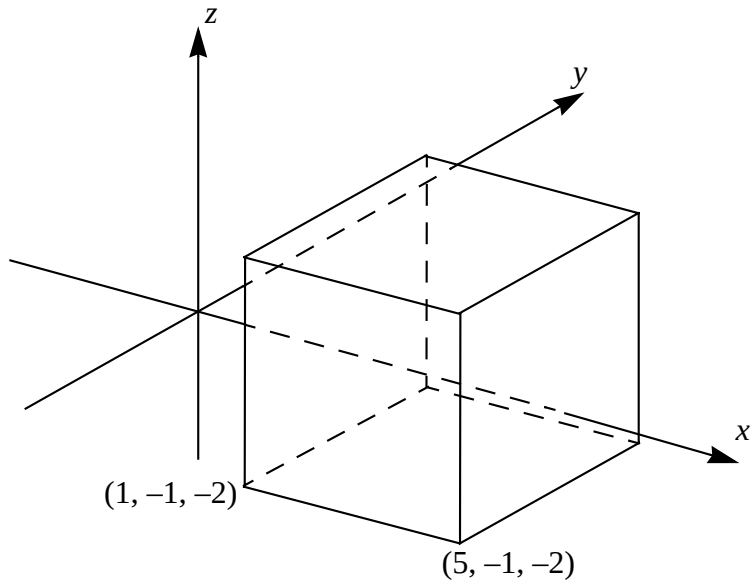


b) What are the co-ordinates of the centres of each of its faces?

44. 3 Dimensional Co-ordinates 2

1) Point A has co-ordinates of (–5, –3, –1).
Point B has co-ordinates of (7, 3, 5).
What are the co-ordinates of point P, the mid point of line AB?

2) a) The cube shown below has two corners of (1,–1,–2) and (5,–1,–2). What are the co-ordinates of the other corners?



b) The cube is moved +2 units in each direction
(i) What are the new co-ordinates of the corners?
(ii) What are the co-ordinates of the centre of the cube?

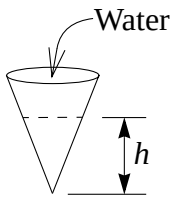
3) The co-ordinates of four of the corners of a cube are (–3,–2,–1), (–3–2,1), (–3,0,1) and (–1,0,1). What are the co-ordinates of the other corners?

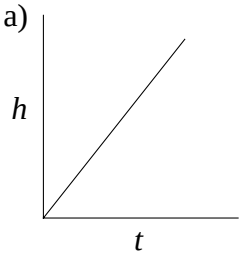
4) A cube has co-ordinates of (–2,–3,–2), (–2,–3,4), (–2,3,4), (–2,3,–2), (4,–3,–2), (4,3,–2), (4,3,4) and (4,–3,4).
a) What is the length of one side of the cube?
b) Write down the co-ordinates of the centres of each of its six faces.
c) What are the co-ordinates of the centre of the cube?
d) If the cube is repositioned so that corner (–2, –3, –2) is moved to (0, 0, 0), what is the new position of corner (4, 3, –2)

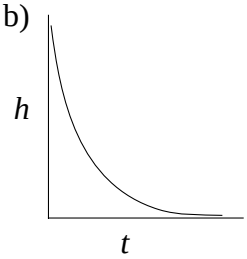
5) A cuboid has four corners of (–7,–6,–5), (–7,–6,–3), (–7,–3,–5), and (–3,–3,–3).
a) What are the co-ordinates of the other corners?
b) What are the co-ordinates of the centres of each of its faces?
c) What are the co-ordinates of the centre of the cuboid?
d) What are the dimensions of the cuboid?
e) If corner (–7, –6, –5) is moved to position (0, 0, 0) to what position will corner (–3, –3, –3) move?

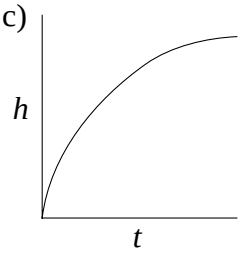
45. Recognising Graphs 1

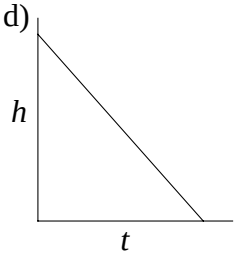
1. Water runs into a conical container. The height (*h)* of the water is plotted against the time (*t*) it takes for the water to flow into the container. Which of the following sketches represents this?



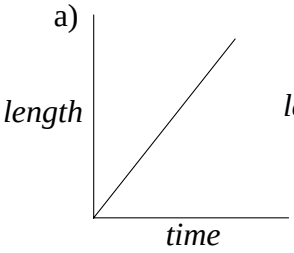
a) 

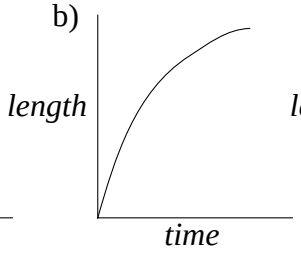
b) 

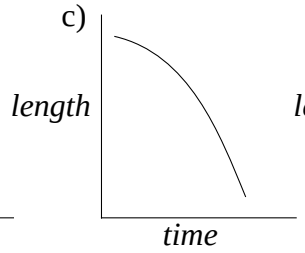
c) 

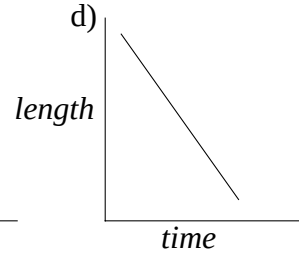
d) 

2. A weight is suspended from the bottom end of a piece of wire. The top end is fixed. The weight makes the wire extend at a constant rate. Which of the following diagrams shows this?

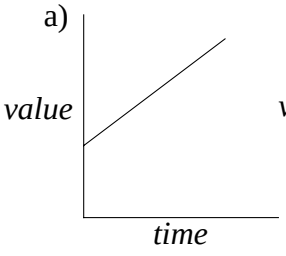
a) 

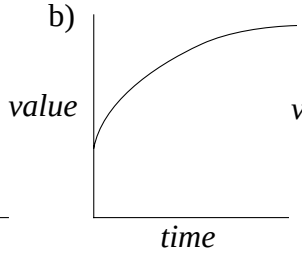
b) 

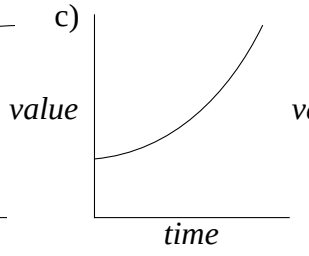
c) 

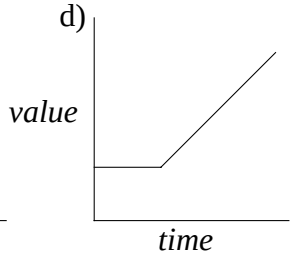
d) 

3. David buys an antique table. He estimates that each year it's value will increase by 20% of it's value at the beginning of that year. Which of the following diagrams represents this?

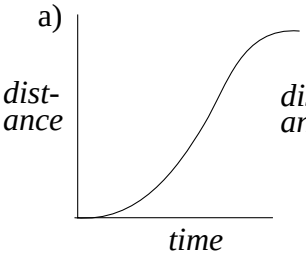
a) 

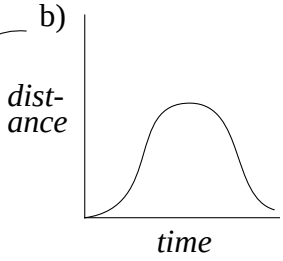
b) 

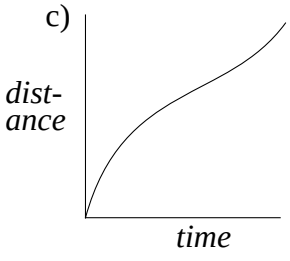
c) 

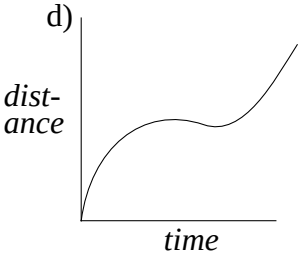
d) 

4. A car sets out from town A and drives to town B. The car is slowed down by traffic at the beginning and the end of the journey, but speeds up in the middle section. Which of the following diagrams shows this?

a) 

b) 

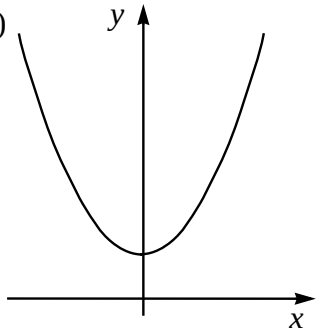
c) 

d) 

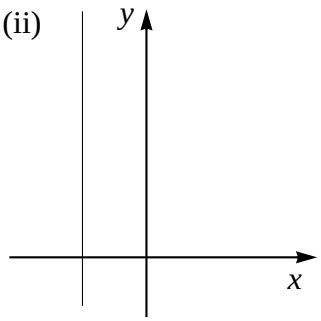
46. Recognising Graphs 2

Below there are 12 sketches of graphs and 12 functions representing them. Write down the letter of the function which goes with its graph.

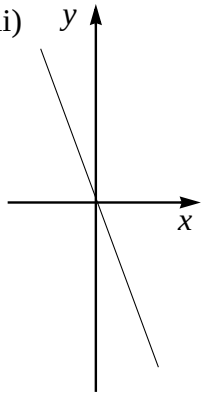
(i)



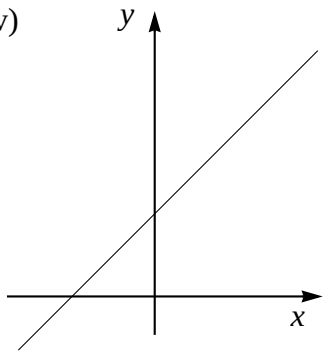
(ii)



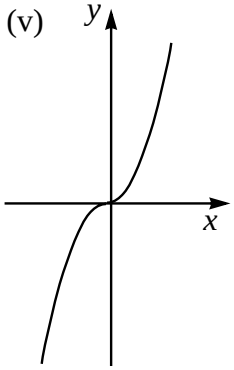
(iii)



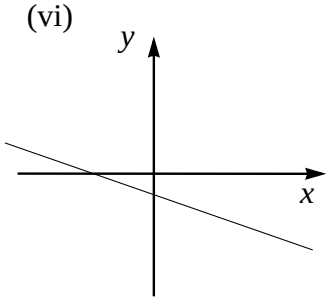
(iv)



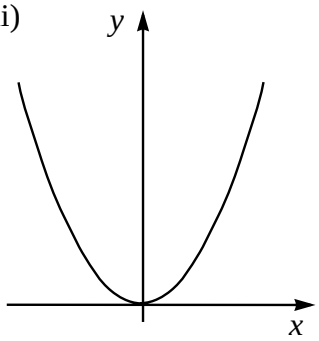
(v)



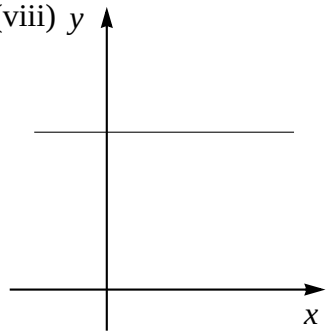
(vi)



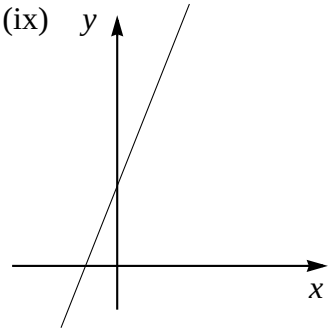
(vii)



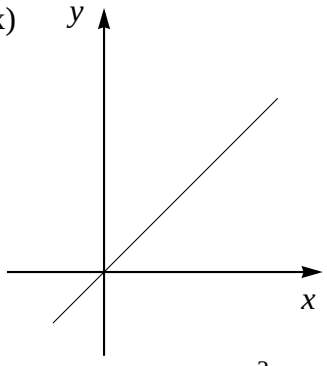
(viii)



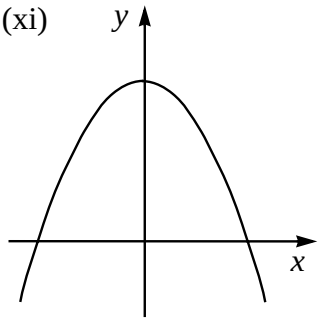
(ix)



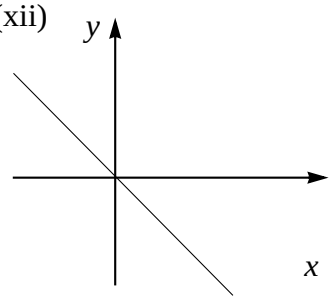
(x)



(xi)



(xii)



a) $y = x$ b) $y = x^2$ c) $y = x + 3$ d) $y = x^3$ e) $x = -3$ f) $y = x^2 + 2$

g) $y = -x$ h) $y = -x^2 + 5$ i) $y = 2x + 4$ j) $y = -\frac{1}{2}x - 1$ k) $y = -2x$ l) $y = 6$

47. Graphs 1

1. a) Complete this table for values of $y = x^2 + 2x - 3$.

$x =$	–4	–3	–2	–1	0	1	2
$y = x^2 + 2x - 3$							

b) Draw the graph of $y = x^2 + 2x - 3$ using a scale of 2cm to represent 1 unit on the x axis and 2cm to represent 1 unit on the y axis.

c) Using the same axis, draw the graph of $y = x + 2$.

d) Write down the two values of x where the two graphs cross.

e) Write down a simplified equation which satisfies these two values of x.

2. a) Complete this table for the values of $y = x^3 - 2x + 2$.

$x =$	–2.5	–2	–1	0	1	2	2.5
$y = x^3 - 2x + 2$	–8.625		3			6	

b) Plot the graph of $y = x^3 - 2x + 2$ using a scale of 2cm to represent 1 unit on the x axis and 2cm to represent 2 units on the y axis.

c) On the same axes, draw the graph of $y = 3x + 2$.

d) Show on the graph that the equation $x^3 - 5x = 0$ has three solutions. From the graphs give approximate values of these solutions.

3. a) Complete the table of values for $y = 2x^2 - 4x - 3$.

$x =$	–2	–1	0	1	2	3	4
$y = 2x^2 - 4x - 3$							

b) Plot the graph of $y = 2x^2 - 4x - 3$ using a scale of 2cm to represent 1 unit on the x axis and 2cm to represent 2 units on the y axis.

c) From the graph write down the solution to the equation $2x^2 - 4x - 3 = 0$.

d) On the graph draw in the line $y = 4$ and from the graph write down the approximate solution to the equation $2x^2 - 4x - 7 = 0$.

4. a) Complete the table of values for $y = (x + 3)(x - 1)$

$x =$	–4	–3	–2	–1	0	1	2
x+3							
x–1							
$y = (x + 3)(x - 1)$							

b) Using a scale of 2cm to represent 1 unit on both axes, draw the graph of $y = (x + 3)(x - 1)$.

c) On the same axes, draw the graph of $y = x + 1$.

d) From your graph, estimate the x co-ordinates of the points of intersection of the two graphs and write down the quadratic equation which these values of x satisfy.

48. Graphs 2

1. a) Complete the table of values of $y = 2x + \frac{8}{x}$ for the values of x from 0.5 to 6.

x	0.5	1.0	1.5	2.0	2.5	3	4	6	7
$y = 2x + \frac{8}{x}$									

b) Using a scale of 2cm to represent 1 unit on the x axis and 1cm to represent 1 unit on the y axis plot the graph of $y = 2x + \frac{8}{x}$.

c) Using the same axes draw the lines representing $y = 14$ and $y = 12 - \frac{x}{2}$.

d) By considering the points of intersection of two graphs write down the approximate solutions to the equation $2x + \frac{8}{x} - 14 = 0$.

e) Show that the intersection of the graphs $y = 2x + \frac{8}{x}$ and $y = 12 - \frac{x}{2}$ gives a solution to the equation $5x^2 - 24x + 16 = 0$. What are the approximate solutions to this equation?

f) What is the gradient of the curve $y = 2x + \frac{8}{x}$ when $x = 4$?

2. a) Complete the table of values of $y = 4 + 3x - x^2$ for values of x from –4 to +4.

$x =$	–4	–3	–2	–1	0	1	2	3	4	5
$y = 4 + 3x - x^2$										

b) Draw the graph of $y = 4 + 3x - x^2$ using a scale of 2cm to represent 1 unit on the x axis and 2cm to represent 4 units on the y axis.

c) On the same axes draw the line $y = 3$ and write down the approximate co-ordinates of the point of intersection of the two graphs.

d) Show that the x co-ordinates at this point are an approximate solution to the equation $3x - x^2 + 1 = 0$.

e) What is the solution to the equation $4 + 3x - x^2 = 0$?

f) By drawing a straight line, find an approximate solution to the equation $8 + 3x - x^2 = 0$.

3. Draw the graphs of $y = (x + 3)(3 - 2x)$ and $y = \frac{3x + 5}{2}$ for values of x from –4 to +2 using a scale of 2cm to 1 unit on the x axis and 1 cm to 2 unit on the y axis. From your graph estimate the solutions to the equations;

a) $13 - 9x - 4x^2 = 0$ and

b) $3 - 3x - 2x^2 = 0$

49. Graphs 3

1. The table below shows values of $y = x^2 + c$. What is the value of c?

x	1	2	3	4	5
y	2.5	5.5	10.5	17.5	26.5

2. The table below shows values of y which are approximately equal to $ax^2 + b$, where a and b are constants.

x	1	2	4	6	8
y	53	62	98	160	240

   a) Plot the values of y against x, using a scale of 2cm to represent 1 unit on the x axis and 2cm to represent 20 units on the y axis.
   b) From your graph, determine the approximate values of a and b.
   c) What is the approximate value of y when $x = 7$?

3. The table shows the approximate values of y which satisfy the equation $y = pq^x$, where p and q are constants.

x	0	0.5	1	1.5	2	2.5	3
y	3	4.24	6	8.5	12	17	24

   a) Plot the values of y against x, using a scale of 4cm to represent 1 unit on the x axis and 2cm to represent 2 units on the y axis.
   b) Use your graph to help you estimate the values of p and q.
   c) What is the approximate value of y when $x = 2.25$?

4. The table below shows the approximate values of y which satisfy the equation $y = a\sin x + b$ where a and b are constants.

$x°$	0	30	60	90	120	150	180
y	–0.5	0.25	0.8	1.0	0.8	0.25	–0.5

   a) Using a scale of 2cm to represent $30°$ on the x axis and 10cm to represent 1 unit on the y axis, plot the graph of $y = a\sin x + b$
   b) From your graph estimate the values of a and b.
   c) What is the approximate value of y when $x = 45°$?

5. The table below shows the approximate values of y which satisfy the equation $y = a^x + b$ where a and b are constants.

x	0	0.5	1.0	1.5	2.0	2.5
y	3.0	3.73	5.0	7.2	11	17.6

   a) Draw the graph of $y = a^x + b$. Allow 4cm to represent 1 unit in the x axis and 2cm to represent 2 units on the y axis.
   b) From your graph estimate the values of a and b.
   c) What is the approximate value of y when $x = 1.3$?

50. Straight Line Graphs

Exercise 1

In each of the following questions say what the gradient of the line is.

1) $y = x + 2$ 2) $y = 3x + 3$ 3) $y = -5x - 2$ 4) $y = 3 - 4x$

5) $2y = 4x + 3$ 6) $3y = 6x - 5$ 7) $3y = -5x + 2$ 8) $4y = 6 - x$

9) $y - 2x + 3 = 0$ 10) $2x - y - 3 = 0$ 11) $y + x + 3 = 0$ 12) $2y + 2x - 2 = 0$

13) $2y + 2x = 1$ 14) $x + y = 3$ 15) $4x - 3y = 4$ 16) $x - 3y = 2$

Exercise 2

1) What is the equation of the line parallel to $y = x$ which goes through the point (3,0)?

2) What is the equation of the line parallel to $y = 2x$ which goes through the point (5,0)?

3) What is the equation of the line parallel to $y = 3x + 2$ which goes through the point (0,0)?

4) What is the equation of the line parallel to $y = -x - 6$ which goes through the point (0,2)?

5) What is the equation of the line parallel to $y = -3x + 2$ which goes through the point (1,1)?

6) What is the equation of the line parallel to $y = 4x - 7$ which goes through the point (4,2)?

Exercise 3

1) The table below shows the relationship between x and y.

x	10	20	30	40	50
y	14	34	54	74	94

a) On graph paper plot the values of x and y and show that this graph is of the form $y = mx + c$ b) What are the values of m and c? c) What is the value of y when $x = 65$? d) What is the value of x when $y = 234$?

2) The table below shows the relationship between two sets of values x and y .

x	2	4	6	8	10
y	30	40	50	60	70

a) On graph paper plot the values of x and y and show that this graph is of the form $y = mx + c$ b) What are the values of m and c? c) What is the value of y when $x = 15$? d) What is the value of x when $y = 105$?

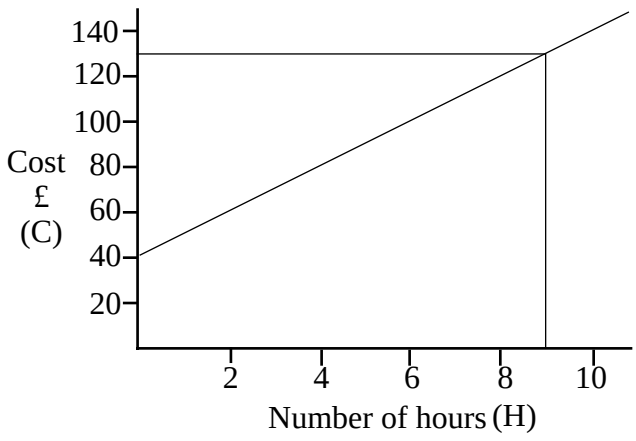
3) A coach company charges for the hire of their coaches according to the graph shown.

a) Write down the equation to the line

b) What is the cost of hiring a coach for 5 hours?

c) What is the cost of hiring a coach for $6\frac{1}{2}$ hours?

d) For how many hours is a coach hired for if it costs £100?



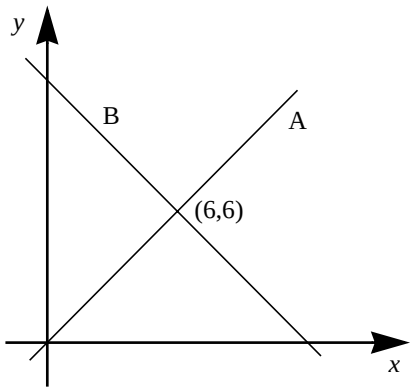
51. Perpendicular Lines

1) a) What is the gradient of a line parallel to $y = 3x$?
b) What is the gradient of a line perpendicular to $y = 3x$?
c) Which of the following equations represent lines perpendicular to the line $y = 3x$?

(i) $y = 4 - \frac{x}{3}$ (ii) $y - 3x = 2$ (iii) $y = -\frac{x}{3} + 7$ (iv) $y + \frac{x}{3} = 6$

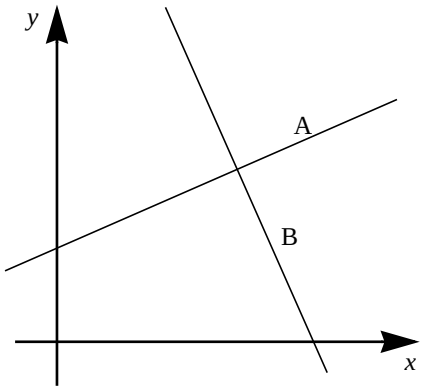
(v) $y = -3x - 1$ (vi) $3y - 9x = 7$ (vii) $3y + x = 9$

2)



The diagram shows two straight lines A and B.
Line A goes through points (0,0) and (6,6).
a) What is the equation of line A in the form $y = mx + c$?
Line B is perpendicular to line A.
Lines A and B meet at (6,6).
b) What is the equation of line B?

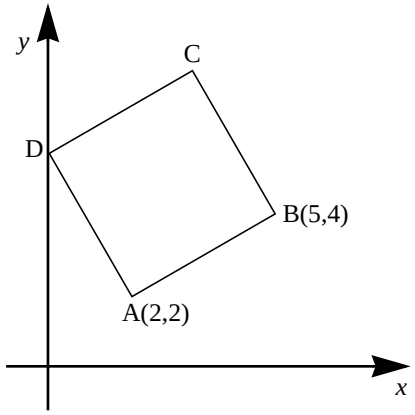
3)



The diagram shows two straight lines A and B.
Line A cuts the y axis at (0,2) and meets line B at point (4,4).
a) What is the equation of line A in the form $y = mx + c$?
Line B is perpendicular to line A.
b) What is the gradient of line B?
c) At what point will line B cut the y axis?
d) What is the equation of line B?

4) The line $y = \frac{x}{3} + 4$ cuts the line $y = mx + c$ at the point (6,2). If the lines are perpendicular to each other, what is the equation of the second line?

5)



The diagram shows a square ABCD with corners A(2,2) and B(5,4).
a) What is the gradient of the line AB?
b) What is the equation of the line AB in the form $y = mx + c$?
c) Write down the gradients of the other three sides.
d) What are the equations of the other three sides?

52. Growth and Decay

1. The relationship between x and y is given by the equation $y = 1.5^{-x}$.
   (a) Complete the table, giving y correct to 3 decimal places where necessary.

x	0	1	2	3	4	5	6
y		0.667			0.198		

   (b) Draw the graph of $y = 1.5^{-x}$, allowing 2cm to represent 1 unit on the x axis and 2cm to represent 0.1 on the y axis.
   (c) From your graph, estimate the following, showing clearly where your readings are taken.
   (i) the value of x when $y = 0.7$.
   (ii) the value of y when $x = 3.5$.

2. The population of a country grows over a period of 7 years according to the equation $P = P_0 \times 1.1^t$ where t is the time in years, P is the population after time t and P_0 is the initial population.
   a) If P_0 = 10 million, complete the table below giving your values correct to 2 decimal places where necessary.

t	0	1	2	3	4	5	6	7
P (million)	10				14.64			19.49

   b) Plot P against t. Allow 2cm to represent 1 year on the horizontal axis and 2cm to represent 1 million on the vertical axis (Begin the vertical axis at 8 million).
   c) From your diagram estimate the population after $5\frac{1}{2}$ years.
   d) How long will the population take to reach 15 million?

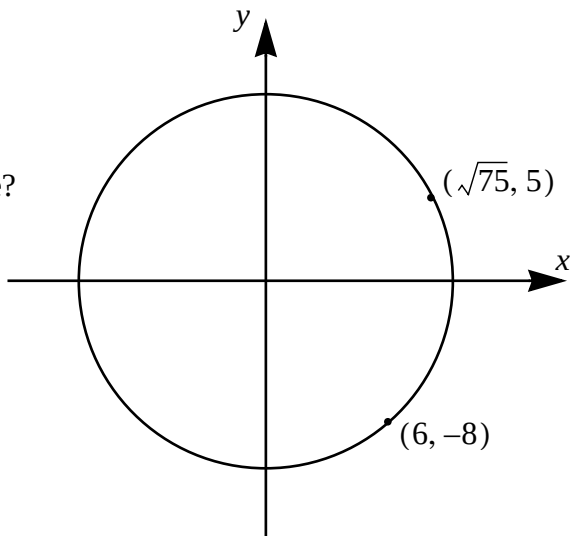
3. The percentage of the nuclei remaining in a sample of radioactive material after time t is given by the formula $P = 100 \times a^{-t}$, where P is the percentage of the nuclei remaining after t days and a is a constant.
   a) Copy and complete the table below for $a = 3$.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
P	100			19.3			3.7		

   b) Draw a graph showing P vertically and t horizontally. Use a scale of 4cm to represent 1 day on the horizontal axis and 2cm to represent 10% on the vertical axis.
   c) From the graph, estimate the following, showing clearly where your readings are taken.
   (i) The half life of the material (ie when 50% of the nuclei remain) correct to the nearest hour.
   (ii) The percentage of the sample remaining after 2.25 days.
   (iii) The time at which three times as much remains as has decayed.

53. Equation of a Circle

1) Two points $(\sqrt{75}, 5)$ and $(6, -8)$ lie on a circle $x^2 + y^2 = r^2$ where r is the radius of the circle with its centre at the origin (0,0), What is the radius of the circle?



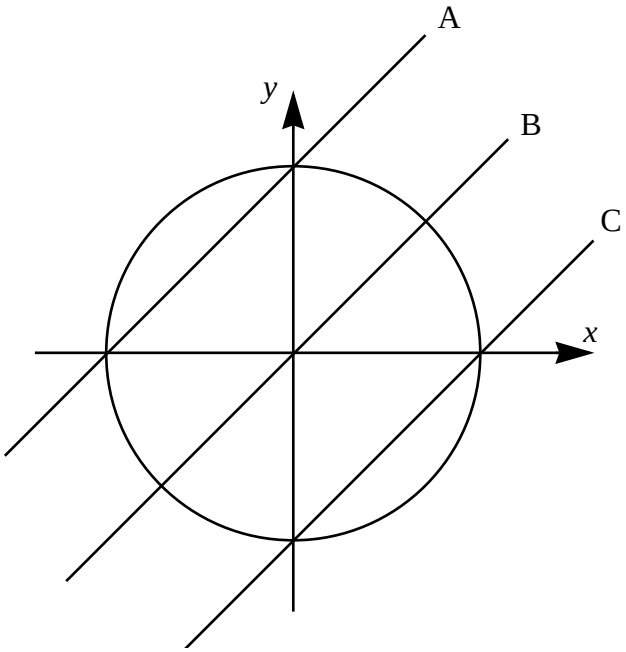
2) The diagram shows the graph of the circle $x^2 + y^2 = r^2$ and three parallel straight lines.

a) Which line has the equation $y = x + 6$?

b) What is the value of r?

c) What are the equations of the other two graphs?

d) At which points does graph B intersect with the circle?



3) At which points does the line $y = 2x - 4$ intersect with the circle $x^2 + y^2 = 100$?

4) a) At which points does the line $y = -7$ cut the circle $x^2 + y^2 = 100$?

b) Write down the co-ordinates of the points at which the line $y = 7$ cuts this circle.

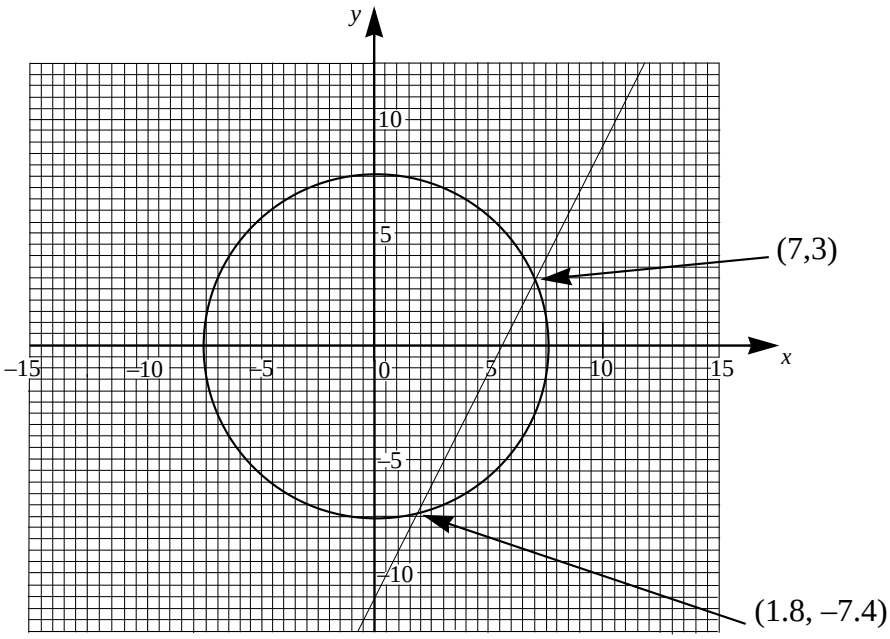
5) One of the points at which the line $y = \frac{1}{2}x - 2$ intersects with the circle $x^2 + y^2 = r^2$ is (–5,–4.5)

a) What is the value of r? Leave your answer as a surd.

b) What are the co-ordinates of the other point of intersection?

54. Simultaneous Equations 2

1) The points of intersection of the straight line $2x - y = 11$ and the circle $x^2 + y^2 = 58$ can be found either graphically, like this:



or by solving the simultaneous equations

$x^2 + y^2 = 58$ (i)

and $2x - y = 11$ (ii)

like this:

Rearrange $2x - y = 11$ to give $y = 2x - 11$ (iii)

substitute (iii) into equation (i) for y

$x^2 + (2x - 11)^2 = 58$

$x^2 + 4x^2 - 44x + 121 = 58$

$5x^2 - 44x + 63 = 0$

$(5x - 9)(x - 7) = 0$

$x = \frac{9}{5}$ or $x = 7$

Substitute into (iii) to find y which gives

$y = -\frac{37}{5}$ and $y = 3$

so the points of intersection of the two graphs are $(\frac{9}{5}, -\frac{37}{5})$ and (7,3)

Exercise

Find graphically the points of intersection of the following circles and straight lines and check the answers by solving the two simultaneous equations.

1) $x^2 + y^2 = 13$ $y = 5 - x$

2) $x^2 + y^2 = 41$ $x - y = -1$

3) $x^2 + y^2 = 29$ $x + y = 7$

4) $x^2 + y^2 = 61$ $2x + y = 16$

5) $x^2 + y^2 = 25$ $2x - y = -11$

55. Trial and Improvement

Exercise 1

By using the method of trial and improvement, calculate the value of x in each of the following equations. In each case show all your workings and give each answer correct to 1 decimal place.

1) $x^3 = 53$ 2) $x^3 = 77$ 3) $x^3 = 101$

4) $x^2 + x = 36$ 5) $2x^2 + x = 41$ 6) $x^2 + 3x = 61$

7) $5x^2 - 3x = 120$ 8) $6x^2 - 2x = 77$ 9) $3x^2 - 5x = 5$

10) $x^3 + 3x = 16$ 11) $x^3 - 3x = 96$ 12) $x^3 - 4x = 12$

13) $2x^3 + 4x = 96$ 14) $3x^3 - 4x = 14$ 15) $2x^3 - 5x = 4$

Exercise 2

1) A square has an area of $45cm^2$. Using the method of trial and improvement, calculate the length of its sides, correct to one decimal place. Show all your calculations.
2) A rectangle has one side 4cm longer than the other. If its area is $100cm^2$, use the method of trial and improvement to calculate the length of the shorter side, correct to 1 decimal place. Show all your calculations.
3) The perpendicular height of a triangle is 2cm greater than its base. If its area is $35cm^2$, use the method of trial and improvement to calculate its height, correct to 1 decimal place. Show all your calculations.
4) A cube has a volume of $36cm^3$. Use the method of trial and improvement to calculate the length of one of its sides, correct to 1 decimal place. Show all your calculations.
5) A cuboid has a height and length which is 3cm longer than its width. Its volume is $150cm^3$. Use the method of trial and improvement to calculate its width, correct to 1 decimal place. Show all your calculations.
6) The solution to the equation $x^2 + 3x = 7$ lies between 1 and 2. Use the method of trial and improvement to calculate its value, correct to 1 decimal place. Show all your calculations.
7) Using the method of trial and improvement, solve the equation $x^3 + 3x - 22 = 0$, correct to one decimal place. Show all your calculations.
8) The solution to the equation $x^3 + 4x = 48$ lies between 3 and 4. Use a method of trial and improvement to find the solution to $x^3 + 4x = 48$, correct to 1 decimal place. Show all your calculations.

56. Inequalities

Exercise 1

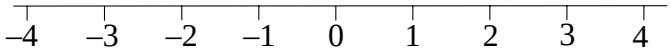
In each of the following inequalities, solve them in the form $x > n$, $x < n$, $x \geq n$ or $x \leq n$ where n is a number.

1) $x + 1 > 7$	2) $x + 4 > 6$	3) $x + 12 > 20$
4) $x + 6 > 3$	5) $x + 5 > 2$	6) $x + 7 > 9$
7) $2x > 12$	8) $3x > 12$	9) $4x > 20$
10) $x + 3 < 5 - x$	11) $x + 4 \leq 10 - x$	12) $x + 3 \leq 13 - x$
13) $3x - 9 \geq x + 7$	14) $5x - 6 < 2x + 3$	15) $6x - 8 < 4x + 2$
16) $2(x - 3) > 3(2 - x)$	17) $3(x + 1) \leq 2(x + 7)$	18) $5(x + 3) \leq 2(x - 4)$
19) $5(2x - 4) \leq 3(3x - 7)$	20) $3(2x - 7) \leq 4(2x + 8)$	21) $3(2x - 6) \geq 2(4x - 7)$

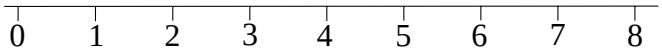
Exercise 2

In questions 1 to 3, copy the number line into your book. Mark on it the integer values of x which satisfy the inequality.

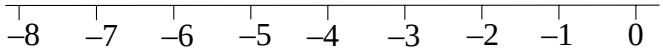
1. $2x + 3 > 3x + 4$



2. $3x + 7 > 4x + 2$

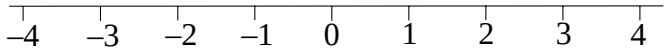


3. $4x + 7 < 2x - 6$

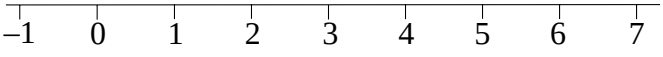


In questions 4 and 5, copy the number line into your book. Mark on the number line the range of values which satisfy the inequality

4. $4x + 3 > 2x - 4$



5. $5x - 6 < 2x + 4$



6. If $3x + 4 > x + 7$, what is the least whole number that satisfies this inequality?
7. If $3x + 4 < x + 7$, what is the greatest whole number that satisfies this inequality?
8. A social club want to hire a bus to take them out for the day. Company A charge £20 per hour. Company B charge £70 plus £10 per hour. Let x be the number of hours they want to hire the bus for.
   a) Write down the inequality satisfied by x where the cost of the number of hours charged by company A is less than the cost charged by company B.
   b) Solve the inequality and explain what the solution tells you.

57. Linear Inequalities

1. Plot the graphs of $x = 6$, $y = 10 - x$ and $y = \frac{x+8}{2}$ for values of x from 0 to 6 and values of y from 0 to 10. Clearly show the area satisfied by the inequalities $x \le 6$, $y \ge 10 - x$ and $y \le \frac{x+8}{2}$. Write down the integer points within this area.
2. Plot the graphs of $y = 6$, $y = 6 - 2x$ and $y = 4x - 2$ for values of x from 0 to 4 and values of y from –3 to 7. Clearly show the area satisfied by the inequalities $y \le 6$, $y \ge 6 - 2x$ and $y \ge 4x - 2$. Write down the integer points within this area.
3. For values of 0 to 5 on the x axis and 0 to 12 on the y axis, plot the graphs of $y + \frac{x}{3} = 4$, $y = \frac{1}{2}x + 3$ and $y = 12 - 3x$. On your diagram clearly indicate the area satisfied by the inequalities $y + \frac{x}{3} \ge 4$, $y \le \frac{1}{2}x + 3$ and $y \le 12 - 3x$. List the integer points lying within this area.
4. Plot the graphs of $y = x$, $y = 12 - 3x$ and $y = 7 - \frac{1}{2}x$ for values of x from 0 to 6 and values of y from 0 to 12. Clearly indicate the area satisfied by the inequalities $y > x$, $y > 12 - 3x$ and $y < 7 - \frac{1}{2}x$. Write down the integer points lying within this area.
5. Plot the graphs representing the equations $x + y = 10$, $y + \frac{1}{2}x = 5$, $y = \frac{1}{2}x + 4$ and $y = 2x - 3$ for values of x from 0 to 6 and values of y from –3 to 10. On your diagram shade in the region representing the inequalities $x + y < 10$, $y + \frac{1}{2}x > 5$, $y \le \frac{1}{2}x + 4$ and $y > 2x - 3$. Write down the integer points lying within this region.
6. On graph paper draw lines and shade in the area representing the inequalities $y \le 8 - 2x$, $y \ge 6 - 3x$, $y \le \frac{1}{2}x - 1$ and $y \ge x - 4$ for values of x from 0 to 6 and y from –4 to 8. Write down the integer points lying within this region.
7. Plot lines and shade in the area representing the inequalities $y + \frac{1}{2}x < 1$, $y \ge x - 3$, $y < 2x - 2$ and $y > -\frac{1}{3}(x + 5)$ for values of x from –5 to 4 and y from –3 to 2. Write down the integer points lying within this region.
8. Plot lines and shade in the area representing the inequalities $y + 2x < 8$, $y < x + 3$, $y + \frac{1}{2}x > 2$ and $y > 2x - 2$ for values of x from –1 to 5 and values of y from –2 to 8. Write down the integer points lying within this area.
9. Plot lines and shade in the area representing the inequalities $y > x + 1$, $y + 3x < 7$, $y < 2x + 6$ and $y > 1 - x$ for values of x from –3 to 4 and values of y from –3 to 8. Write down the integer points lying within this area.

58. Linear Programming

1) A van can carry a maximum load of 400 kg. It carries boxes weighing 20 kg and 40 kg. It carries at least 7 boxes weighing 40 kg. The number of boxes weighing 40 kg is not more than twice the number of 20 kg boxes.
   Let x represent the number of 20 kg boxes and y the number of 40 kg boxes.
   a) Write down three inequalities involving x and y .
   b) Illustrate the three inequalities by a suitable diagram on graph paper. Let 2 cm represent 1 box on both axes.
   c) From the diagram determine the least weight the van carries.
   d) What combinations give the greatest weight?

2) Orange is produced by mixing together red and yellow in certain ratios. David wants to make up to 10 litres of orange paint. He mixes together 200 ml tins of red and yellow. To be classified as orange there must not be more than twice as much of one colour than the other.
   Let x represent the number of red tins and y the number of yellow tins.
   a) Write down 3 inequalities in x and y.
   b) On graph paper, illustrate these inequalities using a scale of 2 cm to represent 10 tins on each axis, showing clearly the area representing the orange mix.
   c) How much of each colour would he use to make 10 litres if he wanted;
      (i) the reddest possible shade of orange.
      (ii) the yellowest possible shade of orange.
   d) What is the maximum amount of orange paint he can make with 14 tins of red?

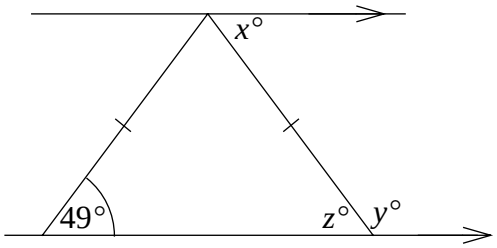
3) A factory produces curtains for large and small windows. Each large curtain requires 10 m^2 of fabric and each small curtain requires 5 m^2 of fabric. There is a total of 500 m^2 of fabric available each day.
   For each of the large curtains the factory makes a profit of £5 and for each small curtain it makes a profit of £8. To cover costs the factory needs to make a profit of at least £400 each day. Due to the type of demand for the curtains, they never make more than twice as many small curtains as large ones.
   Let x represent the number of large curtains and y the number of small curtains.
   a) Write down three inequalities involving x and y .
   b) Represent these inequalities on a graph and clearly indicate the area which satisfies these inequalities. Use a scale of 2 cm to represent 10 curtains on each axis.
   c) From the diagram find the values of x and y which will satisfy all the three conditions and give the greatest profit.

4) The dimensions of a rectangle are such that its perimeter is greater than 20 metres and less than 30 metres. One side must be greater than the other. The larger side must be less than twice the size of the smaller side.
   Let x represent the length of the smaller side and y the length of the larger one.
   a) Write down four inequalities involving x and y.
   b) On graph paper, illustrate these inequalities using a scale of 2 cm to represent 2 metres on each axis, clearly showing the area containing the solution.
   c) What whole number dimensions will satisfy these three inequalities?

59. Angles and Triangles

In each of the following questions write down the sizes of the unknown angles.

1)



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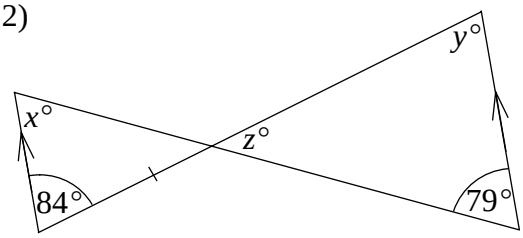
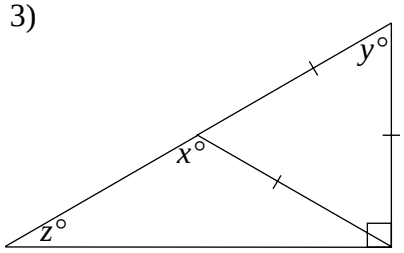
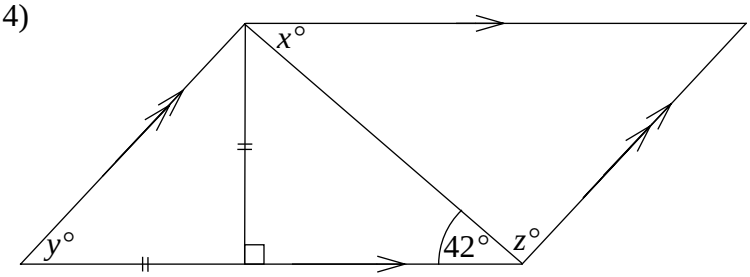


3)



4)



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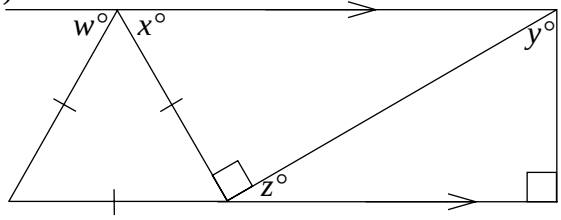
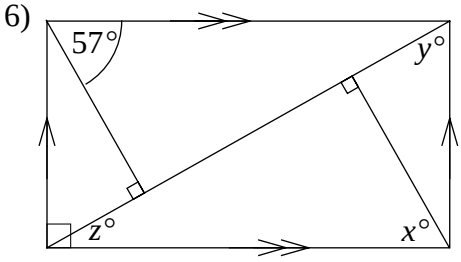
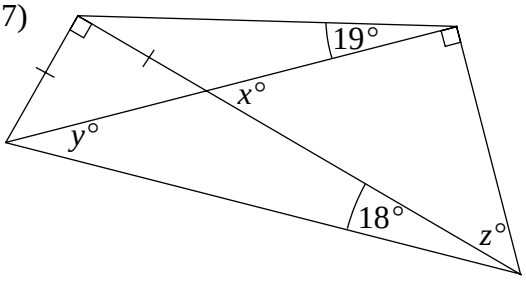


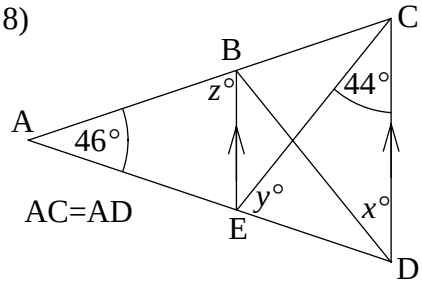
6)



7)



8)



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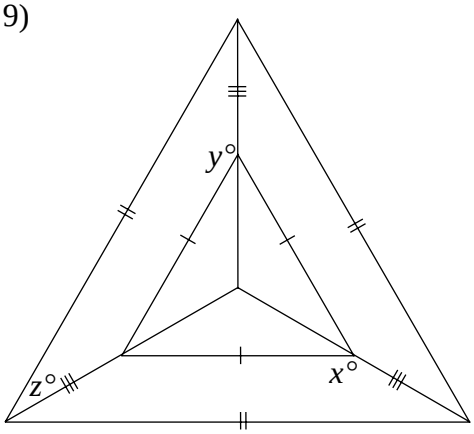

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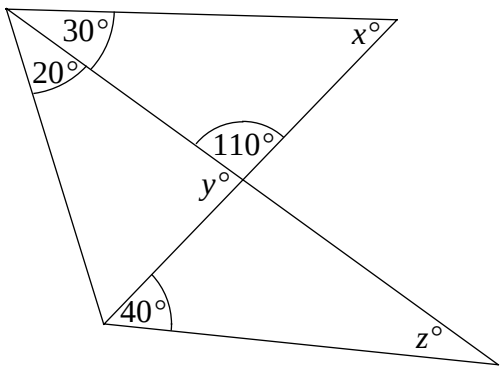


60. Regular Polygons 1

Calculate the interior and exterior angles in each of the regular shapes in questions 1 to 4.

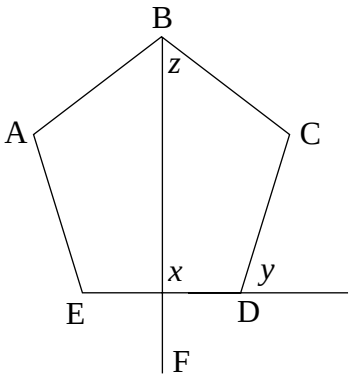
1) A Hexagon

2) A Nonagon (9 sides)

3) A 12 sided figure

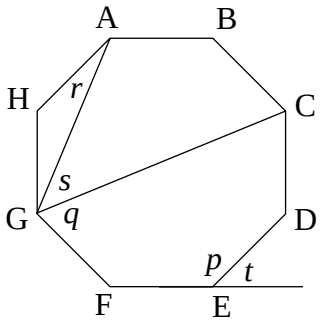
4) A 20 sided figure

5)



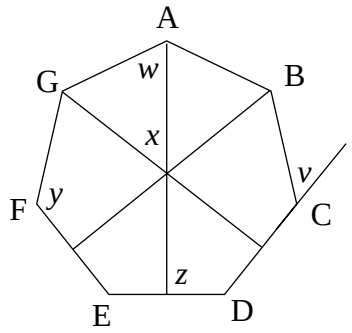
ABCDE is a regular pentagon.
Line BF is a line of symmetry.
a) What is the size of angle *x*?
b) Calculate the sizes of angles *y* and *z*.

6)



ABCDEFGH is a regular octagon.
CG is a line of symmetry.
Calculate the sizes of angles *p*, *q*, *r*, *s* and *t*.

7)



ABCDEFG is a regular heptagon.
Three lines of symmetry are shown.
What are the sizes of angles *v*, *w*, *x*, *y* and *z*?

8) What is the order of rotational symmetry of a regular octagon?

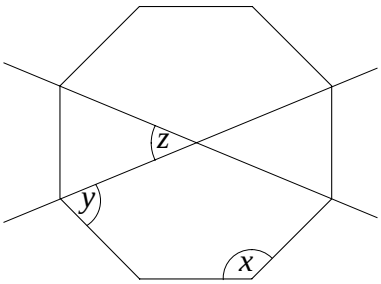
9) Explain why a regular pentagon will not tessellate and a regular hexagon will.

10) How many lines of symmetry has a regular nonagon?

11) ABCDEFGHIJ is a regular 10 sided polygon (decagon) with centre O (where the lines of symmetry cross). Calculate the sizes of the angles ABC and AOC.

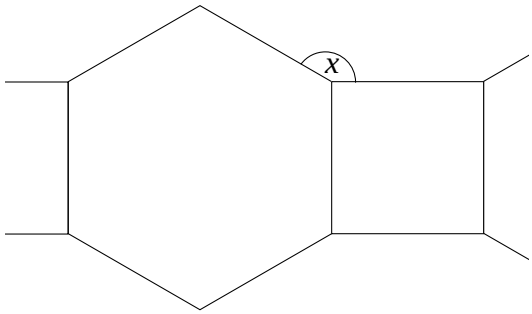
61. Regular Polygons 2

1. a) Calculate the value of the angle *x* in this regular octagon.
   b) What is the size of angle *y*?
   c) Calculate the size of angle *z*.
   In each, clearly explain how you arrive at your answer.

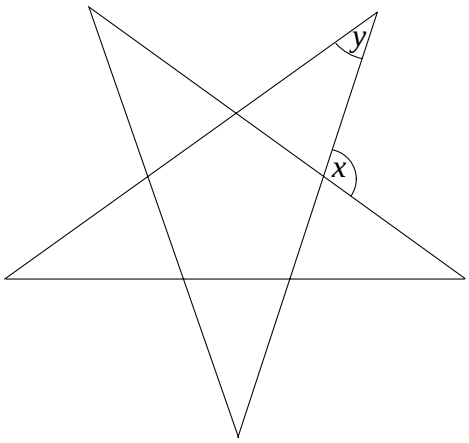


2. What is the sum of all the exterior angles of a polygon?

3. Regular hexagons and squares are put together in a row, as shown in the diagram on the right. Calculate the size of the angle marked *x*. Explain clearly how you arrive at your answer.

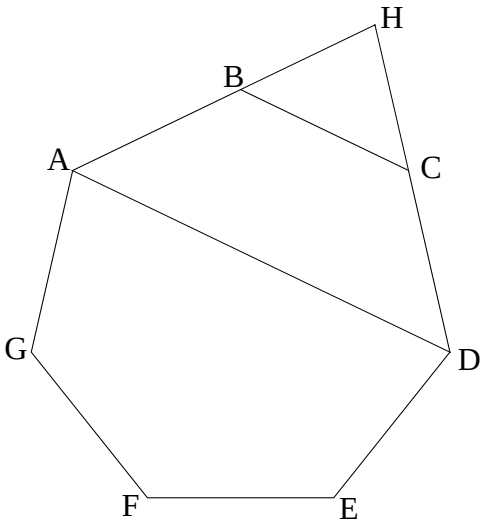


4. A five pointed regular star shape is to be cut from a piece of card. In order to draw it accurately it is necessary to calculate the angles *x* and *y*. Calculate these angles, explaining clearly how you arrive at the answers.



5. What regular shape can be made from a regular hexagon and three regular triangles, all with side lengths of *x* centimetres? What will be the length of one side of the new shape?

6. In the regular heptagon shown on the right AB and DC are produced to meet at H.
   a) Calculate the sizes of angles BCD and BHC, in each case explaining how you arrive at your answer.
   b) Prove that BC is parallel to AD.



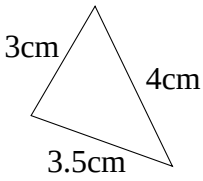
7. Three regular polygons fit together around a point. One is a triangle and one is an octagon. Calculate the number of sides the third shape has.

62. Congruent Triangles

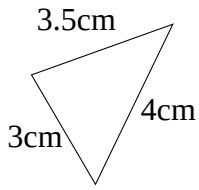
In questions 1 to 4, say whether all, two or none of the triangles are congruent.

1.

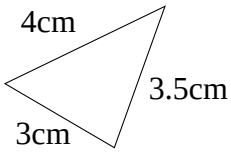
a)



b)

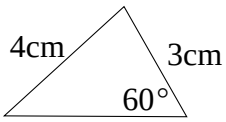


c)



2.

a)



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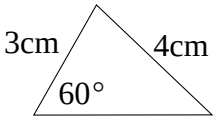

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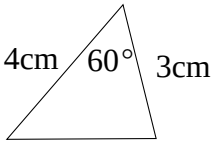
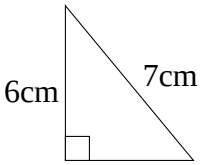
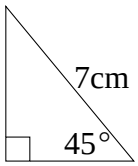


3.

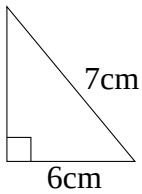
a)



b)

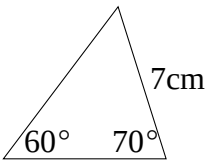


c)

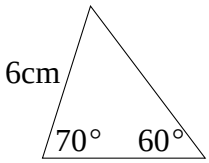


4.

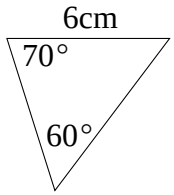
a)



b)

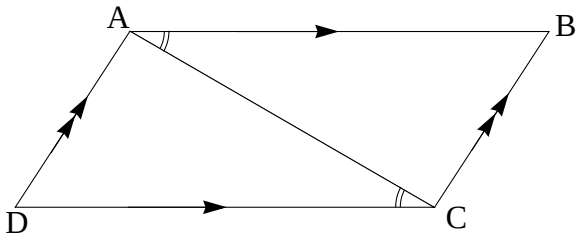


c)

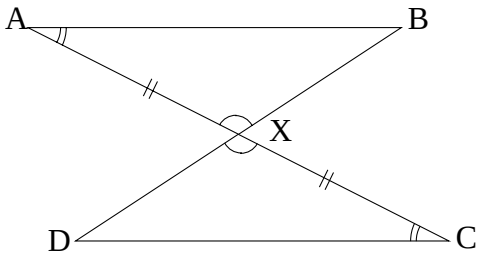


In questions 5 and 6 the triangles indicated are congruent. In each case explain why they are congruent.

5. ΔABC and ΔADC



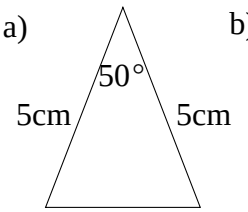
6. ΔABX and ΔCDX



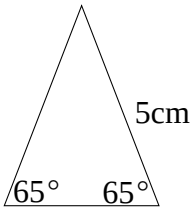
7. Which, if any, of the following statements are true for the triangles on the right

(i) Triangles a and b are congruent
(ii) Triangles a and c are congruent
(iii) Triangles b and c are congruent
(iv) All three triangles are congruent
(v) None of the triangles are congruent.

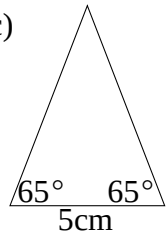
a)



b)



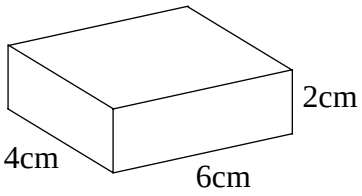
c)



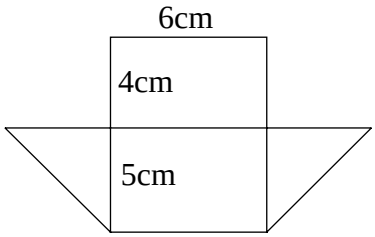
63. Nets and Isometric Drawing

1) Draw the net of a cube whose sides are 4cm.

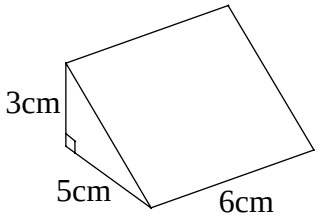
2) Draw the net of this cuboid.
Also draw, on triangular dotty paper or isometric paper, a cuboid whose volume is the same as this one.



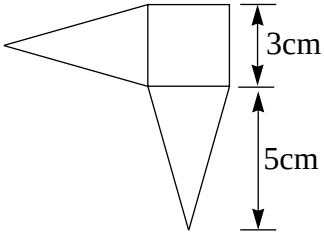
3) This diagram shows part of the net of a triangular prism.
Copy and complete the diagram.
On triangular dotty paper or isometric paper, draw a diagram of the shape.



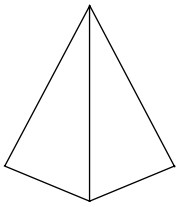
4) Draw the net of this triangular prism



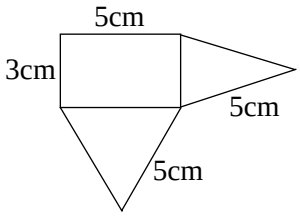
) The diagram on the right shows part of the net of a square based pyramid. Copy and complete the diagram. On triangular dotty or isometric paper, sketch a diagram of the shape.



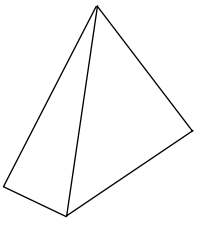
6) The diagram shows a square based pyramid.
Its base edges measure 3cm and its sloping edges are 6cm. Draw the net of its shape.



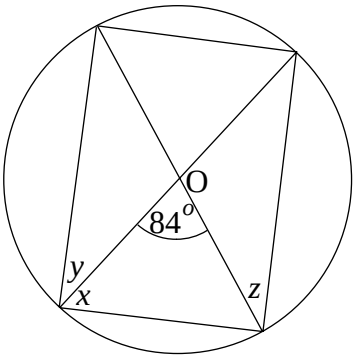
7) The diagram shows part of the net of a rectangular based pyramid. Copy and complete the diagram.
On triangular dotty or isometric paper, sketch a diagram of the shape.



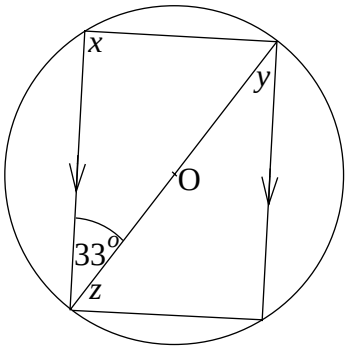
8) The diagram shows a rectangular based pyramid. Its base edges measure 2.5cm and 4cm and its sloping edges are each 5cm.
Draw the net of the shape.



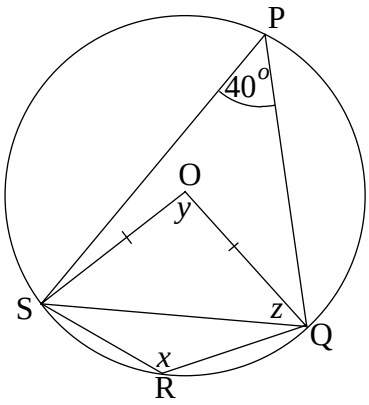
64. Geometry of a Circle 1



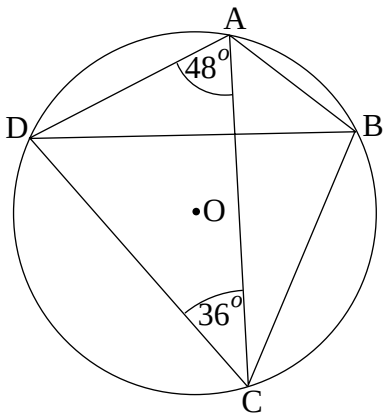
1) Calculate the sizes of the angles *x*, *y* and *z*.



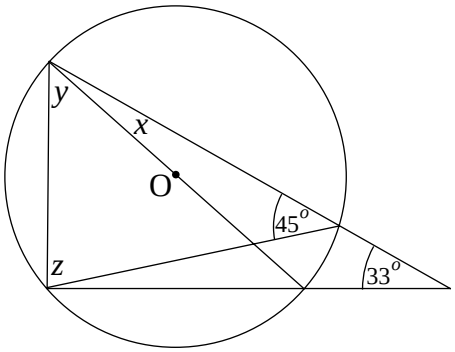
2) Calculate the sizes of the angles *x*, *y* and *z*.



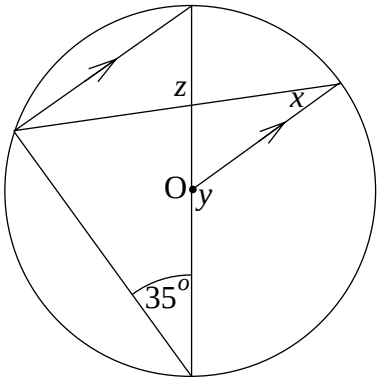
3) O is the centre of the circle. Lines OQ, and OS are equal in length. Calculate the sizes of angles *x*, *y* and *z*.



4) Calculate the sizes of angles ADC ABC and AOD.

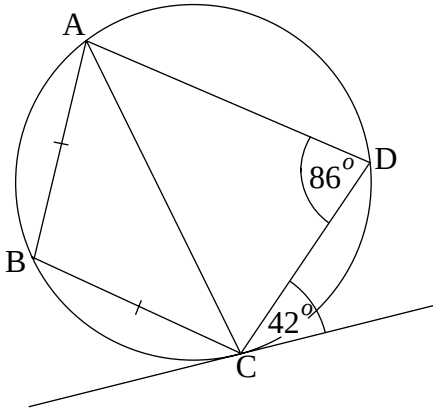


5) Calculate the sizes of angles *x*, *y* and *z*.

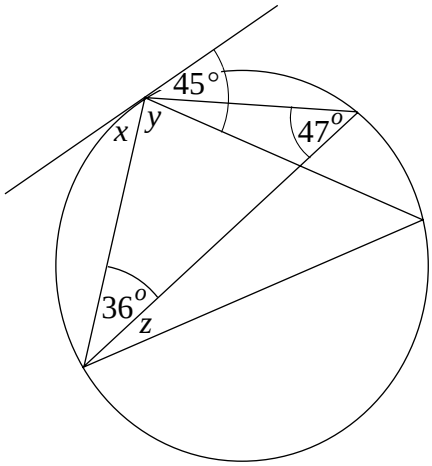


6) Calculate the values of the angles *x*, *y* and *z*.

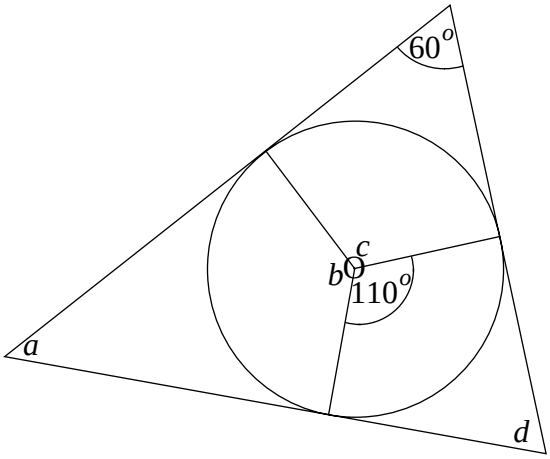
65. Geometry of a Circle 2



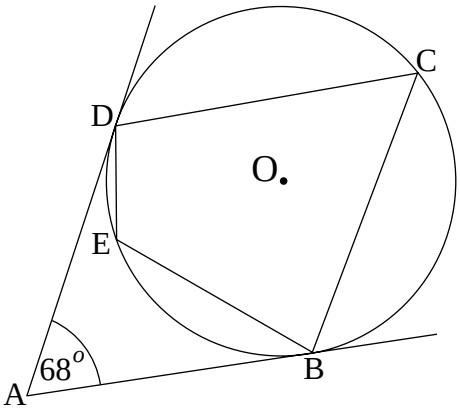
1) If AB=BC, calculate the sizes of the angles ABC, ACB, ACD and DAC.



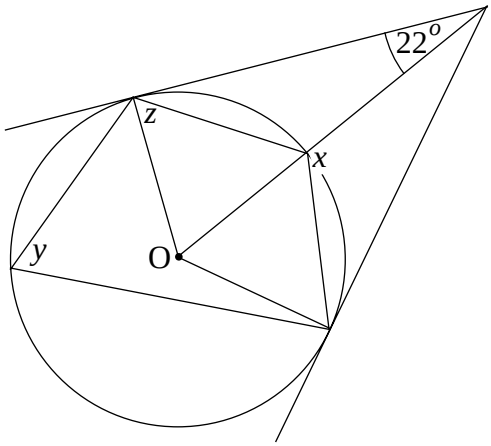
2) Find the values of the angles *x*, *y* and *z*.



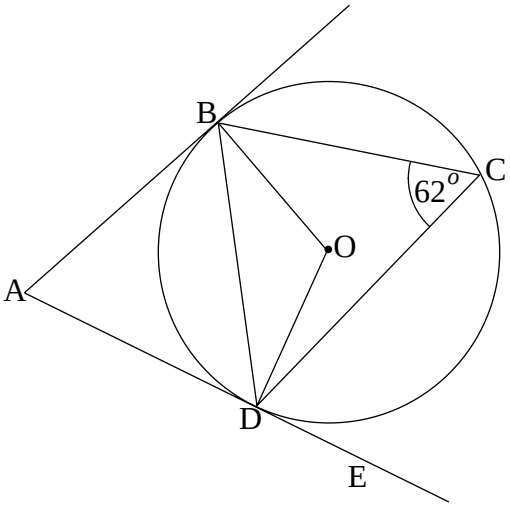
3) O is the centre of the circle. Calculate the sizes of the angles *a*, *b*, *c* and *d*.



4) Calculate the angles DEB, BCD, and DBO.



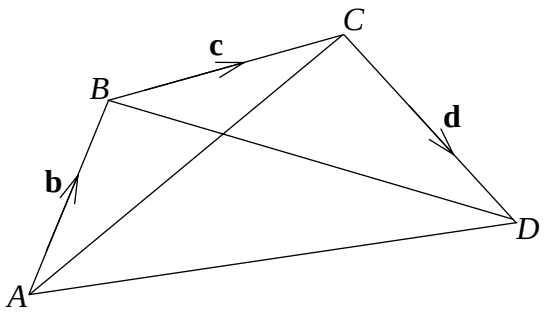
5) Calculate the angles *x*, *y* and *z*.



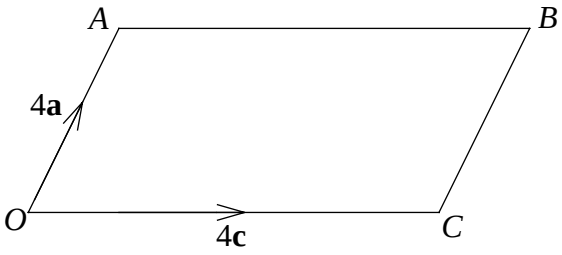
6) Calculate the angles BDA, BOD, BAD and DBO.

66. Vectors 1

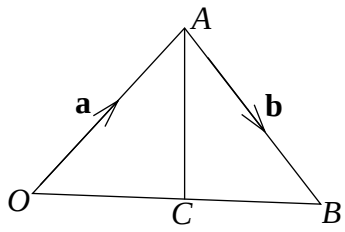
1. In the quadrilateral, $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{CD}$ are represented by the vectors **b**, **c**, and **d**. Find, in terms of **b**, **c**, and **d** $\overrightarrow{AC}$, $\overrightarrow{AD}$, and $\overrightarrow{BD}$



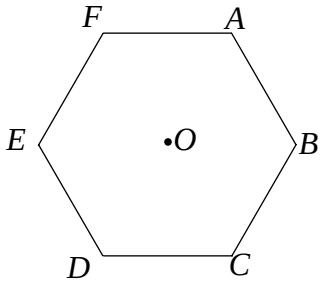
2. $\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}3\\5\end{pmatrix}$ calculate $\overrightarrow{AC}$ in bracket form.
3. Express $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ in its simplest form.
4. *OABC* is a parallelogram. If $\overrightarrow{OA} = 4\mathbf{a}$ and $\overrightarrow{OC} = 4\mathbf{c}$. Find, in terms of **a** and **c** (a) $\overrightarrow{AC}$ (b) $\overrightarrow{AP}$ where *P* is the mid point of *AC* (c) $\overrightarrow{OP}$ (d) If *X* is the mid point of *CB* find $\overrightarrow{PX}$. (e) What is the geometrical relationship between *PX* and *OC* ?



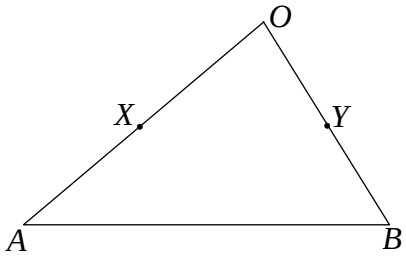
5. In triangle *OAB*, $\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 9\mathbf{b}$. Point *P* is on *AB* such that $AP = \frac{1}{2}PB$. Find, in terms of **a** and **b** (a) $\overrightarrow{AB}$ (b) $\overrightarrow{AP}$ (c) $\overrightarrow{OP}$.
6. *OABC* is a triangle with point *C* half way along *OB*. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$. Find, in terms of **a** and **b**
(a) $\overrightarrow{OB}$ (b) $\overrightarrow{OC}$ (c) $\overrightarrow{AC}$



7. *ABCDEF* is a regular hexagon with centre *O*. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Express in terms of **a** and **b** the vectors $\overrightarrow{OD}$, $\overrightarrow{OE}$, $\overrightarrow{DA}$, $\overrightarrow{BC}$, $\overrightarrow{OC}$ and $\overrightarrow{CF}$.



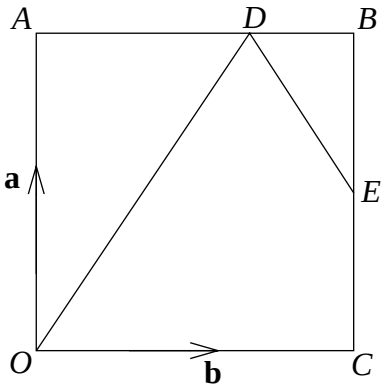
8. (a) If $\overrightarrow{AB} = 3\mathbf{a}$ and $\overrightarrow{CD} = 6\mathbf{a}$ write down the geometrical relationship between $\overrightarrow{AB}$ and $\overrightarrow{CD}$.
(b) What is the geometrical relationship between vectors $4\mathbf{a} + 2\mathbf{b}$ and $6\mathbf{a} + 3\mathbf{b}$?
9. *OAB* is a triangle with point *X* half way along *OA* and *Y* half way along *OB*. If $\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$. Find, in terms of **a** and **b,** (a) $\overrightarrow{AB}$ (b) $\overrightarrow{XY}$ and (c) show that *XY* is parallel to *AB*.



10. In a rectangle *ABCD*, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$. Find, in terms of **a** and **b** the vectors $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

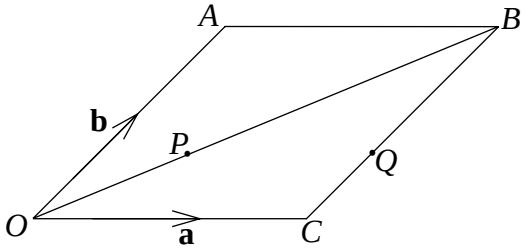
67. Vectors 2

1. *OABC* is a square. $\overrightarrow{OA}$ = **a** and $\overrightarrow{OC}$ = **b**. Point *D* cuts the line *AB* in the ratio 2:1.



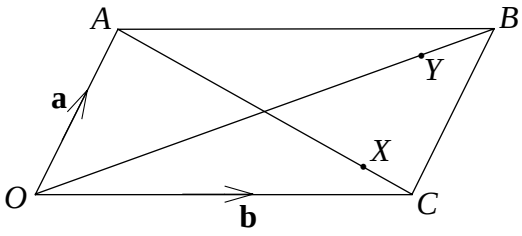
   (a) Find, in terms of **a** and **b** the vector that represents $\overrightarrow{AD}$.
   (b) If point *E* is halfway along *BC*, find the vector that represents $\overrightarrow{DE}$.
   (c) Using your answers to parts a and b show that the vector *OE* is represented by **b** + $\frac{1}{2}$**a**.

2. *OABC* is a rhombus with $\overrightarrow{OA}$ represented by **b** and $\overrightarrow{OC}$ represented by **a**.



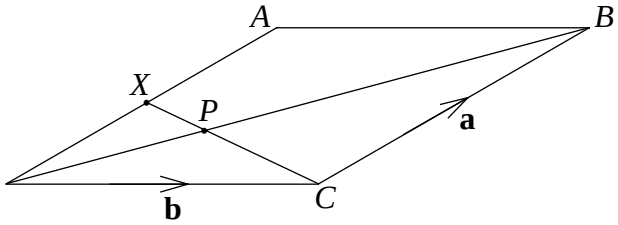
   (a) What is the vector $\overrightarrow{OB}$ represented by?
   (b) What vector represents $\overrightarrow{OP}$ if point *P* divides the line *OB* in the ratio 1:2?
   (c) Point *Q* divides the line *CB* in the ratio 1:2. What vector represents $\overrightarrow{CQ}$?
   (d) Show that the line *PQ* is parallel to the line *AB*.

3. In the parallelogram *OABC*, point *Y* cuts the line *OB* in the ratio 5:1. Point *X* cuts the line *AC* in the ratio 5:1 also. Write down in terms of **a** and **b**



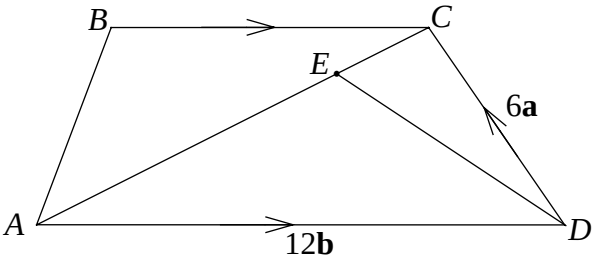
   (a) $\overrightarrow{OB}$ (b) $\overrightarrow{YB}$ (c) $\overrightarrow{AC}$ (d) $\overrightarrow{XC}$ (e) $\overrightarrow{XY}$.
   (f) What is the geometric relationship between lines *XY* and *CB*?

4. *OABC* is a rhombus.
   $\overrightarrow{CB}$ = **a** and $\overrightarrow{OC}$ = **b**.
   Point *X* is halfway along *OA*.
   Point *P* cuts *XC* in the ratio 1:2. Write down, in terms of **a** and **b** vectors for



   (a) $\overrightarrow{OX}$ (b) $\overrightarrow{XC}$ (c) $\overrightarrow{OB}$
   (d) $\overrightarrow{XP}$ (e) $\overrightarrow{OP}$ (f) How far along *OB* is point *P*?

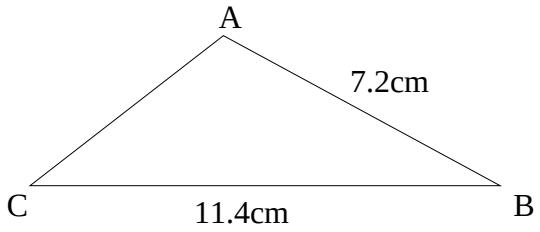
5. *ABCD* is a trapezium with *BC* parallel to *AD* and $\frac{3}{4}$ of its length.
   $\overrightarrow{AD}$ = 12**b** $\overrightarrow{DC}$ = 6**a**.
   Point *E* cuts the line *AC* in the ratio 5:1.
   Find, in terms of **a** and **b**

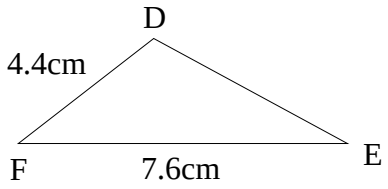


   (a) $\overrightarrow{AC}$ (b) $\overrightarrow{DE}$ (c) $\overrightarrow{AB}$

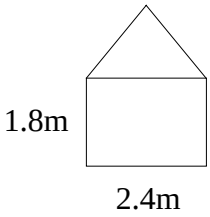
68. Similar Shapes 1

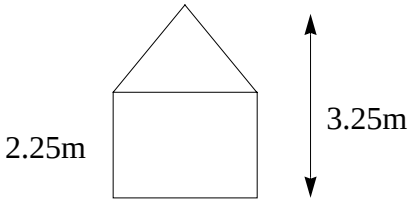
1) The two triangles below, ABC and DEF are similar.
   a) Which angle is equal to angle CAB?
   b) What is the scale factor between triangles ABC and DEF?
   c) Calculate the size of side DE.
   d) Calculate the size of side AC.



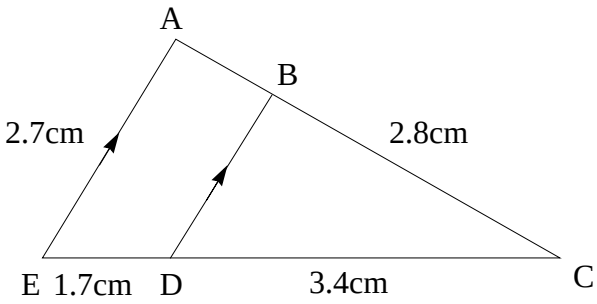


2) The diagrams below show the fronts of two similar garden sheds.
   a) What is the ratio of the lengths of the smaller shed to the larger one?
   b) What is the width of the larger shed?
   c) What is the total height of the smaller shed?

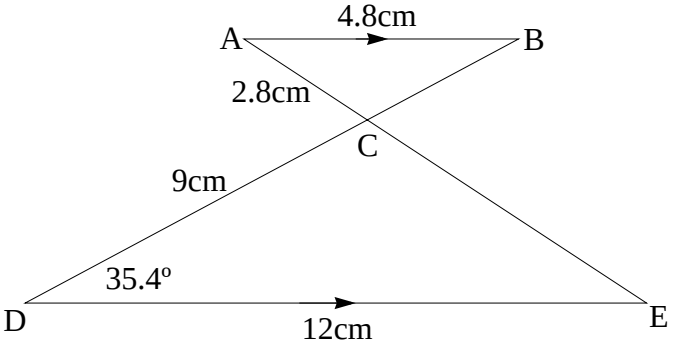




3) In the diagram, AE is parallel to BD.
   a) Which angle is equal to angle EAC?
   b) Calculate the length of BD.
   c) Calculate the length of AB.

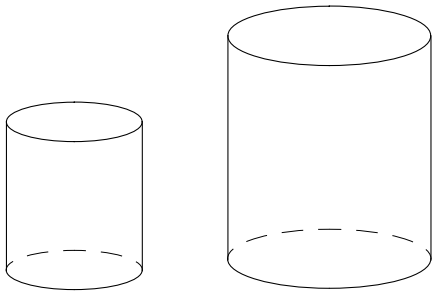


4) In the diagram AB is parallel to DE
   a) Which angle is equal to angle ACB?
   b) What is the ratio of the lengths of triangle ABC to CDE?
   c) Calculate the length of CE.
   d) Calculate the length of BC.

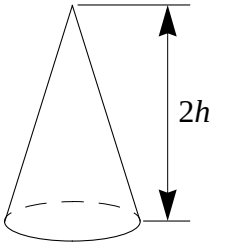


69. Similar Shapes 2

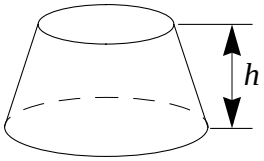
1. Two hollow cylinders are similar in shape. One is 12cm tall and the other is 18cm tall.
   Calculate the ratio of
   a) the areas of their ends
   b) their volumes.
   c) If the smaller one has a volume of 100 cm^3, what is the volume of the larger one?



2. A cone of height 3*h* is cut into two parts to make a smaller cone of height 2*h*.
   a) What is the ratio of the volumes of the two parts?
   b) If the surface area of the top part is 200cm^2, what was the surface area of the original cone?



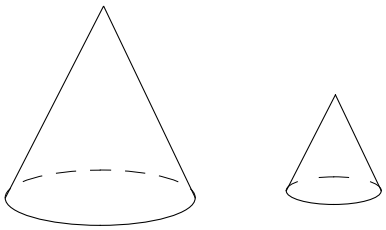
3. Two similar blocks of metal made from the same material have a total surface area of 25cm^2 and 64cm^2. If the smaller one has a mass of 30 grams, what is the mass of the larger one?



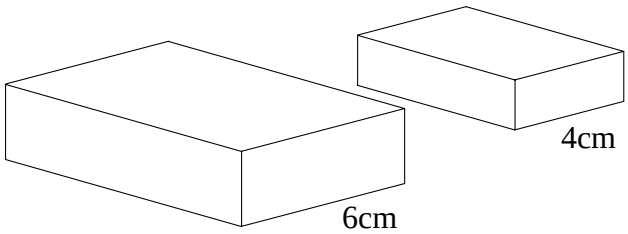
4. A ball has a diameter of 8cm and weighs 200 grams. Calculate the weight of a ball of 10cm diameter made from the same material.

5. A cylindrical can of height 15cm holds one litre of orange juice. What height, to the nearest mm, must a similar can be if it holds 500ml?

6. Two similar cones have heights of 100cm and 50cm. If the volume of the smaller one is 1000cm^3, calculate the volume of the larger one.



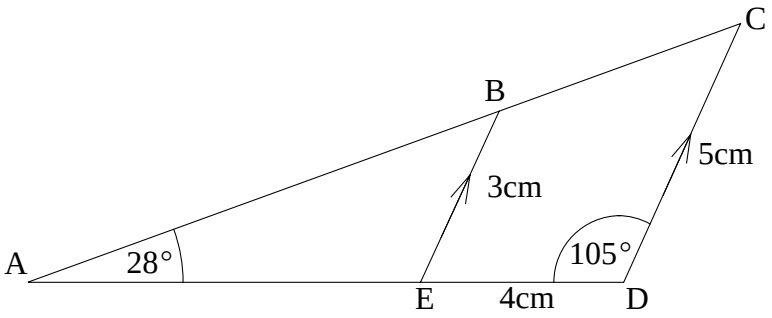
7. Two similar boxes have corresponding sides of 6cm and 4cm.
   a) If the surface area of the smaller box is 50cm^2, what is the surface area of the larger box?
   b) If the volume of the larger box is 50cm^3, what is the volume of the smaller box?



8. Two similar cornflakes packets have widths of 25cm and 20cm. Cornflakes are sold in packets of 750g, 500g and 350g. If the 25cm packet holds 750g, how much does the 20cm packet hold?

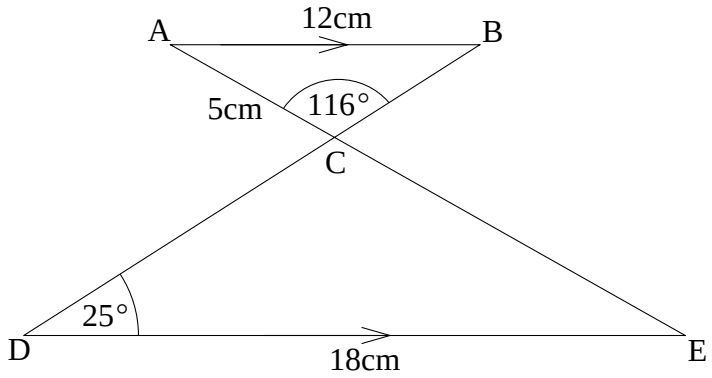
70. Similarity

1. In the triangle ABCDE, BE = 3cm, CD = 5cm , angle CDE = 105° and angle BAE = 28° . Line BE is parallel to line CD.



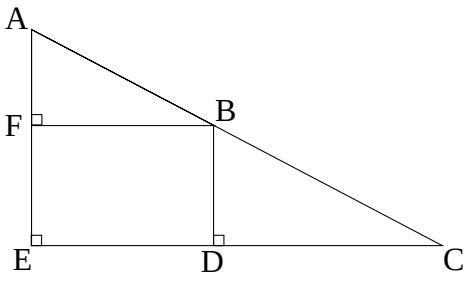
   a) (i) Calculate the size of angle DEB
      (ii) Explain your answer.
   b) Calculate the size of AE.

2. In the diagram, AB = 12cm, DE = 18cm, AC = 5cm, angle ACB = 116° and angle CDE = 25° . Line AB is parallel to line DE.



   a) (i) Calculate the size of angle DEC.
      (ii) Explain how you get your answer.
   b) Calculate the length of CE

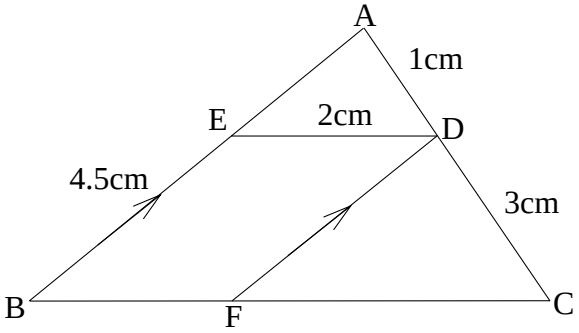
3. In the triangle, AC = 20cm, DC = 10cm and FB = 6cm.



   a) Calculate the length of AB.

   b) If AF = 4.5 cm, calculate the length of FE.

4. In the triangle shown below, AD = 1cm, DC = 3cm, ED = 2cm and BE = 4.5cm. Line AB is parallel to line DF.

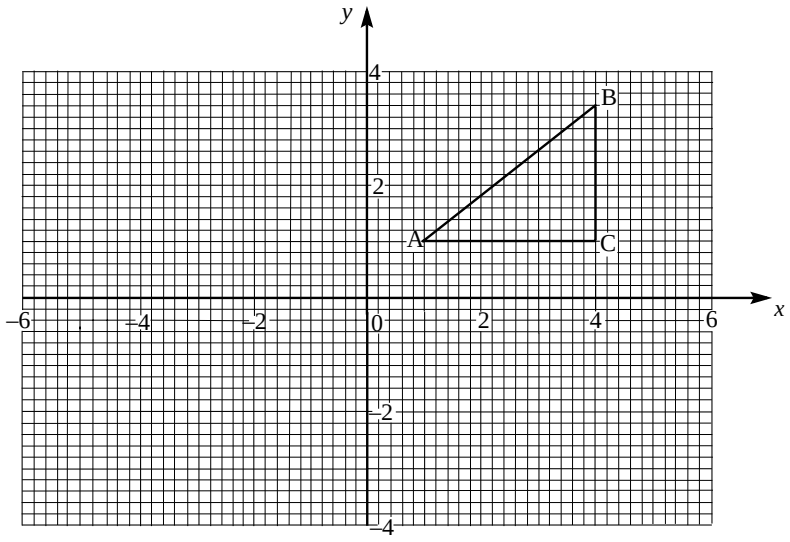


   a) Calculate the lengths of FC and AE.

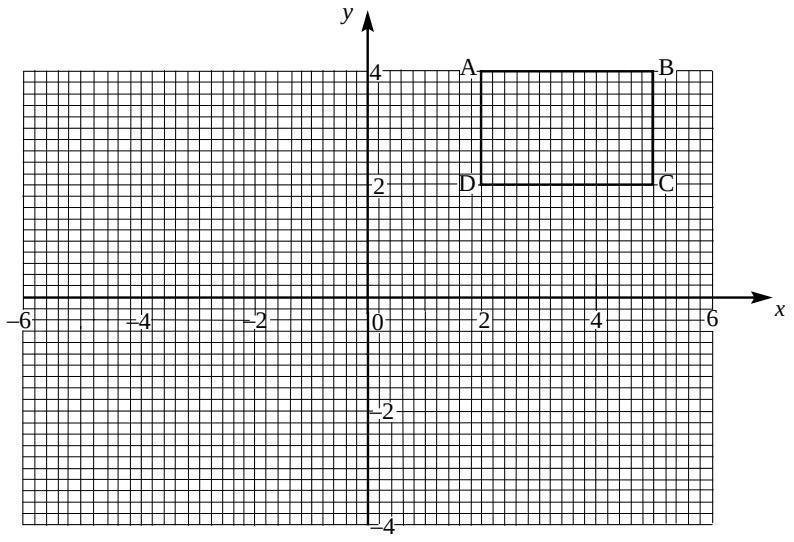
   b) What is the ratio of the areas of triangle AED to that of triangle ABC.

71. Reflections, Rotations and Translations 1

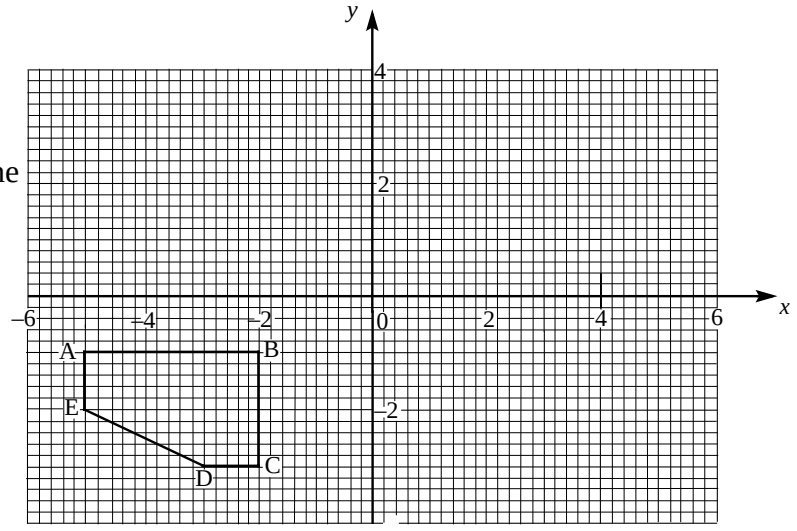
1) The diagram shows a triangle A,B,C. Copy this diagram and show the reflections;
   a) A',B',C' about the *x* axis
   b) A", B", C" about the *y* axis.



2) The diagram shows a rectangle A,B,C,D. Copy this diagram and show the reflections;
   a) A',B',C',D' about the line $x = 1$
   b) A",B",C",D" about the line $y = 1$.

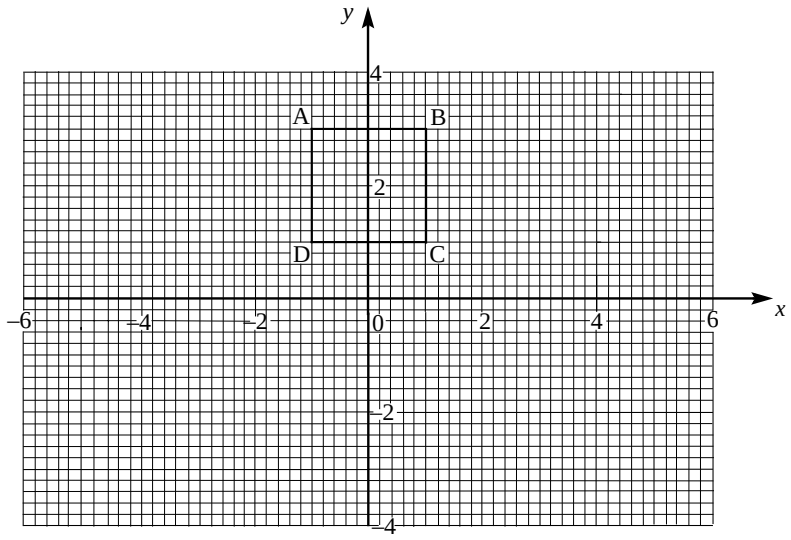


3) The diagram shows a shape A,B,C,D,E. Copy this diagram and show;
   a) the reflection A_1,B_1,C_1,D_1,E_1 about the line $x = -1$
   b) the reflection A_2,B_2,C_2,D_2,E_2 about the line $y = 0.5$
   c) The translation $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ to A_3,B_3,C_3,D_3,E_3.

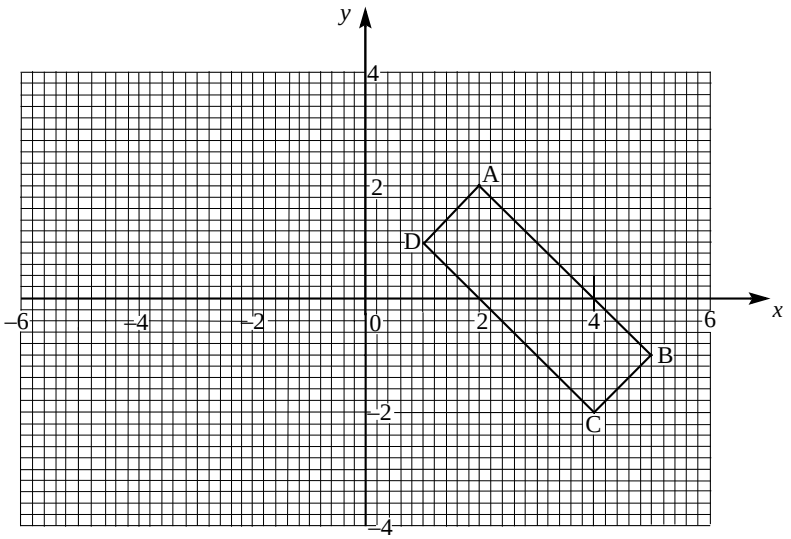


72. Reflections, Rotations and Translations 2

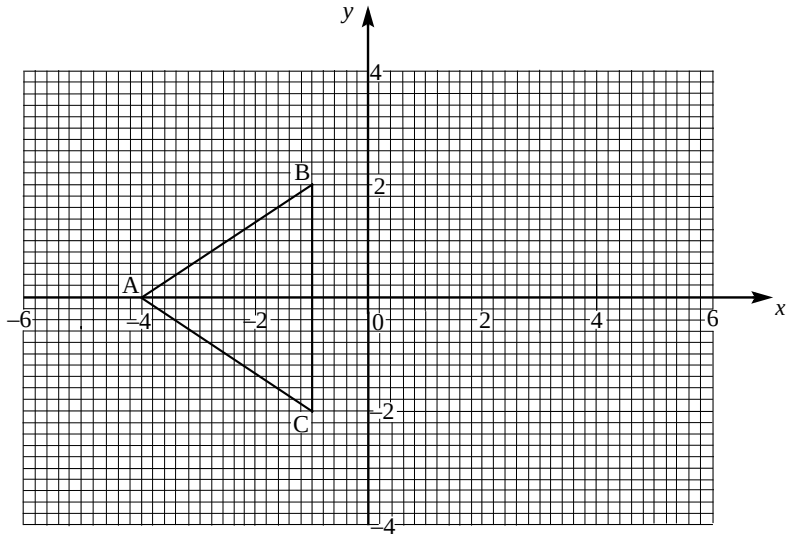
1) The diagram shows a square A, B, C, D. Copy this diagram and show the reflections
   a) A', B', C', D' about the line $y = x$
   b) A'', B'', C'', D'' about the line $y = -x$



2) The diagram shows a rectangle A, B, C, D. Copy this diagram and show the reflections
   a) A', B', C', D' about the x axis
   b) A'', B'', C'', D'' about the y axis

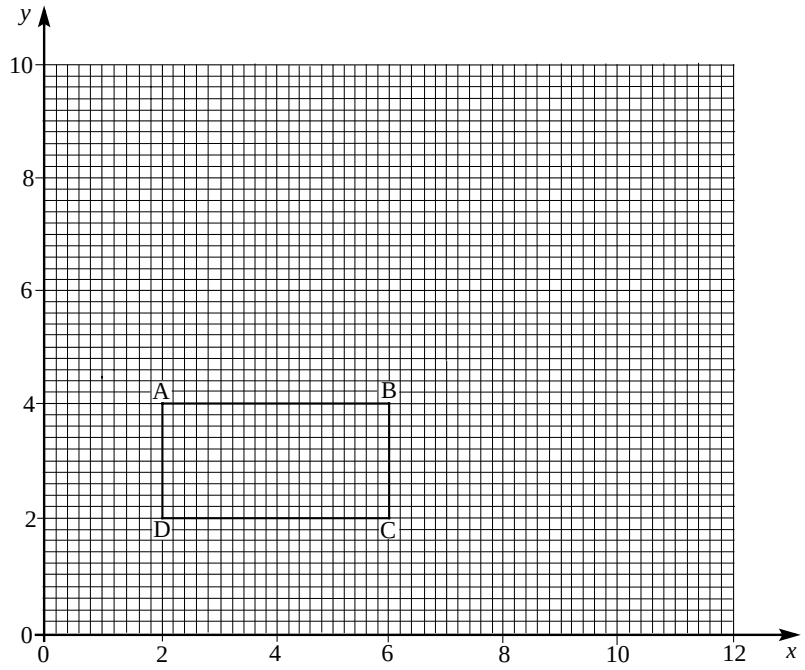


3) The diagram shows a triangle A, B, C. Copy this diagram and show the reflections
   a) A', B', C' about the line $y = x$
   b) A'', B'', C'' about the line $y = -x$

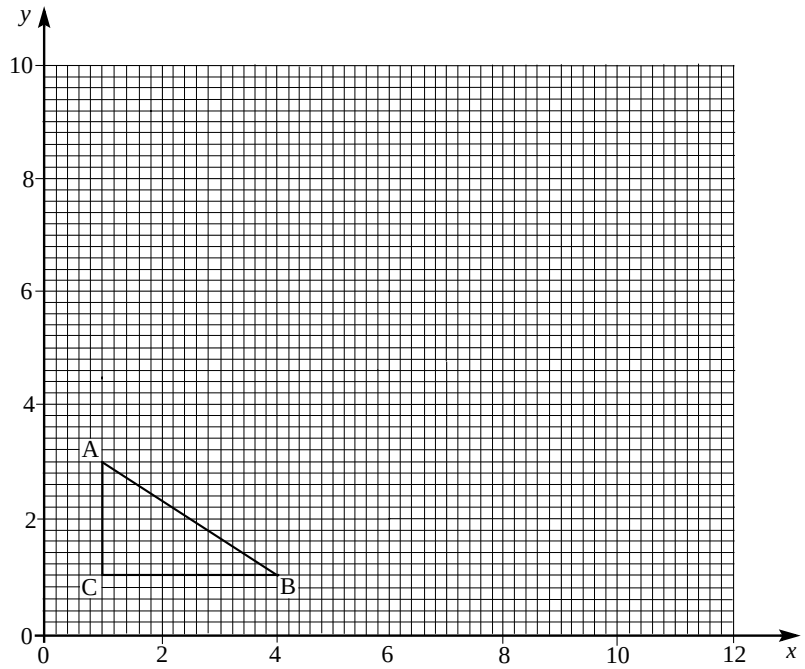


73. Enlargements 1

1) The diagram shows a rectangle A, B, C, D.
Copy this diagram and enlarge it by a scale factor of 2 through the point (0,0)

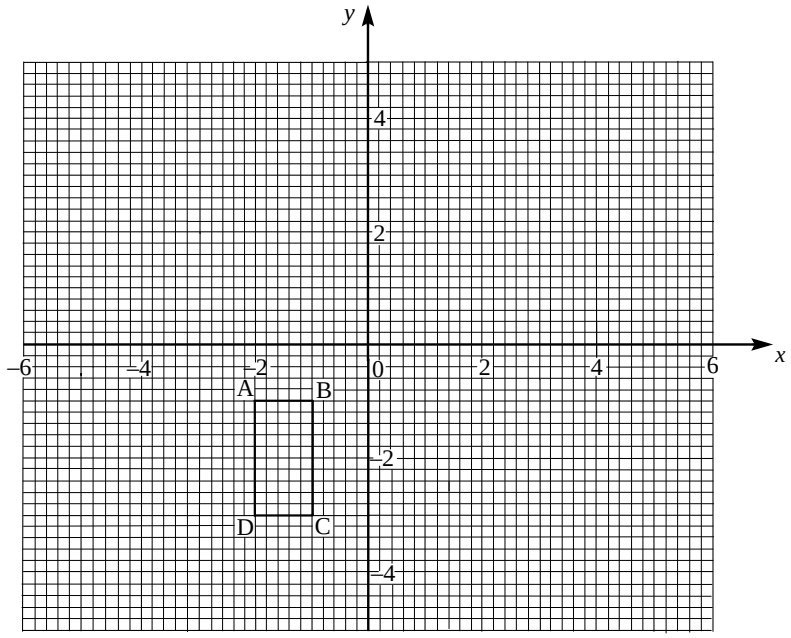


2) The diagram shows a triangle A, B, C.
Copy this diagram and enlarge it by a scale factor of 3 through the point (0,0

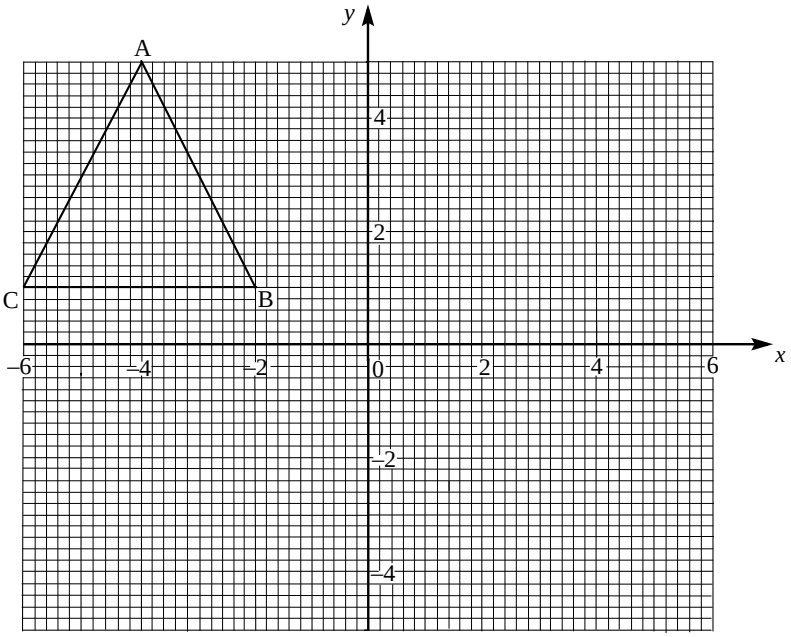


74. Enlargements 2

1) The diagram shows a rectangle A, B, C, D.
Copy this diagram and enlarge it by a scale factor of $2\frac{1}{2}$ through the point (–4, –3)

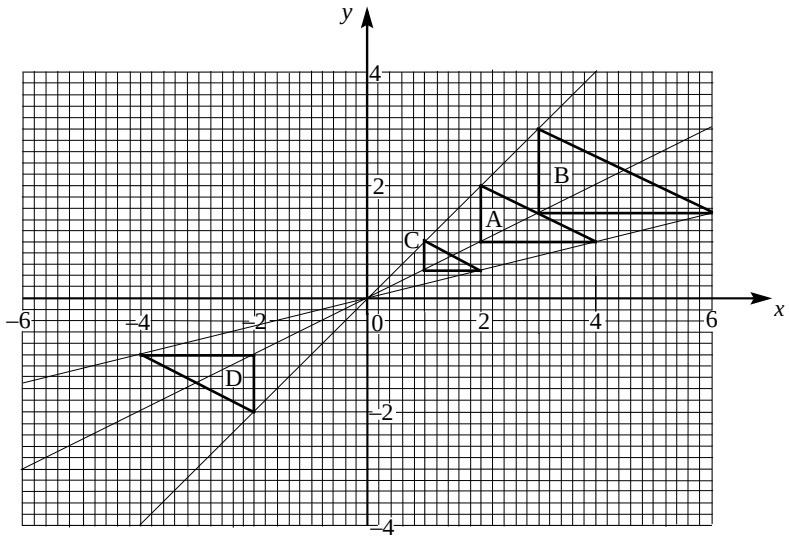


2) The diagram shows a triangle A, B, C.
Copy this diagram and 'enlarge' it by a scale factor of $\frac{1}{2}$ through the point (2,–3)

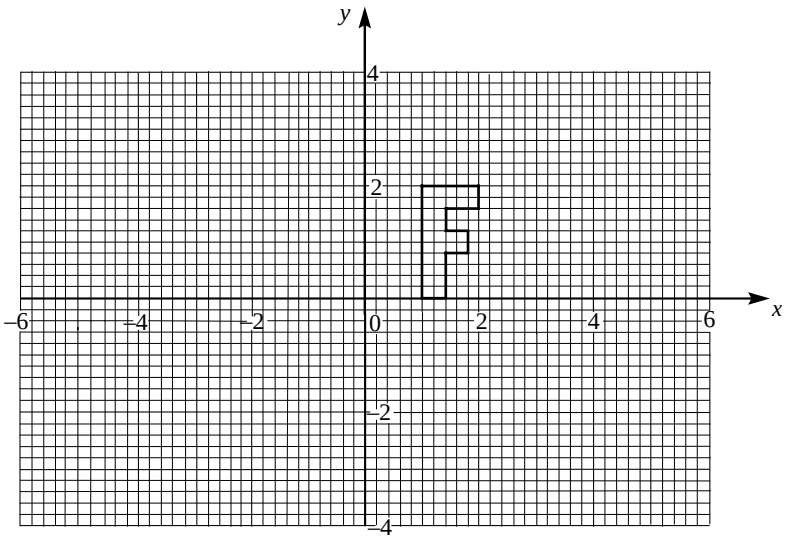


75. Enlargement 3

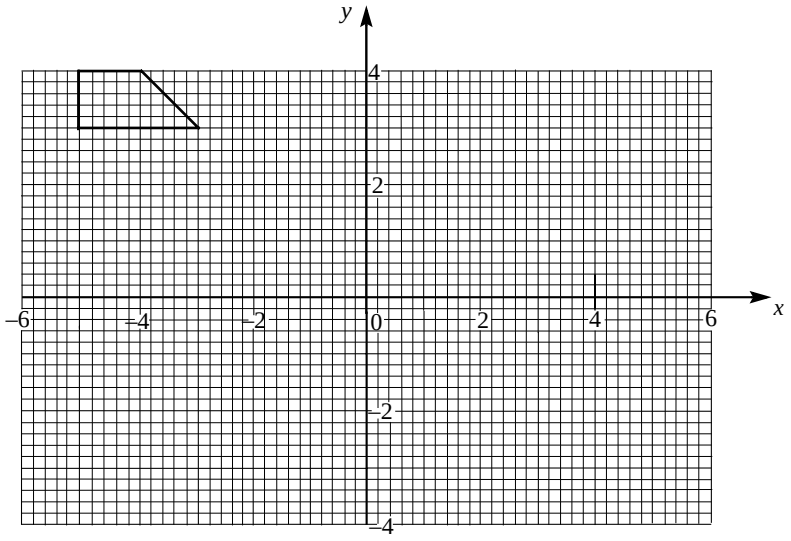
1) Shape A is enlarged three times to make shapes B, C and D. What are their enlargement factors?



2) Enlarge this letter F with a scale factor of –2 through the origin

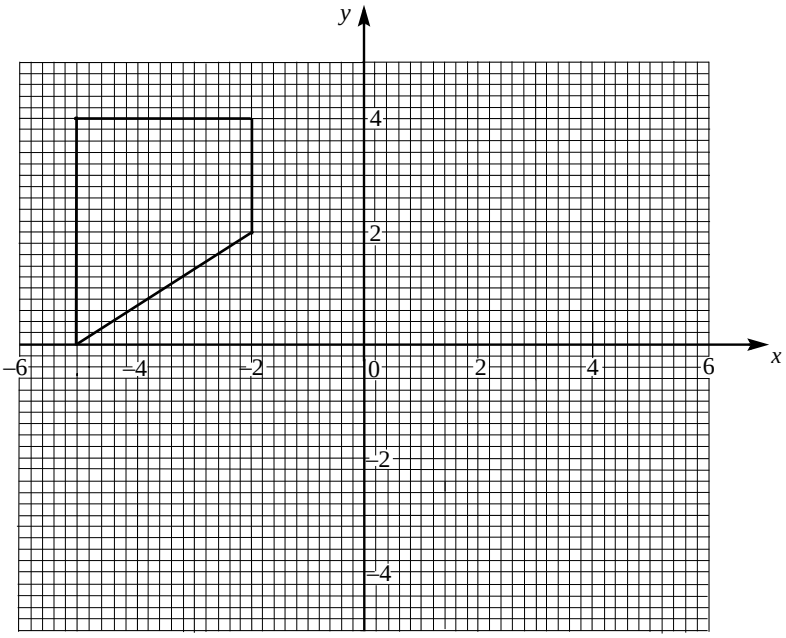


3) Enlarge this shape with a scale factor of –3 through the point (–3,2)

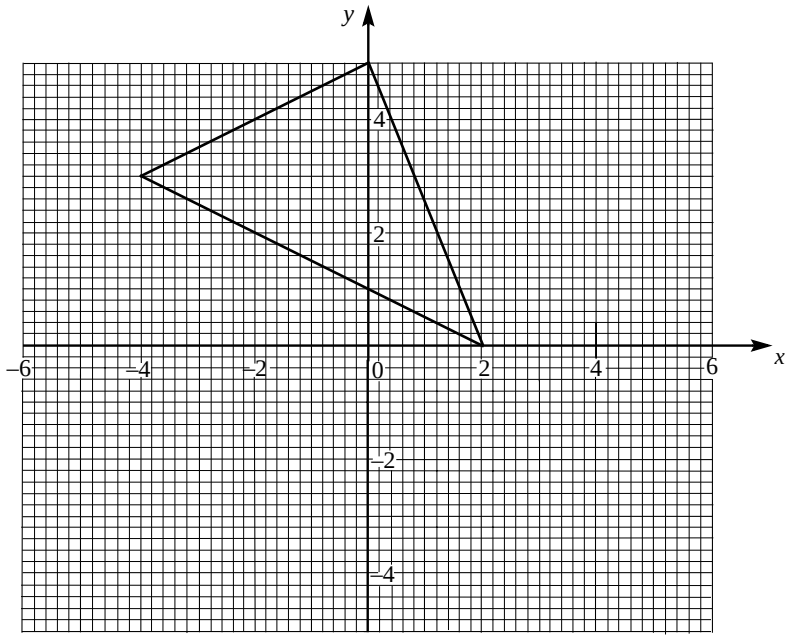


76. Enlargement 4

1) Enlarge this shape with a scale factor of $-\frac{1}{2}$ through the origin.



2) Enlarge this triangle with a scale factor of $-\frac{1}{2}$ through the point (–2,0)

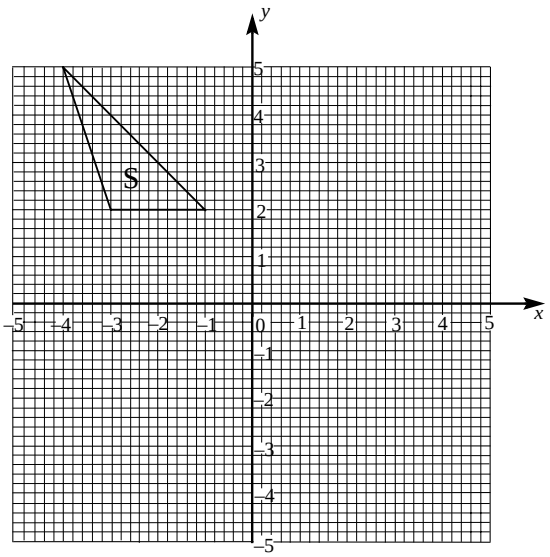


77. Transformations 1

1. a) The triangle S is reflected across the line $y = 0$ to triangle T. Copy the diagram and show triangle T. What are the co-ordinates of triangle T?

   b) Triangle T is reflected across the line $x = 0$ to triangle U. Draw triangle U and write down its co-ordinates.

   c) What is the single transformation that transforms triangle U onto triangle S?



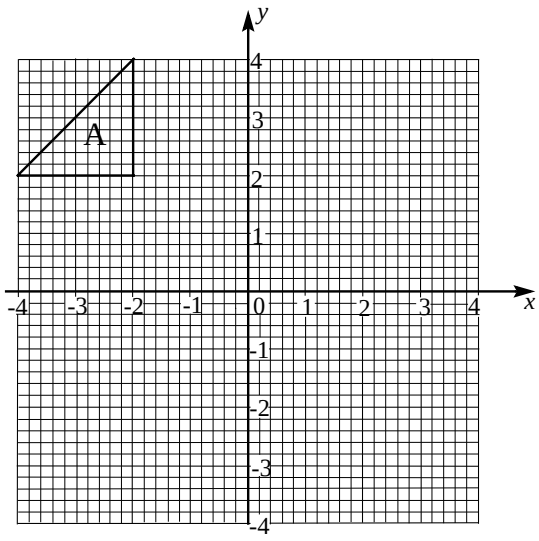
2. Triangle A has vertices at (–2,2), (–4,2) and (–2,4). Copy the diagram and draw each of the following transformations.
   In each case write down their co-ordinates.

   a) A rotation of triangle A of $90°$ clockwise about the origin. Label this B.

   b) A reflection of A about the line $y = x$. Label this C.

   c) An enlargement of A with a scale factor of $\frac{1}{2}$ through the origin (0,0). Label this D.

   d) What single transformation will transform D back into A?

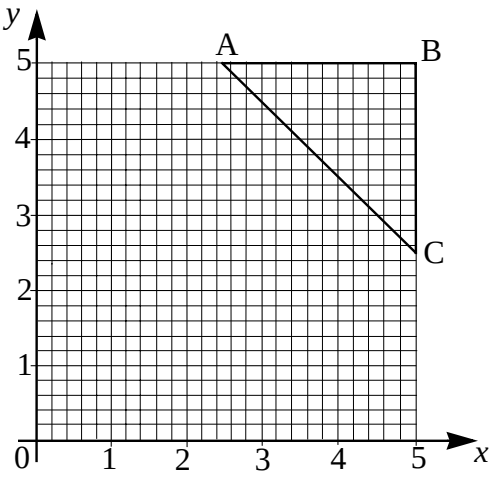


3. Triangle A, B, C is transformed into triangle A_1,B_1,C_1 by an enlargement of scale factor $\frac{2}{5}$ through the origin (0,0).

   a) Draw triangles A, B, C and A_1,B_1,C_1.

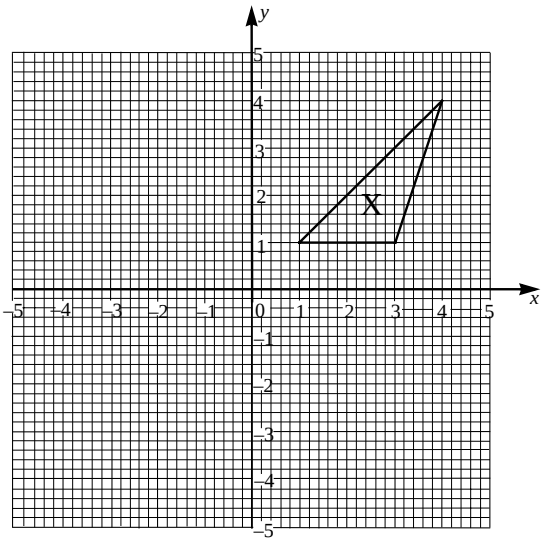
   b) Write down the co-ordinates of triangle A_1,B_1,C_1.

   c) A_1,B_1,C_1 is transformed onto A, B, C. What single transformation will do this?

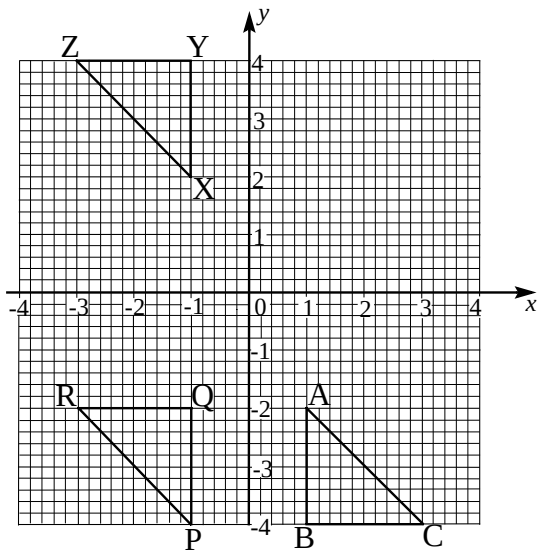


78. Transformations 2

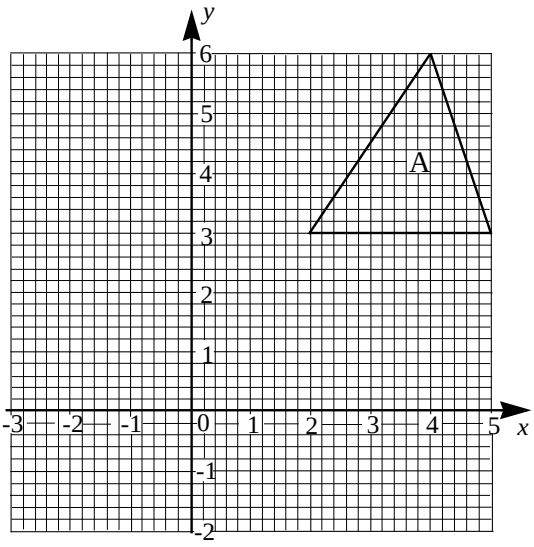
1. Triangle X is rotated 180° anticlockwise about the origin to triangle Y.
   a) Draw triangles X and Y and write down the co-ordinates of Y.
   b) Triangle Y is translated by the vector $\begin{pmatrix}0\\4\end{pmatrix}$ to triangle Z. Draw triangle Z and write down its co-ordinates.
   c) What single transformation will transform triangle X onto Z?



2. a) Triangle ABC is transformed onto triangle XYZ. What single transformation will do this?
   b) Triangle XYZ is transformed onto triangle PQR. What single transformation will do this?
   c) If triangle PQR is rotated through 180° about the point (–1,0) to triangle STU, what are the co-ordinates of the new shape?
   d) What single transformation will transform triangle XYZ onto triangle STU?

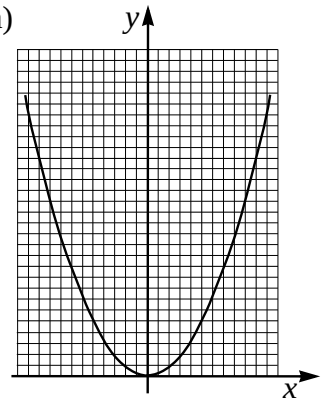


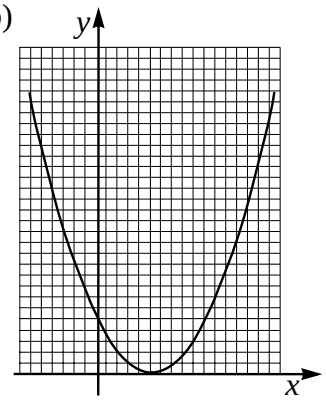
3. a) Triangle A is reflected about the line $y = 2$ to triangle B. Draw triangles A and B and write down the co-ordinates of triangle B.
   b) Triangle B is rotated 90° clockwise about the point (1,2) onto triangle C. Draw triangle C and write down its co-ordinates.
   c) What single transformation will transform triangle A onto triangle C?

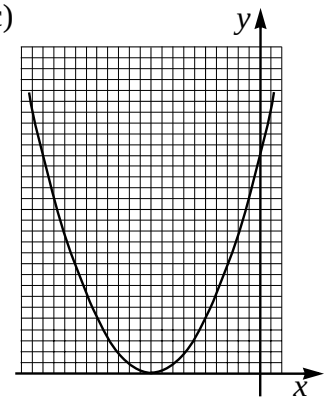


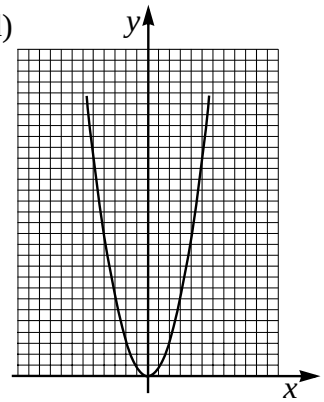
79. Transformations of Graphs

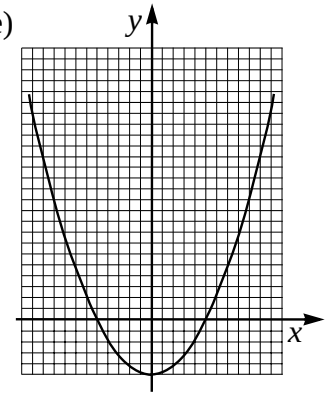
1. Diagram 'a' below shows a sketch of the graph $y = x^2$. The other graphs show (i) $y = x^2 + 1$, (ii) $y = x^2 - 1$, (iii) $y = (x-1)^2$, (iv) $y = (2x)^2$, and (v) $y = (x+2)^2$ but not in that order. Match the diagram to the equation.

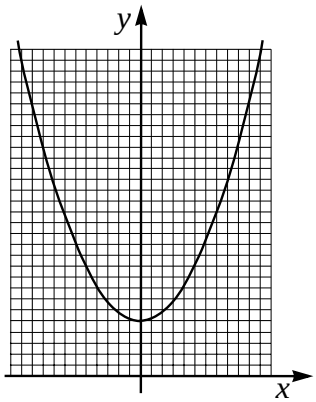
a) 

b) 

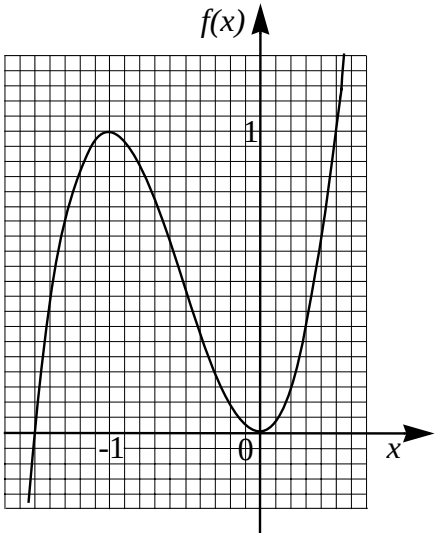
c) 

d) 

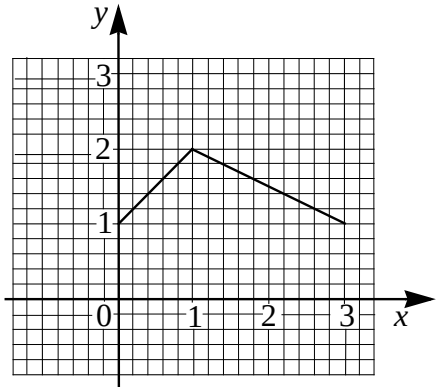
e) 

f) 

2. Explain clearly each of the transformations in question 1.
3. The diagram on the right shows the graph of the function $f(x) = 2x^3 + 3x^2$.
   a) If this graph is reflected in the x axis what will its equation be?
   b) Sketch the graph.
   c) If $f(x)$ is transformed into $f(-x)$ write down its equation and explain the transformation.



4. The function $f(x)$ is defined for $0 < x < 3$ in the diagram below. Sketch the functions
   a) $y = f(x-2)$ and b) $y = f(2x)$



80. Matrix Transformations

1. By first drawing triangle A(1,1), B(3,1), C(3,4) use each of the following matrices to transform it. State the transformation each represents.

   a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ c) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

2. A transformation is given by the matrix $\mathbf{P} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$. Find $\mathbf{P}^{-1}$, the inverse of **P** and use it to find the co-ordinates of the point whose image is (1,3).

3. A' is the image of triangle A after it has been reflected about the *y* axis. A" is the image of the triangle A' after it has been rotated $180°$ about the origin (0,0).
   a) If triangle A has co-ordinates (1,1), (2,1), and (1,3) draw triangle A and the images A' and A".
   b) Find the 2×2 matrix **M** associated with the transformation A to A'.
   c) Find the 2×2 matrix **N** associated with the transformation A' to A".
   d) Calculate the matrix product **NM** and state the single transformation which is defined by this matrix.

4. Triangle T has co-ordinates (1,1), (3,2), and (1,2). Triangle T' is the image of triangle T when rotated through $180°$ about the origin.
   a) What is the matrix **M** associated with the transformation of T to T'?

   Triangle T' is further transformed to T" by the matrix $\mathbf{N} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
   b) What transformation does matrix **N** perform?
   c) State the single transformation which is defined by the matrix product **NM** and write down this matrix.

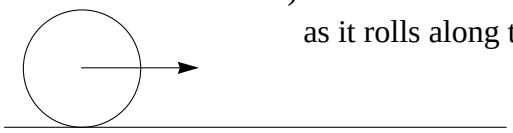
5. Triangle A(1,1), B(2,2), C(1,4) is transformed onto triangle $A_1B_1C_1$ by the matrix **M**

   where $\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

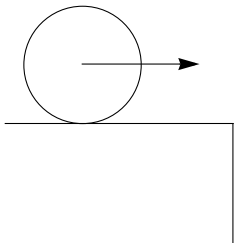
   Triangle $A_1B_1C_1$ is transformed by the matrix **N** where $\mathbf{N} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ onto the triangle $A_1B_1C_2$.
   a) Find the co-ordinates of A_1, B_1, C_1 and C_2 and draw the three triangles.
   b) What single transformation will transform triangle ABC onto triangle $A_1B_1C_2$?
   c) By calculating the matrix **W** = **NM** show that this matrix represents the transformation found in part 'b'.
   d) Calculate $\mathbf{W}^{-1}$, the inverse of **W**. Use it to find the co-ordinates of the point whose image is (3,–5).

81. Loci 1

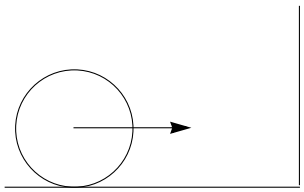
1) Plot the centre of this circle, with a diameter of 4cm, as it rolls along the line to the right.



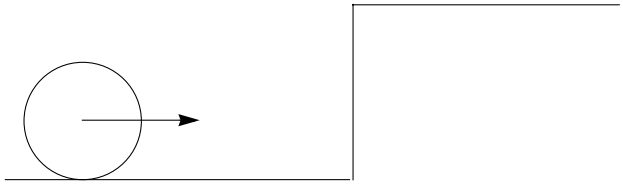
2) Plot the locus of the centre of the circle, diameter 4cm, as it rolls around the outside corner.



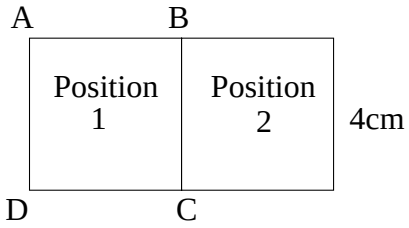
3) Plot the locus of the centre of the circle, diameter 4cm, as it rolls around the inside corner.



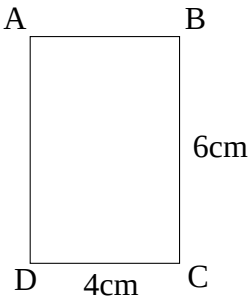
4) Plot the locus of the centre of the circle, diameter 4cm, as it rolls around the two corners.



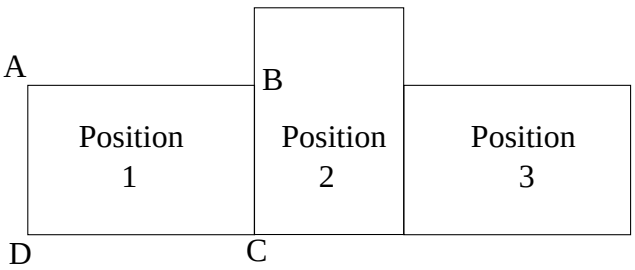
5) A box, 4cm square, is turned about corner C until it moves from position 1 to position 2. Plot the locus of corner D.



6) Plot the path of corner D as the box is pushed over onto its side CB.



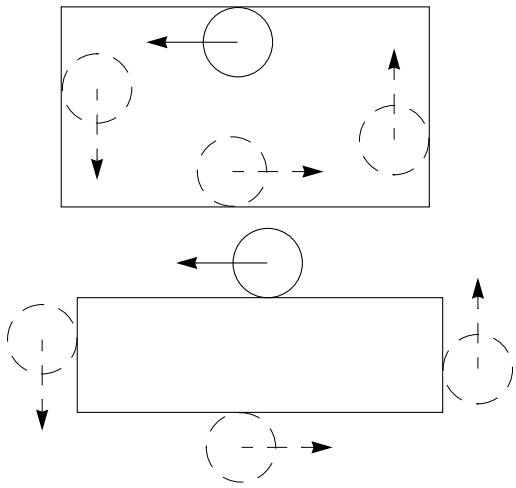
7) Box ABCD, dimensions 3cm by 5cm, is rolled over from position 1 to position 2 then position 3.
a) Which side is now on the bottom?
b) Plot the path of corner C.



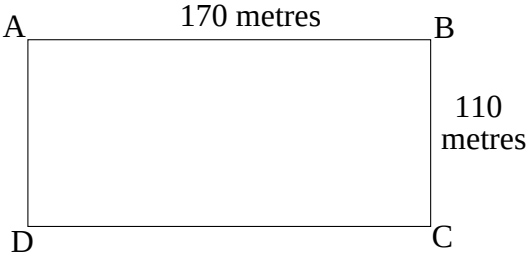
82. Loci 2

1. A wheel of diameter 3cm rotates around the inside of a rectangle measuring 8cm by 11cm. Draw accurately the locus of the centre of the wheel.

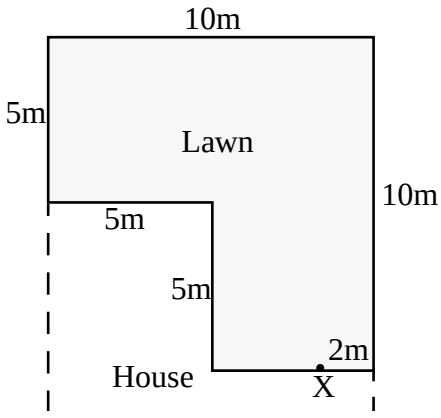
2. A wheel of diameter 3cm rotates around the outside of a rectangle measuring 10cm by 4cm. Draw the locus of the centre of the wheel.



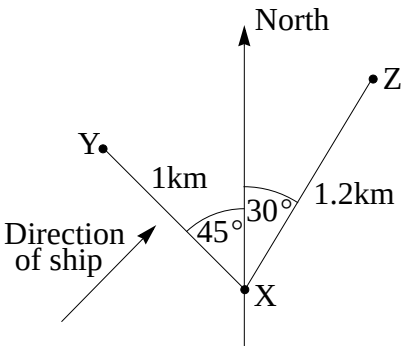
3. A rectangular field ABCD is to have a pipe line laid underneath it. The pipe enters the field through the side AD, is parallel to the side AB and 40 metres from it. After some distance its route turns at a point X towards corner C. Between X and C it is always equidistant from the sides BC and CD. It finally leaves the field by the corner C.
Copy the plan of the field, using a scale of 1cm to represent 20 metres. On the plan clearly indicate the path of the pipe line. Measure and record the distance BX.



4. The diagram shows the plan of a garden lawn. Draw an accurate diagram using the scale of 1cm to represent 1 metre. Point X represents an electrical socket on the side of the house. An electric lawn mower has a cable attached to it allowing it to travel up to 9 metres from the socket. Show clearly on the diagram the area of the lawn which can be mown.



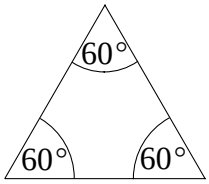
5. A ship sails between two rocks X and Y. The ship is at all times equidistant from both rocks. At point P, when it is the same distance from rocks Y and Z, it alters course, keeping to a path which is equidistant from Y and Z. Draw an accurate diagram to a scale of 5cm to represent 1km to show the route of the ship. What is the distance XP?



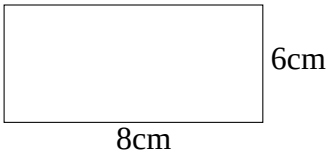
83. Constructions

In the questions below, use only a ruler and a compass, not a protractor.

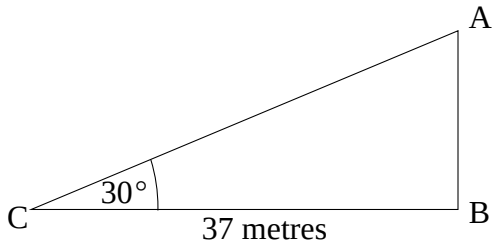
1. Construct an angle of $60°$ and bisect it to make an angle of $30°$.
2. Construct an angle of $90°$ and bisect it to make an angle of $45°$.
3. Construct this equilateral triangle, with sides of 5cm.



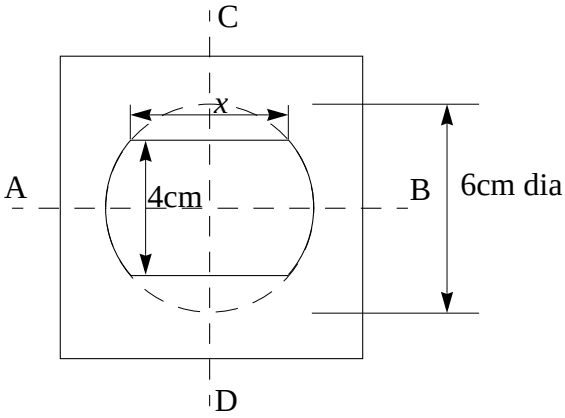
4. Construct this rectangle

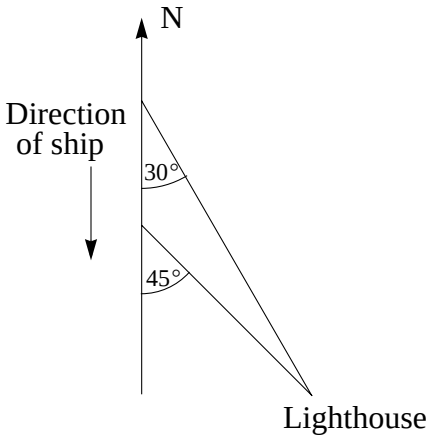


5. Draw a line 12cm long and bisect it by construction.
6. Joanna wants to measure the height of a tower. She measures out a distance from the bottom of the tower (BC) until the angle between the ground and the top of the tower is $30°$. She measures BC to be 37 metres. By making a scale drawing and letting 1cm represent 5 metres, estimate the height of the tower.



7. The diagram shows the sketch of an 8cm square metal plate with a hole at the centre.
   The shape is symmetric about the lines AB and CD.
   Make an accurate drawing of the hole.
   What is the length of dimension *x*?





8. A ship is sailing due south when it detects a light house at an angle of $30°$ from its direction of travel. It carries on for a further 1200 metres and now finds that the lighthouse is at an angle of $45°$ to its direction of travel. Draw an accurate diagram to a scale of 1cm to represent 200 metres. By measuring on your diagram, estimate the closest distance the ship will be to the lighthouse if it continues on this course.

84. Ratios and Scales

1. Divide £450 between Amy, Ben and Charles in the ratio 4:5:6.
2. An amount of money is divided between three people in the ratio 5:6:7. If the first gets £125, what do the others get?
3. £560 is shared among three people. The second gets twice as much as the first who receives twice as much as the third. a) Into what ratio is it divided? b) How much do they each get?
4. Megan makes 1.5 litres of lemon squash. The instructions say that the juice should be mixed with water in the ratio three parts juice to seven parts water. How much juice is needed?
5. An architect draws the plans of a house to a scale of 1:20. Complete the table below.

	Dimensions on drawing	Actual dimensions
Width of house	cm	10 metres
Height of door	11cm	m
Area of dining room floor	cm^2	$20m^2$
Angle of staircase to floor	°	40°

6. The plans of a small housing estate are drawn to a scale of 1:500.
   Copy and complete the following table.

	Dimensions on drawing	Actual dimensions
Length of street	cm	0.5km
Angle between two streets	°	80°
Area of school field	cm^2	$3000m^2$
Width of street	2.5cm	m

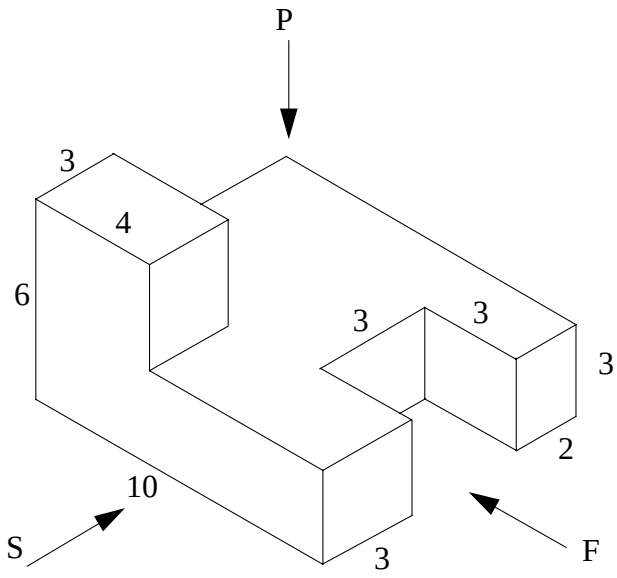
7. Mr. Jones makes a doll's house for his grand-daughter. He makes it a model of his own house to a scale of 1:8. Copy and complete the following table.

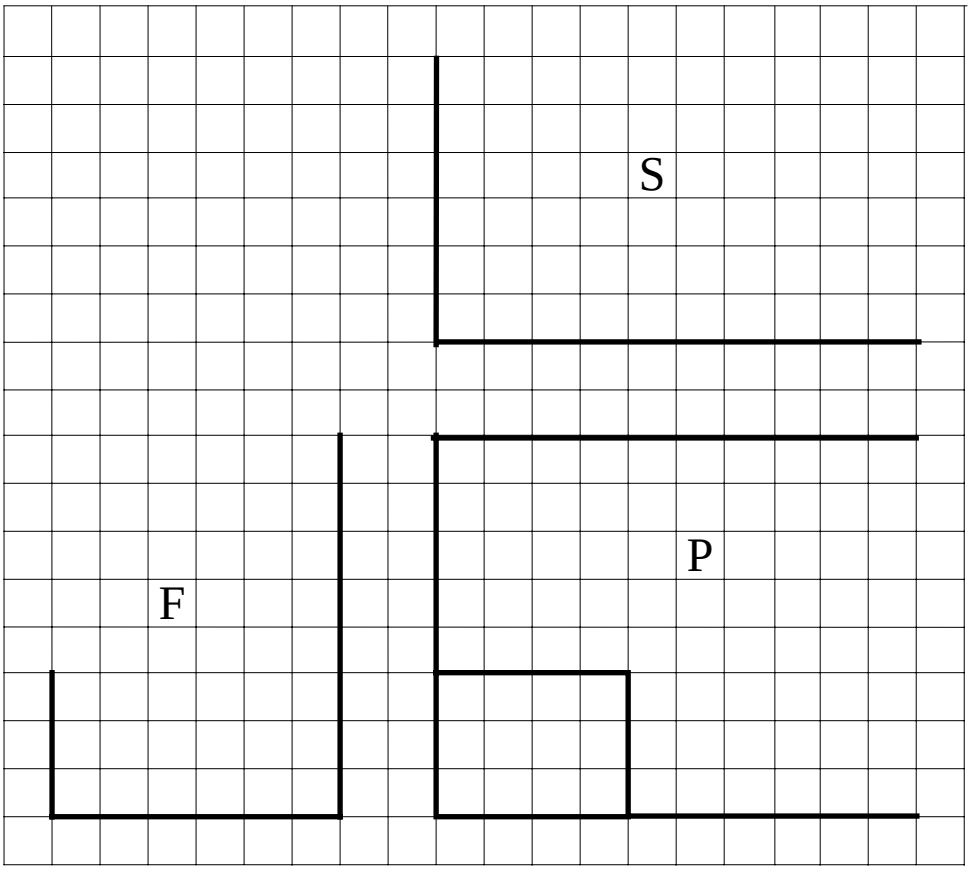
	Dimensions on doll's house	Actual dimensions
Total height	cm	12m
Area of hall floor	$1200cm^2$	m^2
Volume of dining room	cm^3	$48m^3$

   Mr Jones sees some model tables to put into the house. There are three designs. They are 5cm, 10cm and 15cm high. Which one should he buy?
8. A model aeroplane is made to the scale of 1:40.
   a) If the wingspan is 20 metres, what will it be on the model?
   b) If the wing area is 500 square centimetres on the model, what is its actual area?
   c) The volume of the fuselage is 240 cubic metres. What is its volume on the model?

85. Plans and Elevations 1

The diagram below shows an engineering component. In the grid below it three views of the component have been started, two elevations in directions S(side) and F(front), and a plan P. Finish off these three elevations.

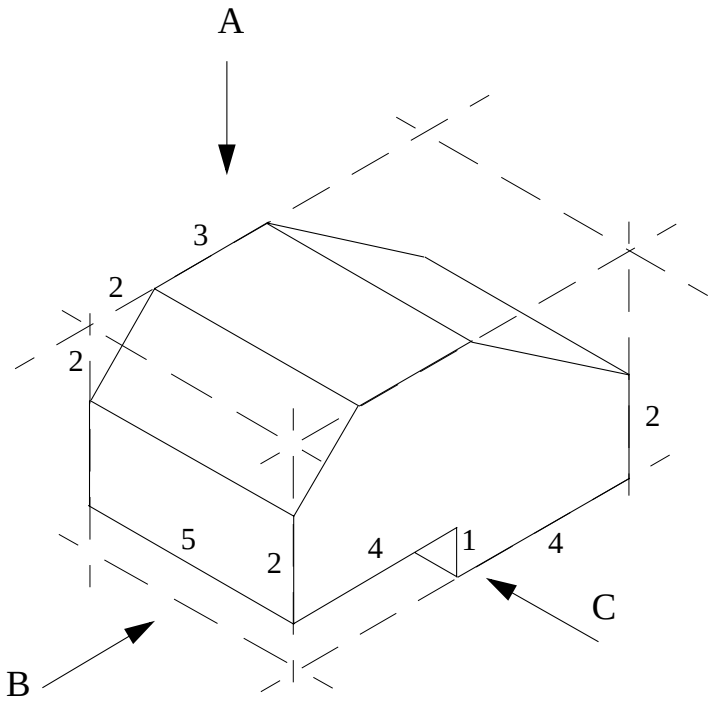




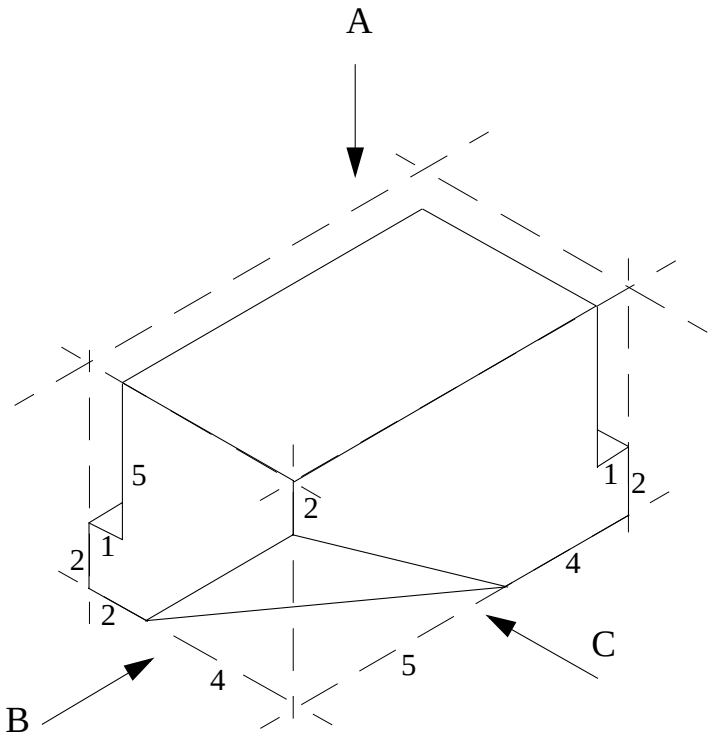
86. Plans and Elevations 2

Below are shown two engineering components. Draw diagrams of each shape when viewed from the directions A, B and C.

1)



2)



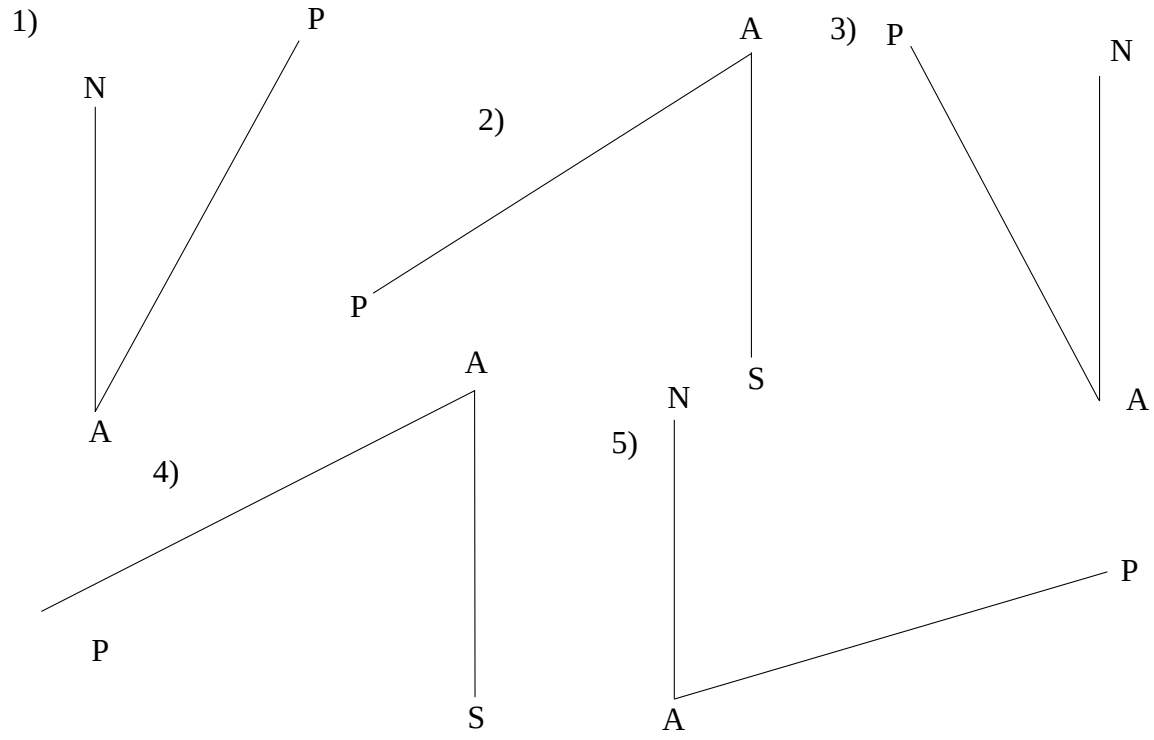
87. Bearings 1

Exercise 1

Draw diagrams to show the following bearings.
1) A is 030˚ from B
2) C is 315˚ from D
3) G is 153˚ from H
4) J is 232˚ from K
5) L is 317˚ from M
6) P is 208˚ from Q
7) R is 098˚ from T
8) U is 076˚ from V

Exercise 2

By measuring these angles, write down the bearing of point P from point A in each case.



Exercise 3

1) If the bearing of A from 324˚ what is the bearing of B from A?
2) If the bearing of C from D is 234˚ what is the bearing of D from C?
3) A ship sails from port P on a bearing of 035˚ for 6km until it reaches point X. It then changes course onto a bearing of 132˚ for a distance of 8km until it reaches point Y. Draw the ship's path accurately using a scale of 1cm to 1km. What is the bearing and distance of point Y from the port P?
4) An aeroplane flies from airport A on a bearing of 202˚ for 75km until it reaches point B. It then changes course onto a bearing of 222˚ for a distance of 80km until it reaches point C. Draw the aircraft's path accurately using a scale of 1cm to 10km. What is the bearing and distance of point C from the airport A?

88. Bearings 2

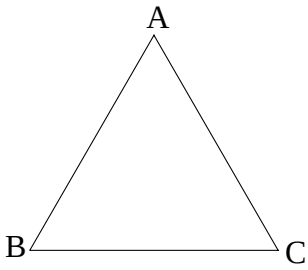
1. An aeroplane sets off on a bearing of N 28° E (028°) but after some time has to turn back to the airport it came from. On what bearing must it travel?
2. A ship sets sail on a bearing of S 17° E (163°). It then turns through an angle of 90° anticlockwise. What is its new bearing?
3. ABC is an equilateral triangle with line BC pointing due east.

   Write down the following bearings:

   a) A from B

   b) C from A

   c) A from C

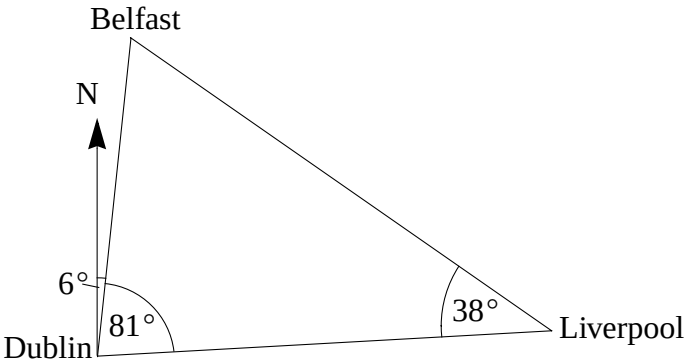


4. The diagram shows the approximate relative positions of Belfast, Dublin and Liverpool. Calculate the bearings of:

   a) Belfast from Dublin

   b) Belfast from Liverpool

   c) Liverpool from Belfast.



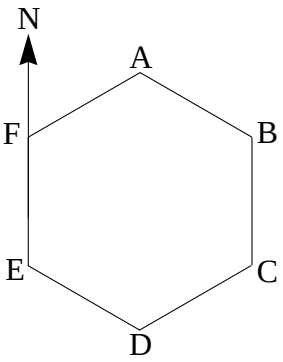
5. The diagram shows a regular hexagon ABCDEF with one side pointing due north. Calculate the bearings of:

   a) A from F

   b) C from B

   c) D from C

   d) E from B



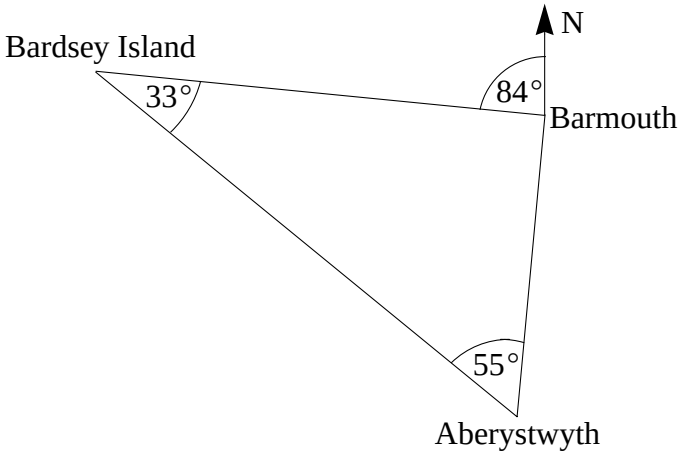
6. The diagram shows the approximate relative positions of Bardsey Island, Barmouth and Aberystwyth.

   Calculate the bearings of:

   a) Bardsey Island from Aberystwyth

   b) Barmouth from Aberystwyth

   c) Bardsey Island from Barmouth.



89. Degree of Accuracy 1

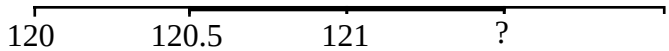
Exercise 1.

1) Round off each of the following numbers to the accuracy stated.
   a) 4321 to the nearest thousand. b) 5226 to the nearest hundred.
   c) 457 to the nearest ten. d) 784 to the nearest hundred.
   e) 14640 to the nearest thousand. f) 23457 to the nearest thousand.
2) Round off the number 23297.
   a) to the nearest ten. b) to the nearest hundred.
   c) to the nearest thousand. d) to the nearest ten thousand.
3) Which of these numbers can be rounded off to 25000?
   a) 25432 b) 24953 c) 24436 d) 25537
   e) 25500 f) 25499 g) 24500 h) 24499
4) State the limits between which the following whole numbers lie. Each has been rounded off in the way shown in the brackets.
   a) 1300 (to the nearest 100) b) 2500 (to the nearest 100)
   c) 4200 (to the nearest 100) d) 23000 (to the nearest 1000)
   e) 70000 (to the nearest 10000) f) 7000 (to the nearest 1000)
   g) 205000 (to the nearest 1000) h) 240 (to the nearest 10)
   i) 750 (to the nearest 10) j) 1350 (to the nearest 10)

Exercise 2.

Copy these diagrams into your book and fill in the blanks with either a number, < or ≤.

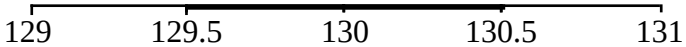
1)



A length of 121 cms to the nearest centimetre

120.5 ≤ Length < ....

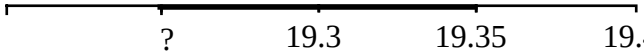
2)



A length of 130 cms to the nearest centimetre

129.5 .... Length 130.5

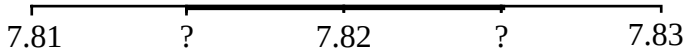
3)



A time of 19.3 seconds, to the nearest 0.1 second.

.... ≤ time 19.35

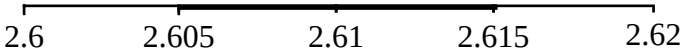
4)



A mass of 7.82 Kg to 2 decimal places

.... ≤ Mass < ....

5)



A capacity of 2.61 litres correct to 2 d.p.

2.605 .... capacity 2.615

Exercise 3.

Each of these values has been rounded to the last figure shown.Write down their limits.

1) 9.4 seconds 2) 62.3 mm 3) 19.5 kg 4) 27.6 kilometres 5) 19.62 metres 6) 25.64 seconds
7) 16 mg 8) 17.3 litres 9) 37.3 centimetres 10) 6.48 tonnes 11) 9.34 ml 12) 1.33 seconds.

90. Degree of Accuracy 2

1. A length of wood measures 3 metres long by 10cm wide. Smaller pieces of wood are to be cut from it, each measuring 10cm by 35mm, the width correct to the nearest millimetre. Calculate the maximum and minimum number of pieces that can be cut from it.
2. Mugs are made in the shape of a cylinder, with internal dimensions of 9.3cm tall and 7.5cm diameter, both measurements correct to the nearest millimetre.
   a) What is the maximum amount of liquid it will hold?
   b) What is the minimum amount of liquid it will hold?
3. A garden is 15 metres long, correct to the nearest metre. A path is made with cobbles measuring 22cm long by 8.5cm wide, both dimensions correct to the nearest millimetre. The cobbles are laid side by side without any space between them, in the way shown in the

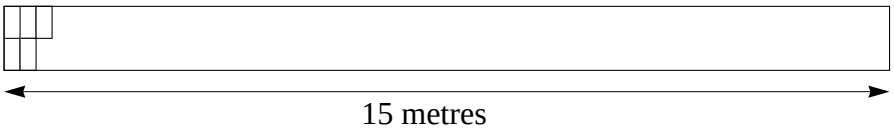
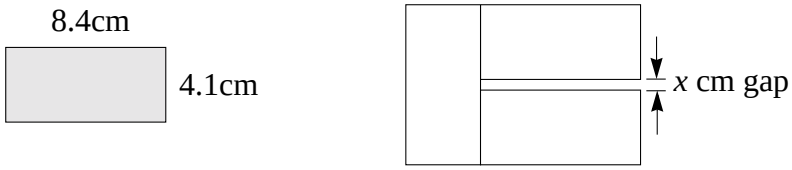


   diagram with the long sides touching. How many cobbles must be ordered to ensure the path is completed?
4. Tiles are made to the dimensions shown in the diagram, correct to the nearest millimetre.



   They are fitted together in the way shown with two shorter sides against a longer side, and the top and bottom edges level. Calculate the maximum and minimum values of the gap, x .
5. Erasers are made in the shape of a cubiod measuring 1cm by 2cm by 4.5cm. All dimensions correct to the nearest millimetre.
   a) Calculate the maximum volume an eraser can be.
   b) What is the minimum volume it can be?
   c) If the manufacturer produces all the erasers to the lower dimensions, what is their percentage saving in rubber, over the maximum dimension?
6. The plans of a new warehouse show that the floor measures 50 metres by 26 metres, correct to the nearest metre. The concrete floor is to be covered with paint . The paint covers 15m^2 per litre.
   a) What amount of paint needs to be ordered to ensure that the floor is covered?
   b) What is the maximum amount of paint that will be left over if this is bought?
7. Floor tiles measure 20cm square, correct to the nearest millimetre. A rectangular floor measures 4.21 metres long by 3.84m wide, both measurements correct to the nearest centimetre.
   a) What is the maximum number of tiles needed to fit on one row along the length of the floor?
   b) What is the maximum number of tiles needed to fit along the width of the room?
   c) What is the maximum number of tiles needed for the whole of the room?
   d) Ian buys the maximum number of tiles needed, but finds that they have been made to their maximum size and the room measured to its minimum size. How many tiles does he have left over?

91. Circumference of a Circle.

In each of the following questions use $\pi = 3.142$ or use the π button on your calculator.

Exercise 1

Calculate the circumference of each of the following circles

1) Radius 4cm	2) Radius 6cm	3) Radius 10cm
4) Radius 18 metres	5) Radius 8 metres	6) Radius 7 metres
7) Diameter 12cm	8) Diameter 16cm	9) Diameter 24cm
10) Diameter 2.3m	11) Diameter 17m	12) Diameter 23m

Exercise 2

Calculate the diameters of circles with the following circumferences (correct to 4 significant figures)

1) 20cms	2) 105 cms	3) 2.3metres
4) 15metres	5) 256cms	6) 176metres

Exercise 3

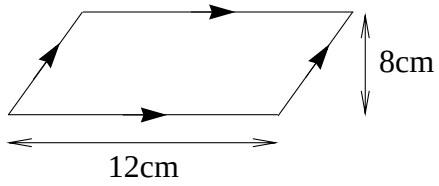
1) A car wheel has a diameter of 50cm. How far will the car travel if the wheel turns 5 times?
2) If the same car wheel turns 500 times, find the distance travelled correct to the nearest metre.
3) A hose pipe is stored by winding it around a drum of diameter 70cms. If it makes 12 turns, how long is the hose correct to the nearest metre?
4) A car has a wheel diameter of 55cms. How many revolutions does it make while travelling a distance of 1 kilometre? (give your answer correct to the nearest whole number)
5) A length of cotton measuring 2 metres is wound around a cotton reel of diameter 3cms. How many turns does it make? (correct to the nearest turn)
6) A bicycle wheel has a diameter of 65cms. How many turns will it make while travelling a distance of 2km?
7) Another bicycle travels 2km and its wheels each turn 1157 times. Calculate the diameter of its wheels, correct to the nearest cm.
8) A pulley wheel, of diameter 1.3 metres, raises a lift in a hotel from the ground floor to the 9th floor. In doing so it makes 9 complete turns. What is the distance, correct to the nearest centimetre, between each floor?.
9) An artificial lake is in the shape of a circle of diameter 200 metres and has a path running around it. It is planned to hold a 10 kilometre race around the lake. How far apart, to the nearest metre, must the start and finish be?

92. Area and Perimeter 1

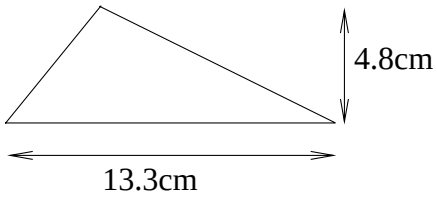
1) Calculate the areas and perimeters of rectangles measuring;
   a) 3cm by 3cm b) 6cm by 8cm c) 9cm by 3.4cm
   d) 8.4cm by 9.3cm e) 1.2 metres by 80cm f) 160cm by 0.9 metres.

2) Calculate the areas of the following shapes;

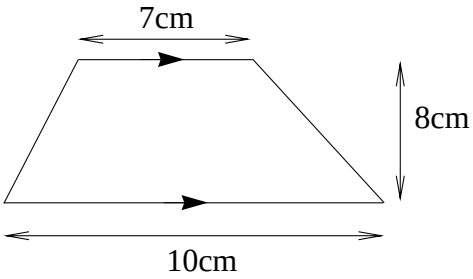
a)



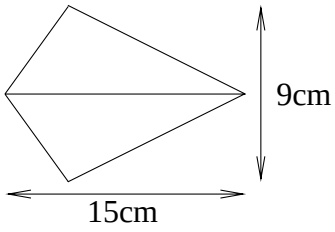
b)



c)



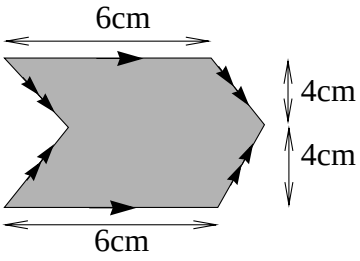
d)



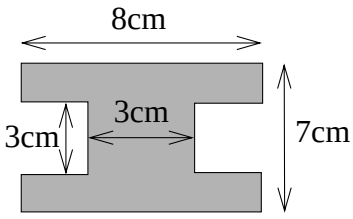
3) Calculate the areas of the following circles. Use $\pi = 3.142$ or the π button on your calculator. Give your answer correct to 1 decimal place.
   a) Radius 3cm b) Radius 7.5cm c) Radius 19cm
   d) Diameter 7cm e) Diameter 3.6cm f) Diameter 17.4cm

4) Calculate the areas of the shaded parts of each of the following shapes.

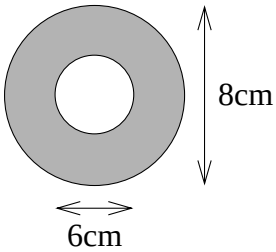
a)



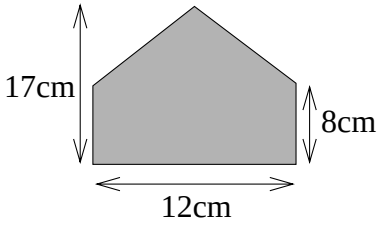
b)



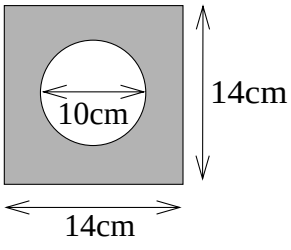
c)



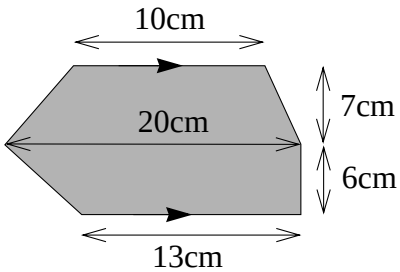
d)



e)



f)

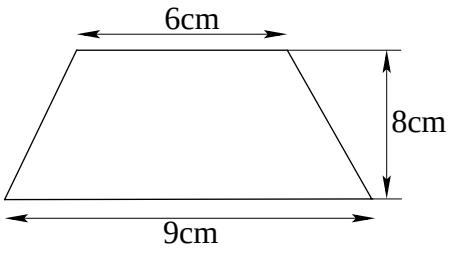


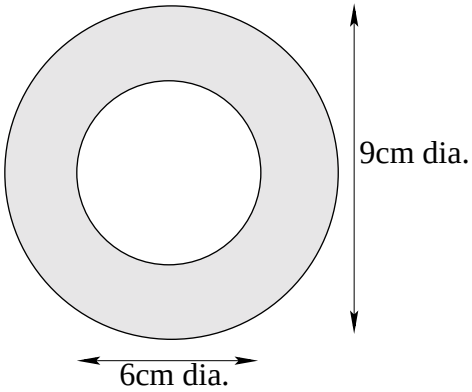
5) How many 30cm square tiles are needed to cover the floor of a room measuring 3 metres by $4\frac{1}{2}$ metres?

6) A lawn is in the shape of a quarter circle. If its radius is 8 metres, calculate its area and perimeter.

93. Area and Perimeter 2

1. Calculate the area of the trapezium on the right.
2. Calculate the area of the shaded part of the ring below.

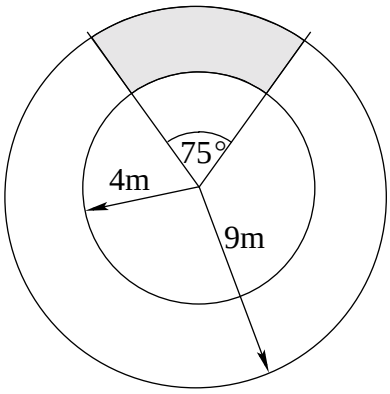


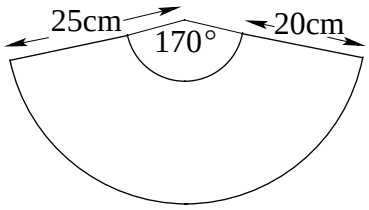


3. Calculate the area and arc length of the following sectors where r is the radius of the circle and θ is the angle at the centre of the circle.

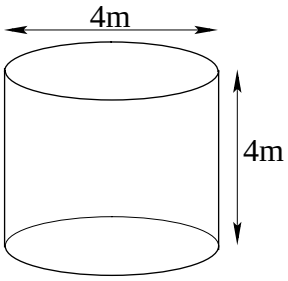
a) r = 6cm, $\theta = 30°$ b) r = 12cm, $\theta = 140°$ c) r = 16.5cm, $\theta = 200°$ d) r = 16.75cm, $\theta = 195°$

4. The shaded part of the diagram shows a flower garden in the shape of an arc. It has an inside radius of 4 metres and an outside radius of 9 metres. a) Calculate the area of the garden. b) If it is completely surrounded by an edging strip, calculate its length.
5. A conical lamp shade is to be made from the piece of fabric shown in the diagram. a) Calculate its area. b) If a fringe is sewn onto it's bottom edge, what length will it be?





6. A water tank is in the form of a cylinder of 4 metres diameter and 4 metres high, open at the top. It is to be made from sheet steel and painted with three coats of paint, both inside and outside. Calculate a) the total area of steel needed and b) the number of tins of paint needed if one tin is sufficient to cover 28m^2
7. The ceiling of a church is in the shape of a hemisphere. If its radius is 8 metres, calculate its surface area.
8. The diameter of the earth is approximately 12734 kilometres. Calculate its surface area.



94. Volume 1

Exercise 1

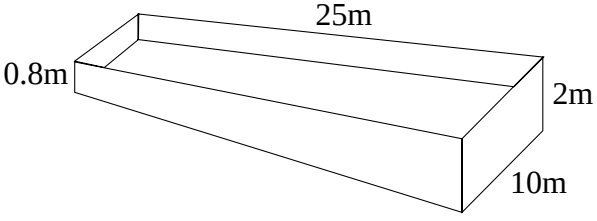
Without using a calculator, change;

1) 2cm^3 into mm^3
2) 0.003cm^3 into mm^3
3) 3.4m^3 into cm^3
4) 0.015m^3 into cm^3
5) 550,000 mm^3 into cm^3
6) 1,200,000 cm^3 into m^3
7) 0.5m^3 into litres
8) 53,000 ml into litres
9) 28 litres into ml
10) 0.003 litres into ml.

Exercise 2

1) A cardboard box is in the shape of a cuboid measuring 6cm by 12cm by 15cm. Calculate its volume.
2) A large cardboard box has internal base dimensions of 80cm by 50cm and a height of 60cm. It is to be packed with smaller boxes measuring 10cm by 12cm by 16cm. How many boxes can be put on the bottom layer and how many of these layers can be put in altogether?
3) A trench 0.7m wide by 1.2m deep and 20 metres long is dug on a building site. Calculate the amount of earth removed.
4) A cylindrical drinks can has a base of 7cm and a height of 10cm. Calculate; a) its volume in cm^3 and b) its capacity correct to the nearest ml.
5) A water tank, in the shape of an open cuboid, has a base measuring 50cm by 60cm and a height of 30cm. How many litres of water will it hold?
6) A circular pond of 4m diameter and 25cm depth is filled with water. How many litres are needed?
7) A rolling pin is made from 3 pieces of wood as shown below. The thicker piece is 5cm in diameter and 35cm long. The two end pieces are each 2.3cm in diameter and 10cm long. If 1cm^3 of this wood weighs 0.75g, find its total weight.

8) A beaker is in the shape of a cylinder with a base diameter of 5cm and a height of 9cm. How many times can the beaker be completely filled from a jug holding 2 litres?
9) A metal tube has an outside diameter of 1.5cm and a thickness of 4mm. If its length is 5m, calculate the volume of metal it contains to the nearest cm^3.
10) An open tank, in the form of a cuboid, can hold 400 litres of water. If its base has dimensions of 50cm by 80cm, what is its height?
11) A water tank, in the shape of an open cuboid, has a base measuring 50cm by 70cm and a height of 30cm. It has water in it to a depth of 20cm. A metal cube of sides 12cm is lowered into the water. By how much will the water rise?
12) A swimming pool is 0.8m deep at the shallow end and 2m deep at the other end. If its length is 25m and its width is 10m, calculate its capacity in litres.



95. Volume 2

1. Calculate the volume of a 12cm square based pyramid with a height of 20cm.

2. Calculate the height of a 4cm square based pyramid whose volume is $40cm^3$

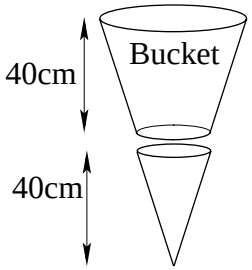
3. A rolling pin is in the shape of a cylinder with hemispherical ends. Its total length is 40cm

and its diameter is 5cm. Calculate a) its volume and b) its weight if $1cm^3$ of wood weighs 0.75 grammes.

4. A marble paperweight is in the shape of a hemisphere of radius 4cm. Calculate its volume. A supplier packs them into boxes of 50. Calculate their weight, correct to the 0.1kg, if $1cm^3$ of marble weighs 2.7 grammes.

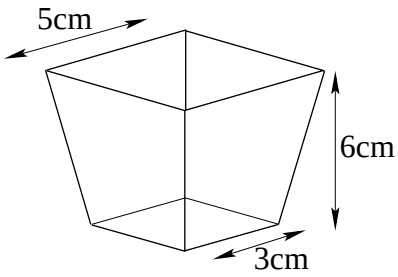
5. A metal sphere of diameter 10cm is lowered into a cylindrical jar of 16cm height and 12cm diameter which contains water to a depth of 10cm. How far up the side of the jar will the water rise?

6. A bucket is made by cutting a cone into two parts, as shown in the diagram. If the rim of the bucket measures 30cm diameter, calculate the amount of water it will hold, correct to the nearest litre.



7. A piece of metal, in the shape of a square based pyramid of height 10cm and base sides of 5cm, is melted down and re-cast into spheres of diameter 3mm. How many spheres can be made?

8. Plastic plant pots are made in the shape of inverted truncated pyramids. The top is a square of 5cm sides, the bottom a square of 3cm sides and its height is 6cm.
   a) Calculate its volume b) How many can be filled from a bag containing 2 litres of compost.?

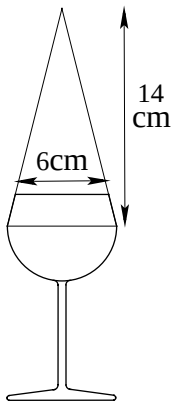


9. Tennis balls of 6cm diameter are packed into cubic boxes which hold 27 balls. Calculate a) the volume of the box b) the percentage of the box filled by the tennis balls.

10. A measuring cylinder with a base diameter of 3.8cm contains water. 500 spheres of diameter 5.3mm are dropped into it and are completely covered by water. Calculate the height by which the water rises.

11. A hollow metal sphere has an outside diameter of 10cm and is 1cm thick. Calculate the volume of the metal.

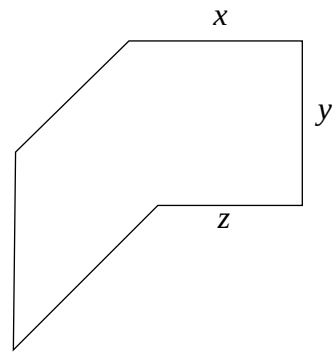
12. The diagram shows a wine glass. The bowl is in the shape of a hemisphere with part of a cone on top of it. The diameter of the hemisphere is 7cm and the height of the cone is 14cm. The diameter of the rim of the glass is 6cm and is 2cm above the hemisphere. Calculate the amount of wine the glass will hold, correct to the nearest millilitre.



96. Formulae for Area, Volume and Perimeter

With each of the following shapes a number of formulae are given.
Decide which formula best satisfies the situation given.

1) Which formula could be used to calculate (i) the perimeter (ii) the area?

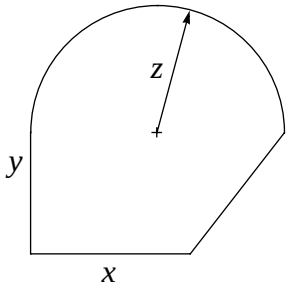


a) $4x + 3y + zx$ b) $2x + 2y + 2z$

c) $xy + yz$ d) $x^2 + 3y + 2z$

e) $x(y + z) + x$ f) $2x^2 + 2y + 2x^2$

2) Which formula could be used to calculate (i) the perimeter (ii) the area?

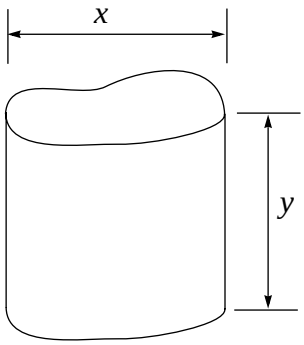


a) $x + \frac{2\pi z^2}{3}$ b) $xyz + \frac{\pi z^2}{2}$

c) $\pi z + xy + 2.1y$ d) $\frac{\pi z}{2} + x + 2.1y$

e) $\frac{\pi z^2}{2} + 1.2xy$ f) $2\pi z + x + y^2 z$

3) Which formula could be used to calculate (i) the surface area (ii) the volume?

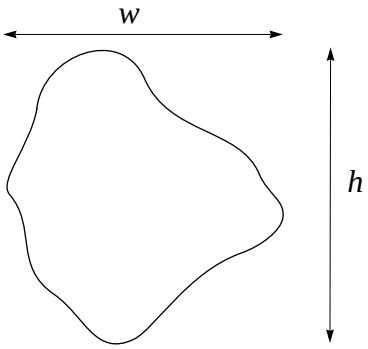


a) $x^2 y + \frac{2\pi y^2}{3}$ b) $\frac{\pi x^2 y}{5}$

c) $\pi y x^2 + xy$ d) $\frac{\pi x^2}{3} + y$

e) $\frac{\pi x^2}{2} + \pi xy$ f) $\pi x^2 y + \pi xy$

4) Which formula could be used to calculate (i) the area (ii) the perimeter?



a) $\frac{wh^2}{4}$ b) $\frac{2wh}{3}$

c) $2w + h^2$ d) $wh + w + h$

e) $\frac{3}{2}(w + h)$ f) $wh^2 + wh$

97. Formulae

Exercise 1

In each of the questions below, taking l, b, h, r and d to be length, classify each of the following formulae into length, area, volume or those making no sense.

1) $4(r^2 + lb)$ 2) $3(l + d)b$ 3) $\pi(l + b)^2$

4) $3bd + c$ 5) $3(l \times b)$ 6) $3b(l + h)$

7) $\dfrac{3(l + 2b)}{2}$ 8) $\dfrac{l^2 + b^2}{r}$ 9) $5\sqrt{r^2 + h^2}$

10) $2lbh$ 11) $3lr^2$ 12) $5b^2(l + h)$

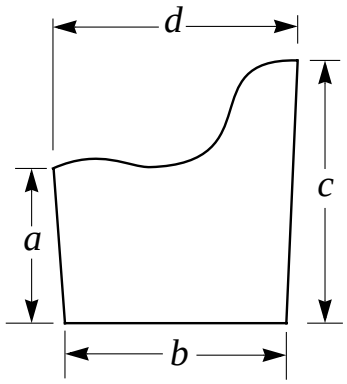
13) $4b^2 + r$ 14) $5b(d^2 + c)$ 15) $6(l + b)$

16) $\frac{1}{2}(l + b)$ 17) $\frac{1}{2}(bh)$ 18) $\dfrac{bdh}{3}$

19) $a^2bd + e$ 20) $d\sqrt{ab} + lr$ 21) $a^2b + lrd$

Exercise 2

1) For the diagram shown on the right, choose one of the formulae listed below that you think best suits a) its area and b) its perimeter

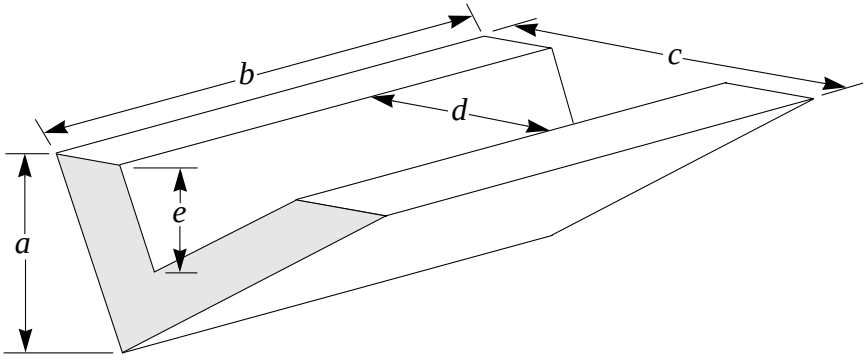


(i) $a + b + c + 1.3d$ (ii) $\frac{1}{3}a^2b$

(iii) $\dfrac{b(c + a)}{2}$ (iv) $\sqrt{abcd}$

(v) $\frac{1}{2}abc$ (vi) $\dfrac{(a + b + c)}{2}$

2) For the diagram shown below, choose one of the formulae listed underneath you think best suits a) the area of the shaded end and b) its volume.



(i) $\frac{1}{2}abc$ (ii) $\frac{1}{2}(ac - de)$ (iii) $\frac{1}{2}(ab^2c)$

(iv) $a^2b - bc$ (v) $\left(\dfrac{ac - de}{2}\right)b$ (vi) abc

98. Compound Measure - Speed and Density

Exercise 1. Speed

1) A car travels at the following speeds (a) 40 mph (b) 30 mph (c) 60 mph. In each case say how far the car travels in

(i) $\frac{1}{2}$ hour (ii) 2 hours (iii) $\frac{1}{4}$ hour

2) A train goes from Chester to London, a distance of 200 miles. It travels at a speed of 80 mph.

(a) How far does it travel in 2 hours?

(b) How far will it travel in $\frac{1}{2}$ hour

(c) How long will it take to travel from Chester to London?

3) (a) A train travels between two towns, A and B. Its average speed is 60mph.

The train takes $1\frac{1}{2}$ hours. How far apart are the towns?

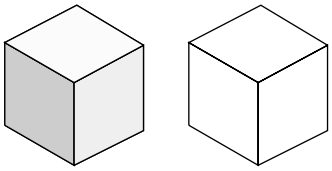
(b) Another train makes the same journey. This train takes 2 hours. What is its average speed?

4) Jane travels to London down the motorway. She travels the first 75 miles at an average speed of 50 mph. She then travels the remaining 20 miles at an average speed of 40 mph. How long did her journey take?

5) A car travels at an average speed of 60 miles per hour down the motorway.
How far will the car travel in (a) 15 minutes (b) 1 hour 10 minutes

Exercise 2. Density

1) These cubes measure 1cm by 1cm by 1cm.



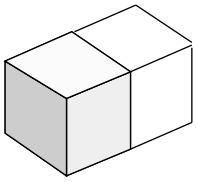
Their volumes are each $1cm^3$.
The weight of each green cube in 4g. The weight of each white cube is 2g.
Calculate the average weight of
(a) 2 green and 2 white cubes (b) 3 green and 5 white cubes

2) Red centimetre cubes weigh 3g and blue centimetre cubes weigh 5g. Find an arrangement of cubes which give an average density of 3.5g per cm^3.

3) Using red and blue cubes find an arrangement with an average density of 4.5g per cm^3.

4) Can you list other arrangements of blue and red cubes which give an average density of 4.5g per cm^3?

5) Green cubes weigh 6 gram and white cubes weigh 2 grams.
(a)



A green cube and a white cube are put together. What is their average density?
(b) If one green cube and 3 white cubes are put together, what is their average density?

99. Compound Measure - Best Buy and a Mixed Exercise

Exercise 1. Finding the best buy

Calculate which of the following sizes give the best buy.

1) Toothpaste costing
85p for a 75ml tube,
£1.15 for a 125ml tube
£1.90 for a 200ml tube

2) Shampoo costing
50p for a 250ml bottle
95p for a 500ml bottle
140p for a 800ml bottle

3) Baked beans costing
18p for a 250g tin
25p for a 450g tin
36p for a 700g tin

4) Paint costing
£2.50 for a 1 litre tin
£5.25 for a $2\frac{1}{4}$ litre tin
£7.95 for a 4 litre tin

5) Oil paint is sold in 3 different sizes.
A 75ml tube costs £1.20.
A 150ml tube costs £2.10.
A 200ml tube costs £3.10.

Exercise 2. Mixed

1) Decide which one of the following pairs of cars is the most economical.
(a) Car A travels 180 miles and uses 11 litres of petrol.
Car B travels 120 miles and uses 7 litres of petrol.
(b) Car C travels 160 miles and uses 9 litres of petrol.
Car D travels 96 miles and uses 5 litres of petrol.

2) Rachel has to paint the ceilings of her house. She needs to give each two coats and the total area of the 8 ceilings is 110 square metres. 1 litre of paint will cover $12m^2$.
If she can buy the paint in 5 litre and $2\frac{1}{2}$ litre tins, how much paint will she buy?

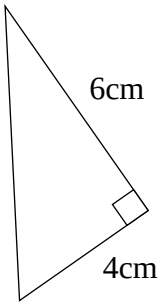
3) David plans to spread "lawn care" over his lawn. The instructions say that it has to be used at the rate of $1\frac{1}{4}$ ounces per square yard. His lawn is in the shape of a rectangle measuring 10 yards by 8 yards. How much "lawn care" will he have to buy if it is sold in 2lb packets?
(Note 16 ounces (oz) = 1 pound (1lb))

4) Jane needs to paint the walls in her bedroom. The room measures 4m by 3m and is $2\frac{1}{2}$ metres high. However $3m^2$ is taken up by the door and a window. Paint is sold in 5 litre and 2 litre cans and it will cover the walls at a rate of $11m^2$ per litre. She gives the walls 2 coats of paint.
How many cans of paint will she have to buy?

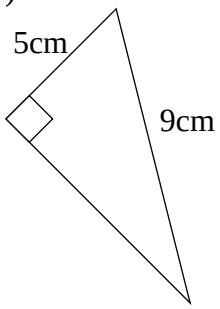
100. Pythagoras Theorem

1. Calculate the length of the unknown side in each of the following diagrams

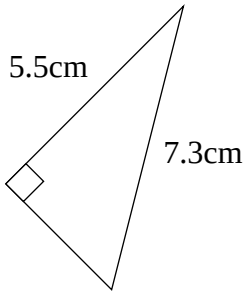
a)



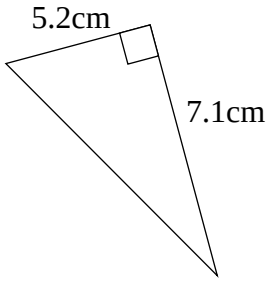
b)



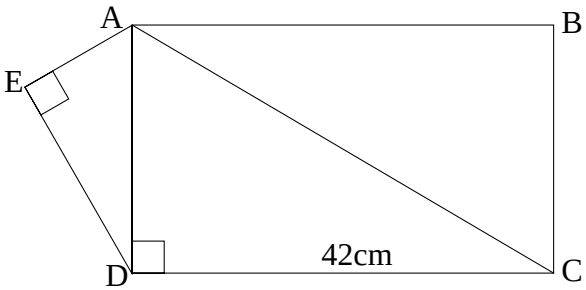
c)



d)

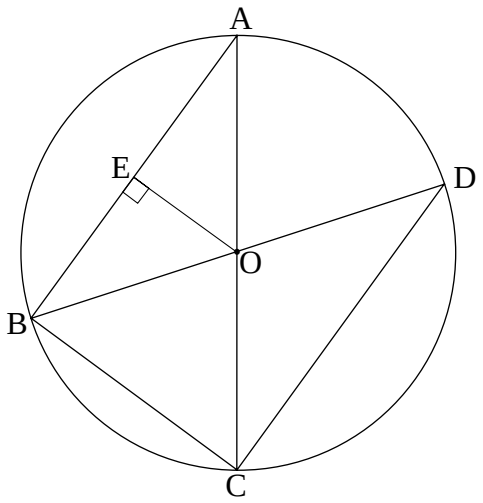


2. The diagram shows a rectangle ABCD with CD = 42cm, and the triangle ADE with AE = 10.5cm and DE = 14cm. Calculate the length of AC.



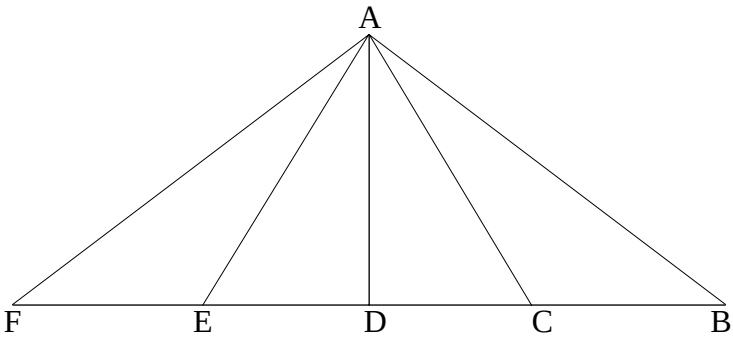
3. In the diagram on the right, AOC and BOD are diameters of a circle of radius 4.5cm. If DC = 7cm, calculate the lengths of BC and OE.

4. A rhombus has diagonals measuring 10cm and 8cm. Calculate the lengths of its sides.



5) A window cleaner positions the base of his ladder 2 metres from the bottom of a block of flats. He opens his ladder up to 4 metres to clean the first floor window and 6.5 metres to clean the second floor window. On both occasions the ladder just touches the window sill. Calculate the distance between the two window sills.

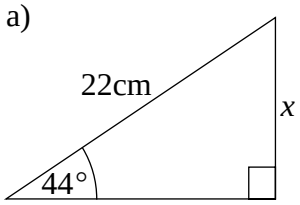
6) A Radio mast AD is held in a vertical position by four wires, AF, AE, AC and AB. AF = AB and AE = AC. If AF = 50 metres, AE = 35 metres and AD = 25 metres, calculate the distance CB.



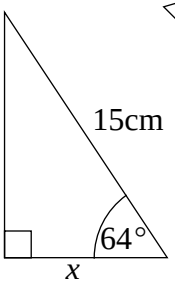
101. Sine, Cosine and Tangent Ratios 1

1) Calculate the length of the unknown side, *x*, in each of the following triangles.

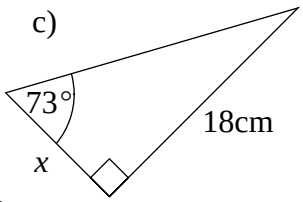
a)



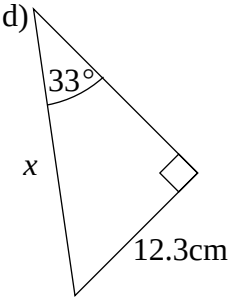
b)



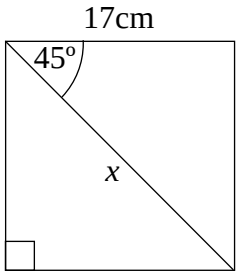
c)



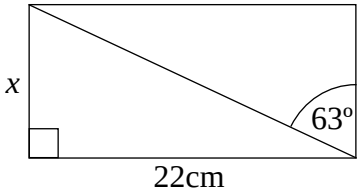
d)



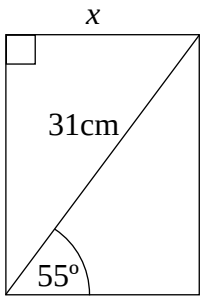
e)



f)

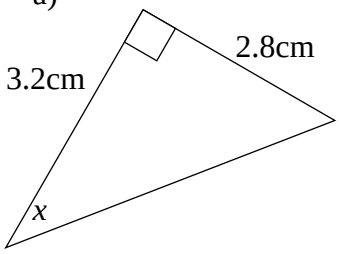


g)

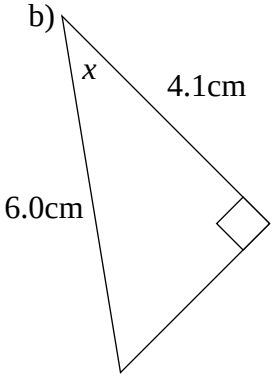


2) Calculate the size of the unknown angle, *x*, in each of the following diagrams.

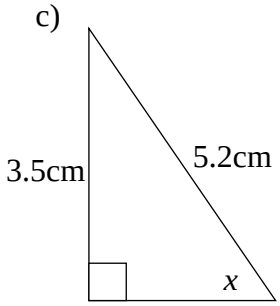
a)



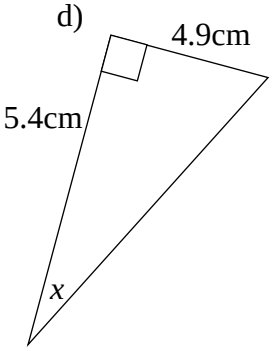
b)



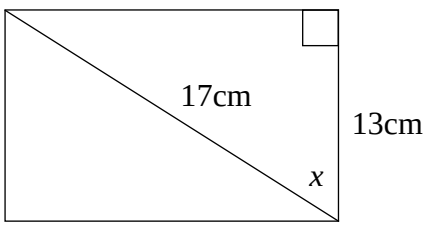
c)



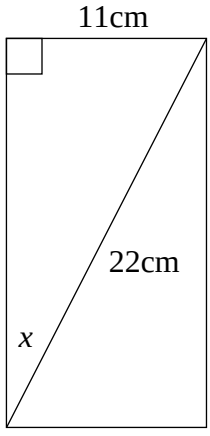
d)



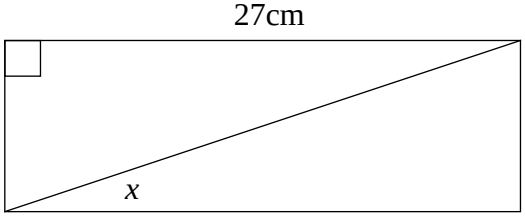
e)



f)

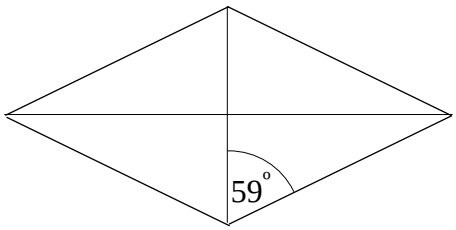


g)

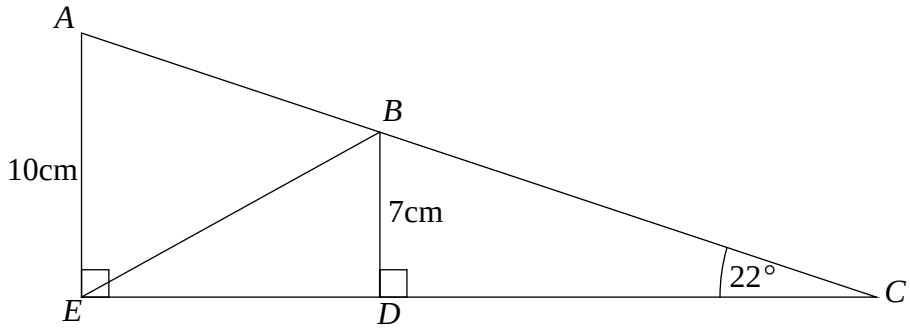


102. Sine, Cosine and Tangent Ratios 2

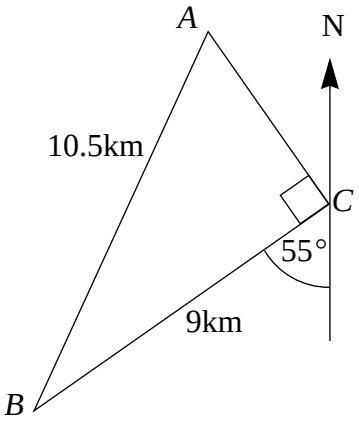
1) The diagram shows a rhombus with sides of 15cm. Calculate the lengths of the two diagonals



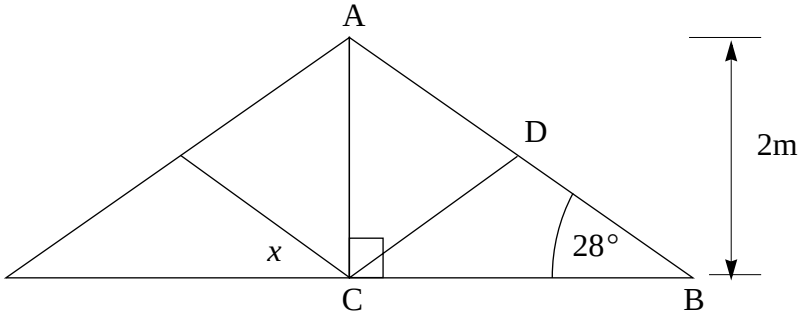
2) In the diagram below, AE = 10cm, BD = 7cm and $\angle ACE = 22°$. Calculate the sizes of the lines CD, AC and EB. Calculate also the size of angle DEB.



3) The diagram shows the positions of three towns, A, B and C. The bearing of town B from town C is $235°$. The distance between towns A and B is 10.5km, and the distance between towns B and C is 9km. Calculate a) the bearing of town A from town B and b) the distance from town A to town C.



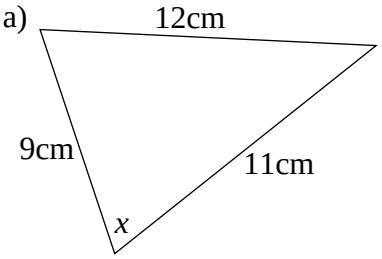
4) The diagram below shows the roof truss of a house. Its height in the middle is 2 metres, and the angle between the horizontal and the roof is $28°$. It is symmetrical about the line AC. Calculate a) the width of the truss and b) the angle x if point D is half way along AB.



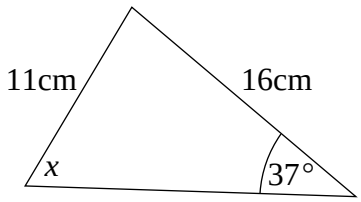
103. Sine and Cosine Rules

1. Calculate the sizes of the unknown value of x in each of the following triangles.

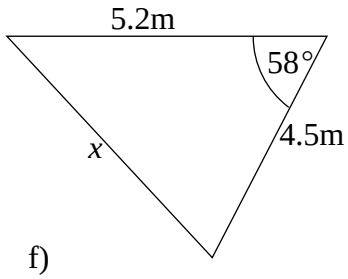
a)



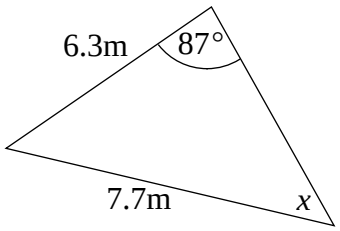
b)



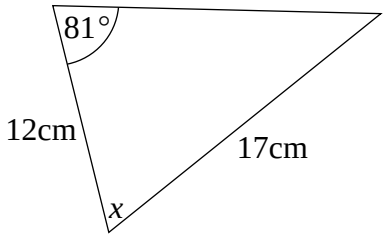
c)



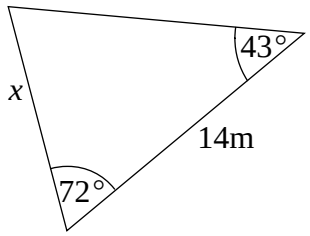
d)



e)

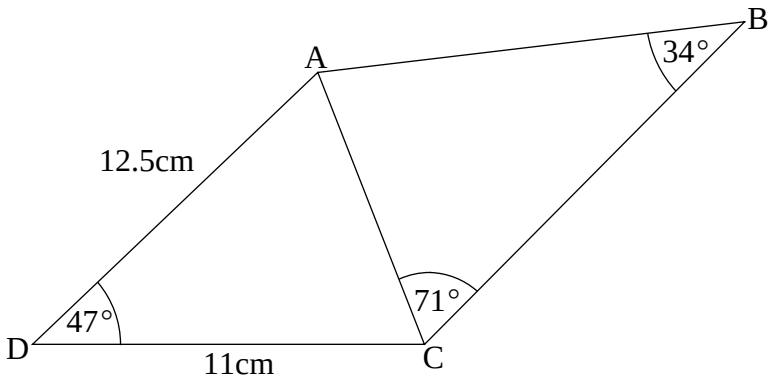


f)



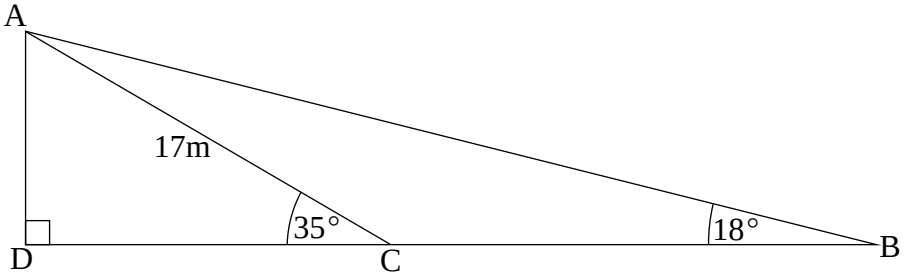
2. In the diagram on the right, calculate the length of the line AB.

3. In a parallelogram ABCD, AB = DC, and AD = BC. The diagonal BD = 20cm and the diagonal AC = 14cm. If angle ACB = $85°$, calculate the sizes of the two interior angles of the parallelogram.

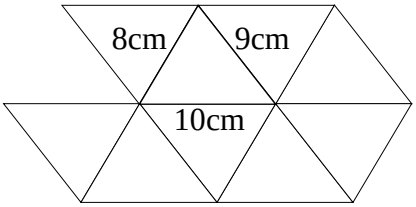


4. The bearing of a beacon from an aeroplane is $160°$ (S$20°$ E). The aeroplane flies due south for 5km and now finds that the bearing is $135°$ (S$45°$ E).What is now the distance of the beacon from the aeroplane?

5. In the diagram below, calculate the length of CB



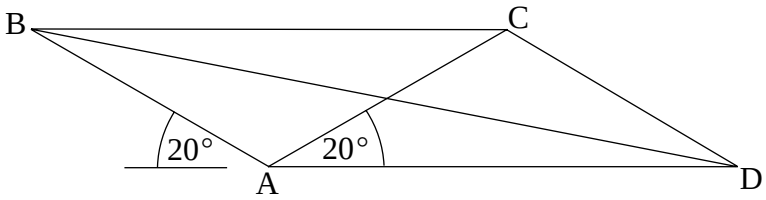
6. The diagram shows a series of congruent triangles used in the making of a patchwork design. If the sides of the triangle measure 8cm, 9cm and 10cm, calculate its angles.



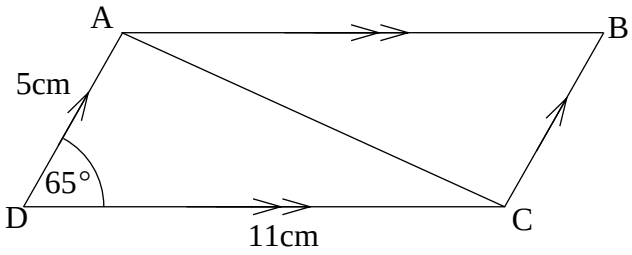
104. Areas of Triangles

In questions 1 to 5 calculate the areas of these triangles

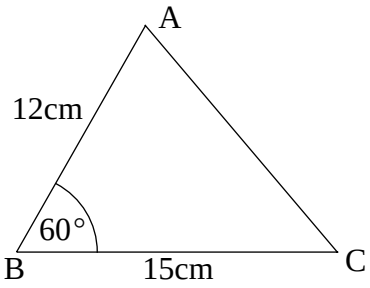
1. In triangle *ABC*, $\angle ABC = 61°$, *AB* = 6cm and *BC* = 8cm.
2. In triangle *XYZ*, $\angle XYZ = 77°$, *XY* = 8.5cm and *YZ* = 13cm.
3. In triangle *CDE*, $\angle CDE = 110°$, *CD* = 5.6cm and *DE* = 8.5cm.
4. In triangle *PQR*, $\angle PQR = 119°$, *PQ* = 7.2cm and *QR* = 8.7cm.
5. In triangle *RST*, $\angle RST = 47.7°$, *RS* = 22.3cm and *ST* = 16cm.
6. In the diagram below both AB and AC are equal to 9cm. Show that the area of triangle ACD is equal to the area of triangle ABD.



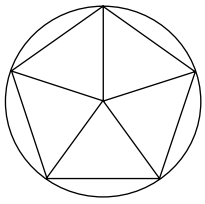
7. By calculating the area of triangle ADC, calculate the area of the parallelogram ABCD.



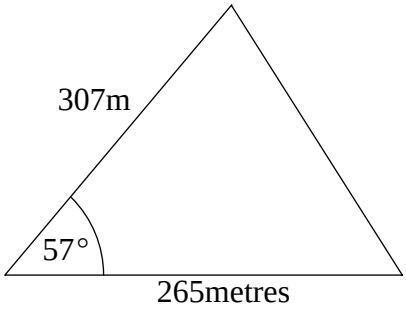
8. A patchwork bed cover is to be made from 300 pieces of fabric cut into the shape of the triangle shown. Calculate the area of the bed cover, in square metres, correct to one decimal place.



9. A regular pentagon lies inside a circle of diameter 20cm with its vertices touching the circle. Calculate the area of the pentagon.

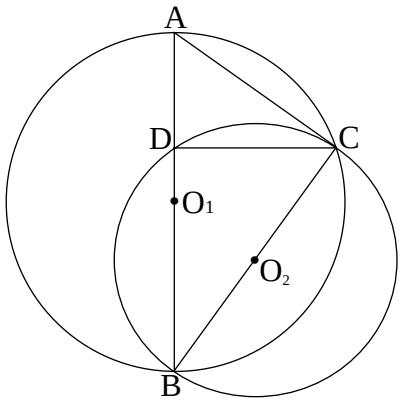


10. A field is in the shape of a triangle. Calculate its area in hectares, correct to 1 decimal place. (10,000 sq. metres is equal to 1 hectare)

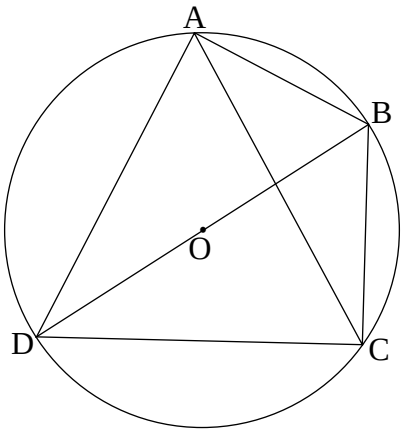


105. Trigonometry Mixed Exercise

1. Two circles with diameters AO_1B and BO_2C intersect at B and C. If AB = 11cm and BC = 9cm, calculate the dimensions of AC and CD.

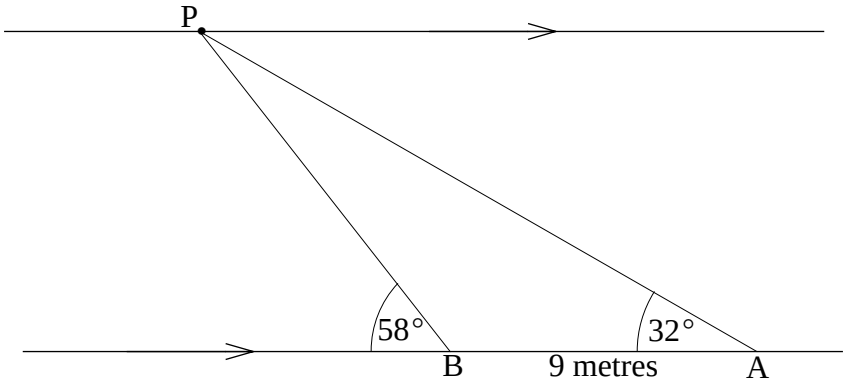


2. BD, the diameter of the circle below, is 12cm in length.

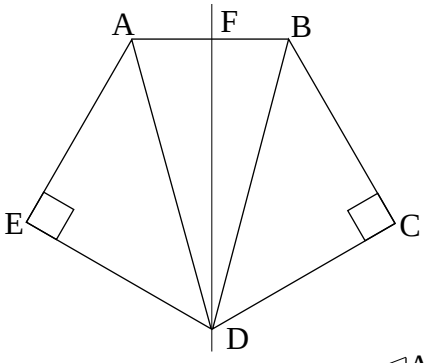


AD = 11cm and BC = 8cm. Calculate the length of the line AC.

3. The diagram shows a river with a post P on its bank. When viewed from two points, A and B, on the other side of the bank, the angles between the lines of sight and the bank are $32°$ and $58°$. If the distance between A and B is 9 metres calculate the width of the river.

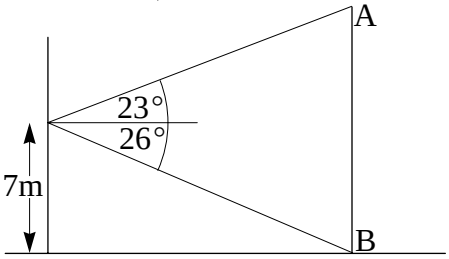


4. The diagram on the right shows a pentagon ABCDE which is symmetrical about the line FD. The lengths of AE and ED are 10cm. If AB = 6cm, calculate the sizes of angles ABC and CDE.



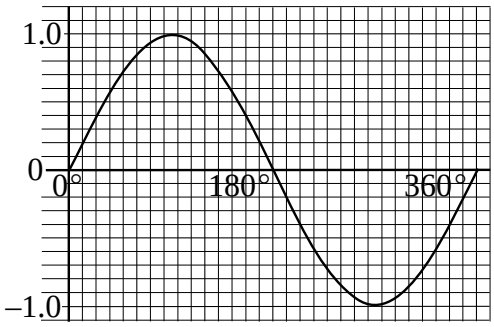
5. In the triangle ABC, AB = AC and angle ABC = $70°$. If BC = 10cm, calculate the length of AC.

6. A building AB, is observed across a street from the window of a house 7 metres above the ground. If the angle of elevation of the top of the building is $23°$ and the angle of depression of the foot of the building is $26°$, calculate the height of the building.



106. Graphs of Sines, Cosines and Tangents

1. The diagram shows the graph of $y = \sin x$



   a) Sketch this into your book and mark on it the approximate solutions to the equation $\sin x° = -0.5$ where x lies between $0°$ and $360°$.

   b) Calculate accurately the solution to the equation $\sin x° = -0.5$ where x lies between $0°$ and $360°$.

2. a) Draw a graph of $y = 2\sin x° + 1$ for $0° \le x \le 180°$ using a scale of 2cm for 1 unit on the y axis and 2cm for $30°$ on the x axis.

   b) From your diagram, calculate the values of x which satisfy the equation $2\sin x° + 1 = 2.5$ for $0° \le x \le 180°$.

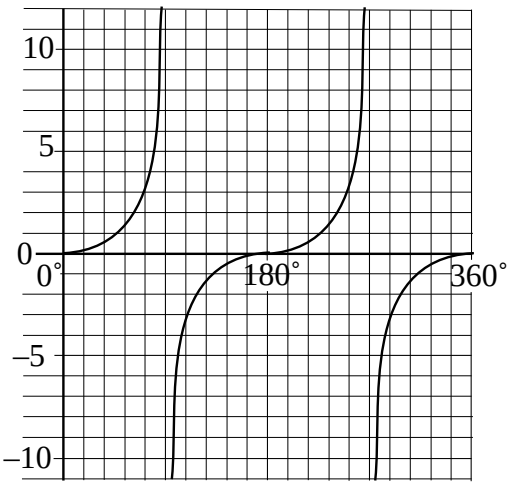
3. a) Sketch the graph of $y = \cos x$ for $0° \le x \le 360°$.

   b) Show on the diagram the approximate locations of the solutions to the equation $\cos x = -0.5$ for $0° \le x \le 360°$.

4. a) Draw the graph of $y = 3\cos x + 2$ for $0° \le x \le 360°$. Use a scale of 2cm for 1 unit on the y axis and 4cm for $90°$ on the x axis.

   b) From the graph, calculate the solutions to the equation $3\cos x° + 2 = 3$ for $0° \le x \le 360°$.

5. The diagram on the right shows a sketch of $y = \tan x$ for $0° \le x \le 360°$. From the graph determine the approximate solutions to the equation $\tan x = 4$ for $0° \le x \le 360°$.



6. a) Sketch the graph of $y = \tan x + 2$, for $0° \le x \le 360°$. Indicate on the graph the approximate location of the solution to the equation $\tan x + 2 = 0$ for $0° \le x \le 360°$.

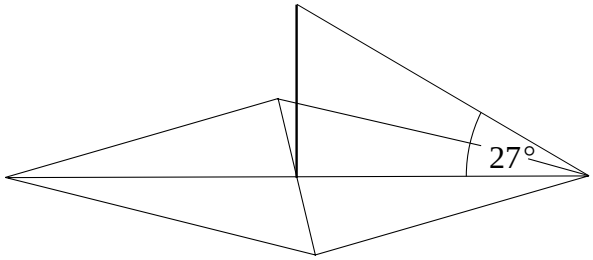
   b) Sketch the graph of $y = 3\tan x$ on the same axis as $y = \tan x$ for $0° \le x \le 180°$ clearly showing the difference between the two graphs.

7. a) Sketch the graph of $y = 2\tan x - 1$ for $0° \le x \le 360°$.

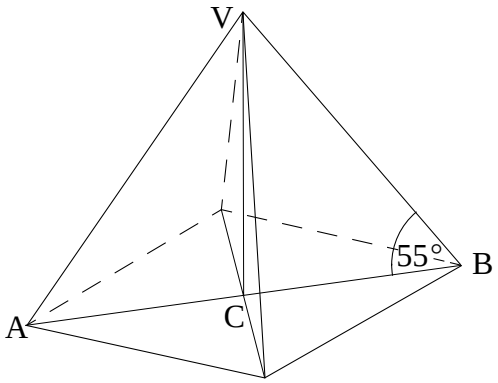
   b) Indicate on the graph the approximate location of the solution to the equation $2\tan x - 1 = 2$ for $0° \le x \le 360°$.

107. Three Dimensional Trigonometry

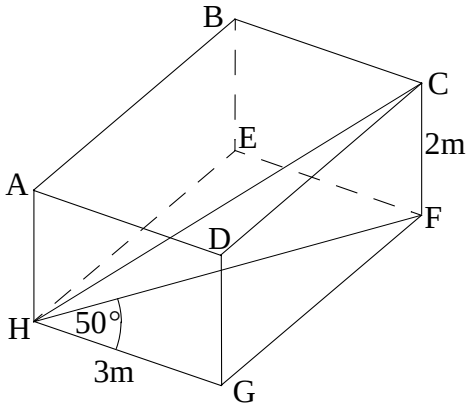
1. A flag pole stands in the middle of a square field. The angle of elevation of the top of the flag pole from one corner is $27°$. If the flag pole is 12 metres high, calculate the lengths of the sides of the field.



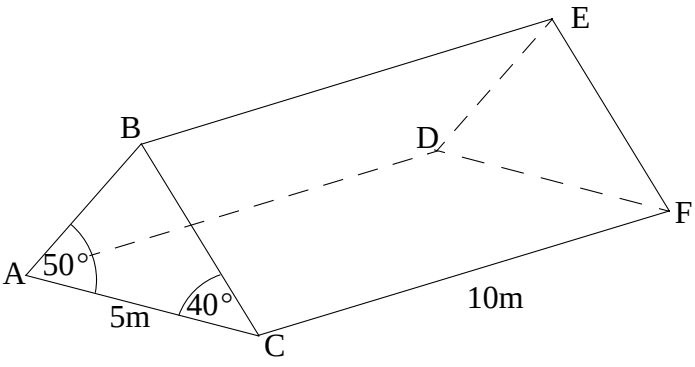
2. The diagram shows a square based right pyramid.
   a) If the base edges are 20cm, calculate the length of the base diagonal AB.
   b) If the angle VBA is $55°$, calculate the height VC.



3. The diagram shows a cuboid with vertices ABCDEFGH. Its height is 2 metres and its width is 3 metres. Angle GHF measures $50°$
   Calculate:
   a) the diagonal of the base, HF
   b) angle CHF
   c) the diagonal of the box HC.



4. The roof of a building is in the shape of a triangular prism and is shown in the diagram.
   Calculate:
   a) the length of BC
   b) the angle BFC.



5. A radio mast is due east of an observer A and 150 metres away. Another observer, B, stands due north of A on level ground. If the angle of elevation of the top of the mast from B is $28°$ and the bearing of the mast from B is $132°$, calculate the height of the mast.

108. Questionnaires 1

1) A large company want to build a supermarket in the town. Paul designs a questionnaire to find out whether local residents want it and if so whether they will use it. Write down two questions he might use, each question having a choice of 3 responses.

2) Bill is keen to have a wine bar in the high street. He wants to find out what local residents think. In order to get an unbiased response, he chooses two of the following groups of people to ask.
(a) the youth club
(b) the old peoples home
(c) the local supermarket
(d) the residents of the high street
(e) the residents of a local housing estate
Which groups do you think he should choose and why?

3) The local council decide to pedestrianise the centre of town (i.e. stop all vehicles using it). They decide to ask the traders in the town centre their opinion and no one else. Is this a good idea? Explain your answer.

4) The school committee decide that the tuck shop is to sell vegetarian snacks. The snacks they want are a) fruit b) yoghurt c) oatmeal biscuits d) nuts and e) wholemeal sandwiches. Devise a questionnaire in which they can determine the snacks pupils like best.

5) A new burger restaurant is to be opened near to Claire's school. Lots of the local people have said that they don't want it. Claire thinks that most people do want it to be opened so she writes a questionnaire to get the necessary evidence.
She decides to give out the questionnaire to members of year 11.
(a) Why is this not a good idea?
(b) Which of the following groups of people would give the least biased replies? Say why.
(i) Members of the youth club
(ii) Customers leaving the local supermarket
(iii) Members of the local golf club
(iv) The people living next to the school

6) The manager of a D.I.Y. store wants her staff to wear a new uniform. She thinks that they should all wear green tops and either a black skirt or trousers. The deputy manager thinks that they would prefer a red top and jeans. The manager designs this questionnaire for the staff

I think that it is a good idea
to have green tops and either a black
skirt or trousers as our school uniform.
Please indicate whether you agree with me
by ticking the appropriate box below

Agree ☐

Disagree ☐

Do you think that this is a good questionnaire? Make comments

109. Questionnaires 2

1. Amy is following a course in tourism. As part of the course she has to do a statistical survey. She decides to find out where most tourists in Britain come from. She thinks that the USA will be her answer.
   Her home town lies more than 300 miles from London and is very historical. She carries out a survey on the main street of her town at 12:00 midday each day during the month of July.
   a) Will her survey be biased? Explain your answer.
   b) How could she improve the survey?
   c) Write down three questions she might ask.

2. James lives at Ayton-on-sea. He works for the local newspaper who want to begin a regular feature for people over the age of 60. To help decide whether the venture is worth undertaking, his editor asks him to do a survey of the over 60's.
   James decides to place a questionnaire in the newspaper and invite people to fill it in and send it back to him. This was the questionnaire.
   'The Ayton-on-sea Observer are thinking of starting a feature for the over 60's. If you are in this age range we would be grateful if you would fill in the following questionnaire and send it back to us.

   Would you regularly read a feature for the over 60's? Please tick the appropriate box.

   Yes ☐ No ☐

   What types of articles would you like included? Please list them in the space below.

   a) What other information do you think is needed?
   b) Is the survey biased in any way?

3. Jessica believes that the life expectancy of a car is 15 years. She carries out a survey by observing cars travelling down her high street and noting their registration number (this tells her how old they are).
   a) Explain why this will only give her an idea of the life expectancy and not the full answer.
   b) In order to add to this data, she did a further survey by questioning people leaving a supermarket. What questions do you think she needs to ask?

110. Pie Charts 1

1) A town council wants to make its transport system more efficient. As a first step they interviewed a sample of 900 people. They were asked how they got into town.
Their results were as follows.

Train 50	Bus 250	Car 400
Bicycle 50	Walked 150	

Show this information on a pie chart.

2) The United Kingdom is made up of the following approximate areas, measured in millions of hectares

England 12.5	Wales 2
Scotland 8	Northern Ireland 1.5

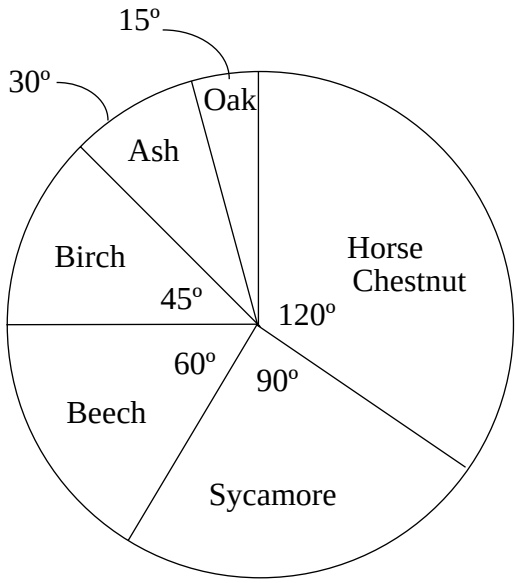
Show this information on a pie chart.

3) Samuel earns £90 a week. His expenses each week are as follows

Lodging/Food £40	Clothes £10	Entertainment £15
Bus fares £12	Savings £5	Other £8

Show this information on a pie chart

4)



The pie chart shows the composition of a mixed woodland area. Altogether there are 2,160 trees.
Calculate
a) how many trees are represented by 1º
b) how many trees there are of each type

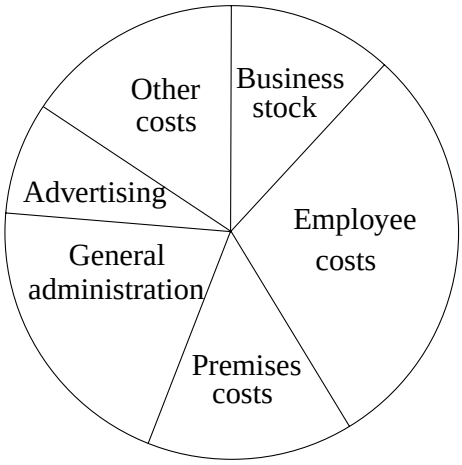
5) The table below shows the number of visitors to Dibchester castle in 1996.

Spring	Summer	Autumn	Winter
700	1300	800	200

Construct a pie chart to show this information. Write down
a) the number of degrees representing 100 people
b) the number of degrees representing each season.

111. Pie charts 2

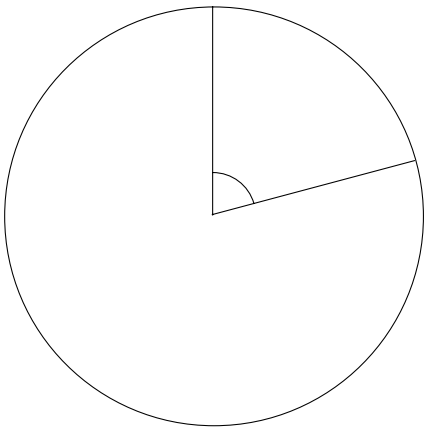
1. The diagram shows the way in which a small company spends its money.
   During a year it spends £50,000.
   By measuring the angles on the diagram, calculate the approximate amount of money spent in each area.



2. An Italian restaurant sells 6 different types of pizzas.
   During one week the manager keeps records of the pizzas sold.
   The table shows the results.
   From the table draw a pie chart to show the data, clearly indicating how the angles are calculated.

Type of pizza	Number sold
Margherita	256
Seafood	97
Hot and spicy	83
Vegetarian	132
Chef's special	185
Hawaiian	47

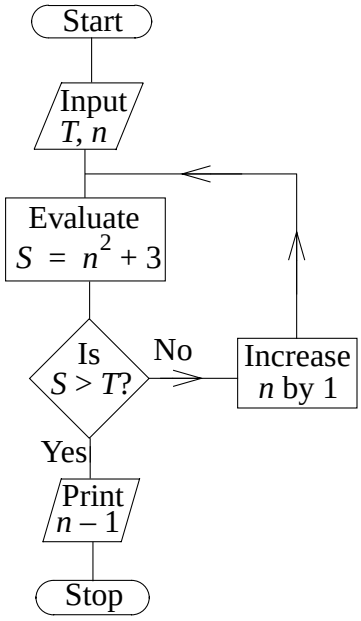
3. Liam asked a number of people how they intended to vote in the forthcoming election, for the Left, Right, Centre or Reform Party.
   He began to draw a pie chart for the data. The angle he has drawn on this pie chart represents the 210 people who voted for the Centre Party.

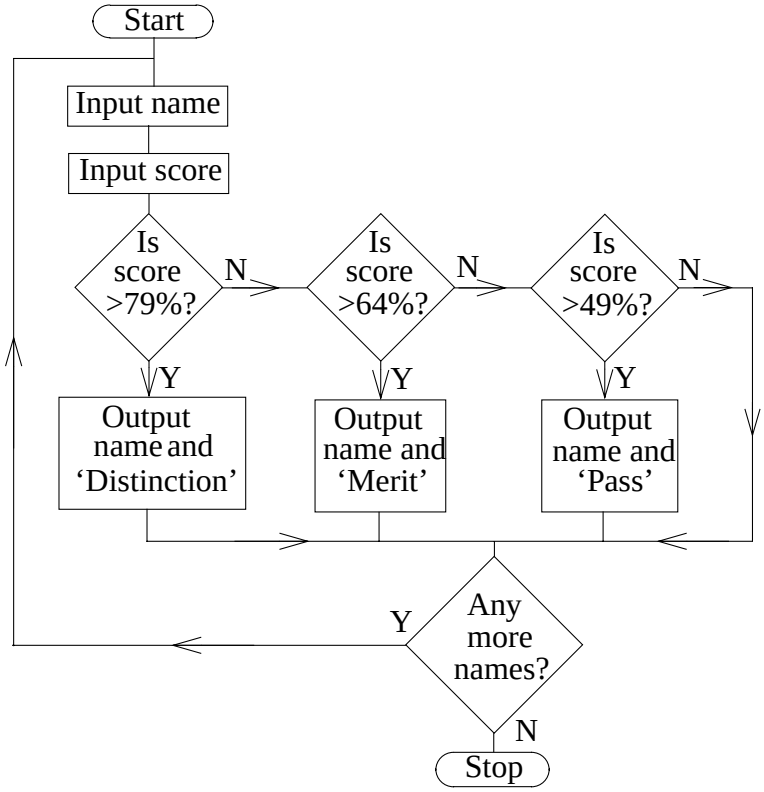


   a) Measure the angle and calculate how many people were interviewed altogether.
   b) If 392 people voted for the Right Party, what angle would be used to show this?
   c) If $100°$ was used to represent the Left Party, show this information on a pie chart, and complete the pie chart.
   d) How many people intended to vote for the Reform Party?

112. Flowcharts 1

1. This flowchart can be used to calculate how many numbers (n) there are in the sequence $n^2 + 3$ smaller than or equal to T. For example, the first number is $1^2 + 3 = 4$, the second is $2^2 + 3 = 7$ and so on. Starting with $n = 40$ and $T = 2000$, list each successive value of n and each corresponding value of S. Find how many numbers there are in the sequence which are less than 2000





2. This flowchart can be used to print out a list of students names and their examination results. If the following are inputted, write down what you would expect to be outputted.

W Jones 67

J Connah 56

C Smith 83

R Rogers 24

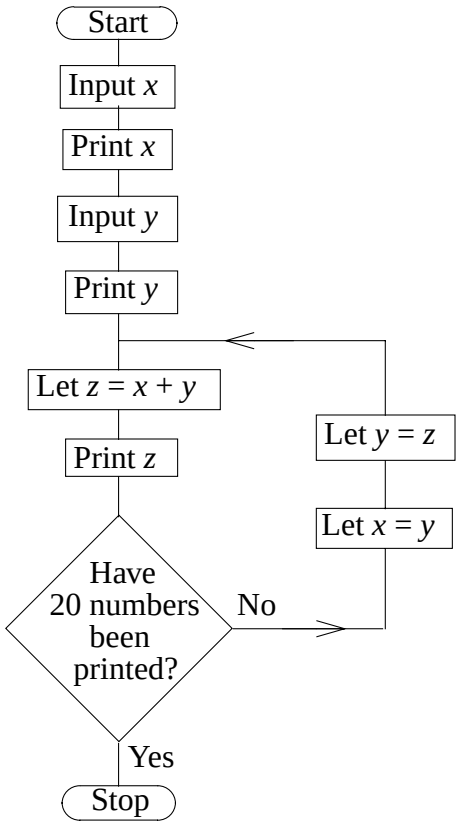
H Patel 52

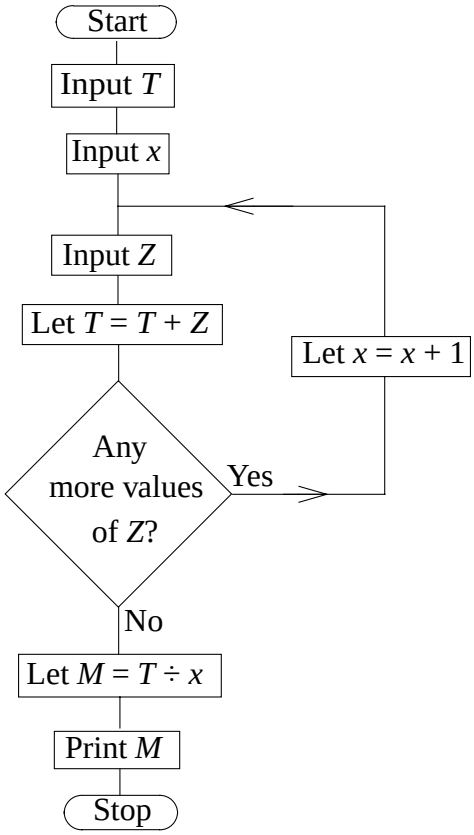
113. Flowcharts 2

1. The flow diagram can be used to calculate the first 20 elements of a number sequence. Use it to calculate the first 20 numbers of the sequence.

   Explain how each number in the sequence is formed.

   Start with $x = 0$ and $y = 1$.





2. This flow chart is used to process the following values of *Z*.

   10, 7, 9, 11, 6, 3, 6, 13, 5, 10, 8.

   Carry out the process.

   Write down the value of *M* and explain what the flowchart does.

   Start the process with $T = 0$ and $x = 1$.

114. Stem and Leaf Diagrams

1) Below is shown a stem and leaf diagram for the number of DVD's sold by a shop over a period of one month .

5	3, 0, 4, 3, 2
5	6, 8, 7, 5, 6, 9, 6, 8, 5, 7
6	1, 4, 2, 0, 1, 4, 2
6	7, 5, 8, 8, 6
7	0, 1
7	6

a) What does the 0 represent in the 5th row?
b) What does the 5 represent in the 4th row?
c) How many days are in the month?
d) What are the highest and lowest temperatures?
e) What is the range for the temperatures?

2) The times taken for 24 marathon runners to complete a race (to the nearest minute) are shown in the table below.

170,	217,	203,	205,	186,	161,	201,	179,
192,	187	161,	185,	210,	173,	186,	204,
191,	185,	179,	168	185	192	179	205

Draw a stem and leaf diagram to show this data.

3) The table below shows the amount of rain (to the nearest 0.1mm) each day over a period of one month for a town on the east coast of England.

1.3,	2.0,	2.3,	3.2,	3.4,	1.8,	3.1,	2.3,	1.9,	2.6,
1.6,	0.0,	2.4,	3.1,	0. 6,	0.7,	0.3,	0.0,	2.2,	1.5,
3.6,	2.3	1.8,	2.7,	3.0,	2.9,	1.2,	2.2,	1.4,	3.3

a) Draw a stem and leaf diagram for the data with each row representing 0.5mm of rain.

The table below shows the amount of rain (to the nearest mm) each day over the same period for a town on the west coast of Ireland.

3.6,	3.0,	4.2,	2.5,	3.6,	3.4,	4.3,	3.5,	4.2,	2.1,
2.6,	2.9,	4.2,	4,0,	2,2,	3.1,	2.4,	4.1,	0.0,	0.0,
4.7,	3.5,	2.9,	4.7,	3.9,	3.4,	2.7,	4.1,	3.0,	4.1

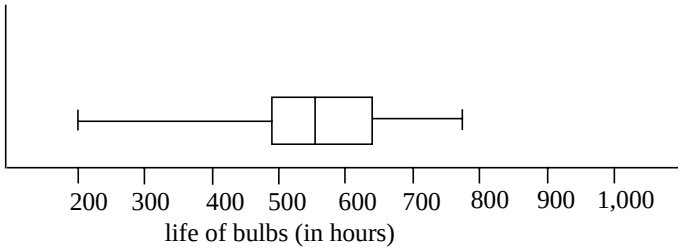
b) Draw a stem and leaf diagram for the data with each row representing 0.5mm of rain.
c) Use your diagrams to comment on the differences between the two sets of data.

115. Box Plots

1) There are 150 apple trees in the cider farmers orchard. The following figures relate to their heights.
Tallest tree 450cm Smallest tree 250cm
Lower quartile 310cm Upper quartile 360cm
Median 334cm

Represent this information on a box plot.

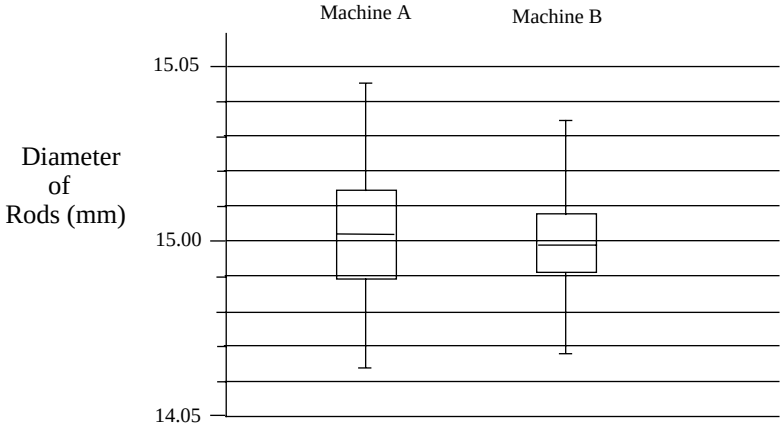
2) The box plot below represents the life of 30 Regular electric light bulbs tested by the Shiny Electric Light Company.



When they test the Super bulbs they get the following results.
Shortest life 350 hours Longest life 950 hours
Lower quartile 650 hours Upper quartile 770 hours
Median life 740 hours.

Copy the diagram and on it show the box plot for the Super bulbs.
Compare the two types of bulbs and make comments.

3) The diagram shows two box plots representing the diameters of rods made on two machines. Two samples of 100 rods were taken from each of the machines.



The rods are produced to a diameter of 15.00 mm and any rod which is more than 0.01mm above or below that value is a) scrapped if it is too small or b) re-machined if it is too large. Any value between these allowances is acceptable.
Use the diagrams to comment on the accuracy of the machines.

116. Sampling 1

1) The table shows the number of pupils in each year at Clivedenbrook Community College.

Year Group	7	8	9	10	11	12	13
Number of Pupils	173	147	166	144	140	49	21

Emma does a survey to find out what types of TV programmes people like to watch. She decides to interview 70 pupils.

a) Use a stratified sampling technique to decide how many pupils should be asked from each year.

b) Without using the names of any specific TV programmes, write down three relevant questions she could ask with a choice of at least 2 replies in each case.

2) The table shows the salaries of the 180 employees at the ACME sausage factory.

Amount Earned	<£10,000	£10,000 to <£15,000	£15,000 to <£20,000	£20,000 to <£30,000	£30,000 and over
Number of Employees	45	65	44	16	10

In order to understand whether they have satisfactory working conditions the management plan to interview 40 employees. They are to be chosen at random using a stratified sampling technique.

a) Calculate how many employees from each group should be interviewed.

b) Write down two questions which could be asked, each with a choice of two responses.

c) What other type of stratification could be used? Give one example.

3) The manager of a fitness centre wants to find out whether the members are satisfied with the amenities available. She would like ideas to improve the facilities. The table shows the age ranges for all the members.

Age Range	Under 25	25 to 40	40 to 60	Over 60
Number of Members	237	463	414	136

She decides to interview 100 members using a stratified sampling technique.

a) Calculate the number of people from each group that should be interviewed.

b) Write down two relevant questions, each with a choice from at least two responses.

117. Sampling 2

1) The table below shows the number of people belonging to a fitness club.

Age	Gender	Number of Members
< 18	F	45
18 ≤ Age < 50	F	233
≥ 50	F	115
< 18	M	65
18 ≤ Age < 50	M	265
≥ 50	M	77

A survey is carried out using a stratified sample of 100 people.
Calculate the number of people selected from each of the six groups.

2) The number of students in the six faculties at a university are shown in the table below.

Faculty	Number of Students	Stratified Sample
Arts	1085	47
Business		16
Education	139	6
Engineering	715	
Law	69	3
Science		27

Some of the figures have been left out.
The administrators decide to ask a sample of the students about the amenities within the college. They ask a stratified sample.
a) Complete the table.
b) How many students are at the university and how many are in the sample?

3) A car-sales company want to ask a sample of their customers about the service the showroom gives. They decide to send letters to 50 of the customers from last year. There were 660 car sales last year. The table below shows the number of cars sold in the different categories.

Type of car	Number sold	Stratified Sample
Mini	226	
Small Family	172	
Large Family	121	
Executive	85	
Estate	34	
4x4	22	

If the sample they use is stratified, calculate the number of people they need to choose from each category of car.

118. Sampling 3

Use this table of random 2 digit numbers to answer the questions below.

11	55	59	38	95	33	25	34	14	74
39	63	20	84	96	85	68	98	66	43
71	26	45	15	48	42	35	37	30	07
10	91	23	50	83	21	02	86	76	72
78	62	01	65	57	67	03	51	87	18

1) a) Explain how you could produce a table of random numbers, similar to the one above, using 10 sided dice like the one below.

b) How would you produce a table of 3 figure random numbers?

2) The table below shows the number of people belonging to a fitness club.

Age	Gender	Number of Members
< 18	F	45
18 ≤ Age < 50	F	233
≥ 50	F	115
< 18	M	65
18 ≤ Age < 50	M	265
≥ 50	M	77

a) A sample of 10 males aged 50 or more is needed for a survey. Describe how you would use the table of random numbers above to do it.

b) Which members would you choose?

c) Explain why you cannot use this table to choose a sample from the 18 to 50 male age group.

d) Explain what you would need to choose a sample from the 18 to 50 age group.

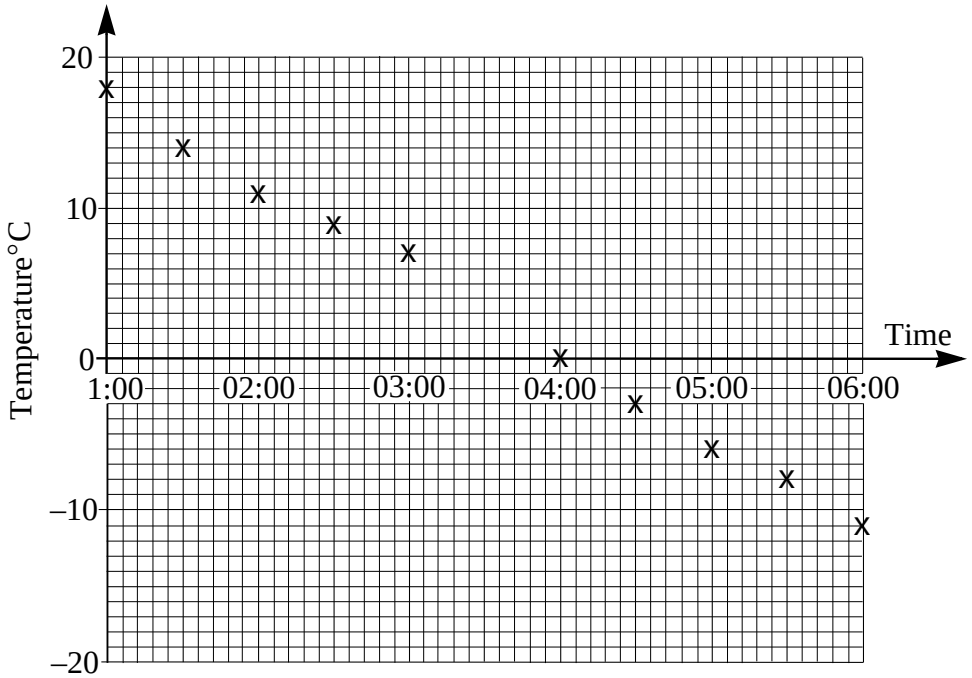
119. Scatter Diagrams 1

1. The table shows the sales of cans of drink from a vending machine, over a period of 12 days in June

Day	S	M	T	W	T	F	S	S	M	T	W	T
Temp. °C	15	17	20	25	21	18	15	20	23	27	30	29
Cans sold	104	113	188	275	212	150	90	205	251	330	425	404

   a) Construct a scatter diagram of the data and draw on it a line of best fit.
   b) Approximately how many cans would be sold on a day when the average temperature is 22°C ?

2. George's old freezer registers –4°C , which is higher than it should be so he buys a new one. The diagram shows the temperature inside the new freezer after he has started it up.



   The freezer is switched on at 1:00pm and he takes the temperature every 30 minutes.
   a) At what time would you expect it to reach its minimum temperature of –20°C ?
   b) He missed taking the temperature at 03:30. What was the approximate temperature?
   c) George transfers food from his old freezer to the new one when both the temperatures are equal. At approximately what time does he do this?

3. A lorry can carry up to 20 tonnes of sand. The lorry moves sand from the quarry to a collection point 2km away. The times it takes for 10 deliveries are shown in the table below. Draw a scatter graph of the data showing a line of best fit.

Time taken (minutes)	5.0	4.5	3.5	3.7	4.8	4.3	4.1	4.0	3.6	4.2
Amount of sand (tonnes)	10	14	20	18	13	15	17	16	19	17

   a) What weight of sand would take approximately 4 minutes 20 seconds to deliver?
   b) How long would a load of 16 tonnes take to deliver?

120. Scatter Diagrams 2

Draw a scatter diagram for each of the following results. Draw a line of best fit and use it to answer each of the questions. You must show clearly on your diagram how you get your answer.

1) A vertical spring, fixed at its upper end, was stretched by a weight at its lower end. The length of the spring for different loads was measured and the results recorded, as follows.

Load (g)	0	10	20	30	40	50	60	70	80
Length (cm)	12.5	13.4	14.7	15.5	16.3	17.7	18.6	19.4	20.6

From your diagram, estimate

a) The load when the length of the spring is 15cm.

b) The length of the spring for a load of 55g.

2) David travels to work each day by car. Most of his journey is down a ten mile stretch of motorway. Over a ten day period, he records the time taken to get to work and the speed he travels down the motorway.

Speed (mph)	65	50	45	50	70	55	60	70	70	45
Time taken (min)	19	24	26	25	17	22	21	19	18	28

From your diagram, estimate how fast he would have to travel down the motorway in order to get to work in 20 mins.

3) At the end of each week, Abigail saves whatever pocket money she has left from that week. She finds that the more times she goes out each week, the less money she has left. Here is a table showing the number of times she went out over a period of 8 weeks, together with the amount of money she saved.

Number of times out	5	3	4	1	1	4	5	2
Money saved (pence)	120	290	210	510	480	180	90	380

Abigail wants to save £2.50 every week for her holidays. From your graph decide on the maximum number of times she can go out each week.

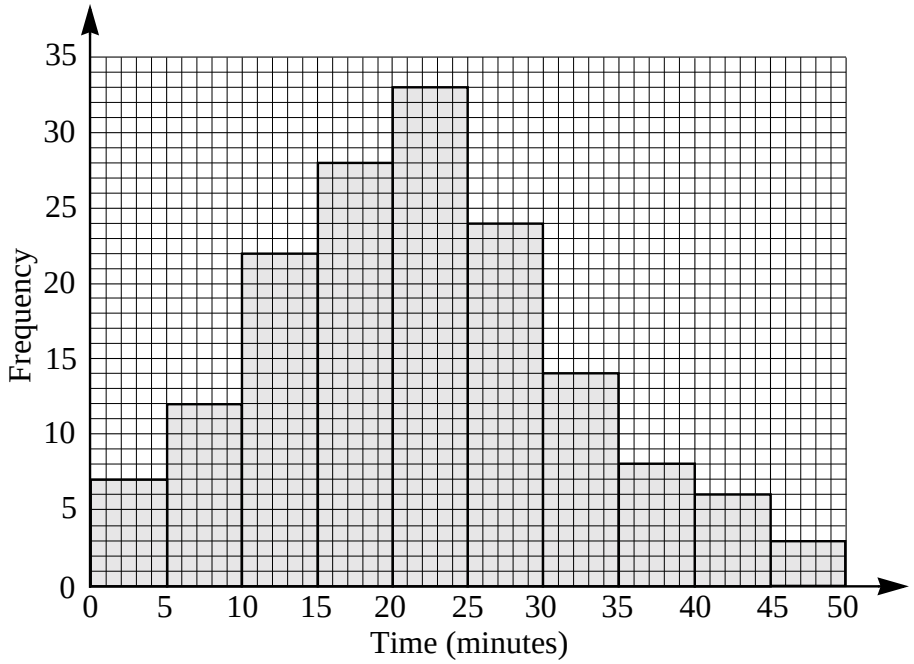
4) In an experiment twelve pupils were weighed and their heights measured. Here are their results.

Weight (kg)	52	59	58.5	63	60.5	66	66	66.5	72	76.5	71	79.5
Height (cm)	122	124	126	127	128	128	132	134	136	139	142	144

It is known that another pupil weights 75kg. Approximately how tall is he?

121. Histograms 1- Bar Charts

1. The time taken for the pupils in year 11 to get to school on Monday morning are shown below in the diagram.



a) Copy and complete this frequency chart.

Time	0-	5-	10-	15-	20-	25-	30-	35-	40-	45-50
Freq										

Use the data to calculate
b) the mean time
c) the percentage of pupils who took 25 minutes or more.

2. The members of a fitness centre were weighed and their masses noted in the table below.

51	53	61	77	82	93	107	67	73	70	69
51	81	99	105	47	64	77	69	82	41	65
79	62	108	98	80	75	61	65	52	41	43
69	73	78	81	73	63	74	86	42	56	58
64	81	76	63	84	92	103	94	85	72	63

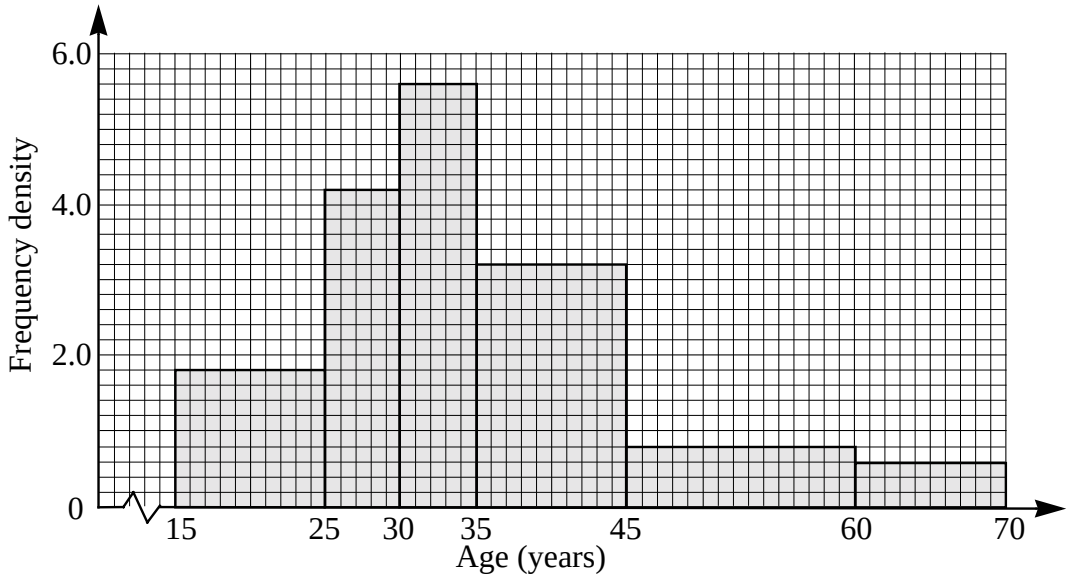
a) Copy and complete this frequency table.

Mass	40-	50-	60-	70-	80-	90-	100-110
Frequency							

b) Show this data on a bar chart
c) What is the modal class?

122. Histograms 2

1. The histogram shows the range of ages of members of a sports centre.

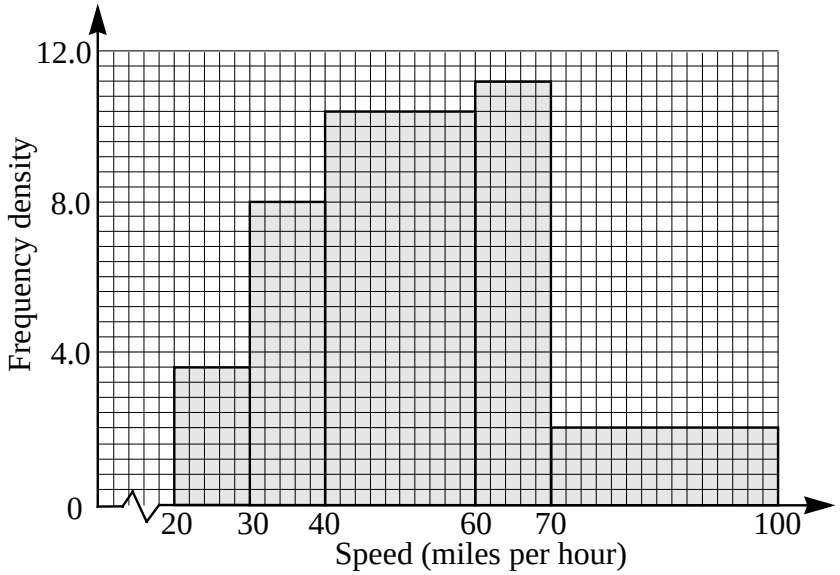


a) Complete this frequency table

Age	15-	25-	30-	35-	45-	60-70
Number of people						

b) Calculate an estimate for the mean age of the members.

2. Harry carries out a survey of the speeds of vehicles passing a point on a motorway. From the data he draws this histogram.



a) Complete this table

Speed	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 100$
Number of cars					

b) Which range of speeds is the modal range?

123. Histograms 3

1. The following list shows the heights of the tomato plants in a greenhouse.

94	124	113	103	127	106	131	132	112	118
106	117	117	123	102	114	133	117	118	101
127	109	119	93	110	126	139	108	97	113
109	119	114	128	101	129	111	121	126	110
91	122	116	99	125	116	108	125	114	107

a) Complete the following frequency table.

Height in centimetres	Number of plants	Frequency density
$90 < x \leq 100$	5	0.5
$100 < x \leq 105$		
$105 < x \leq 115$		
$115 < x \leq 120$		
$120 < x \leq 130$		
$130 < x \leq 140$		

b) Draw a histogram of the data.
c) From your histogram, find the percentage of tomato plants which are greater than 115cm.

2. A battery manufacturer tests the life of a sample of batteries by putting them into electric toys and timing them. The results she gets, in minutes, from 58 tests are shown below.

726	945	863	673	876	842	645	942	621	833
1042	526	735	893	621	773	531	733	635	998
954	763	1073	550	725	1084	747	849	716	1032
721	962	683	768	872	632	787	641	752	800
1063	794	1062	613	714	867	590	749	854	943
1021	681	943	842	841	730	961	982		

a) Complete the following frequency table

Time - Minutes	Frequency	Frequency density
$500 < x \leq 600$	4	0.04
$600 < x \leq 700$		
$700 < x \leq 750$		
$750 < x \leq 900$		
$900 < x \leq 1100$		

b) Draw a histogram to show this data
c) From the frequency table, estimate the percentage of batteries whose life expectancy is less than 800 minutes.

124. Mean 1

1. The table below shows the wages paid to a number of people working in a factory. Complete the table and calculate the mean wage.

Wages, £	Frequency	Mid value	Frequency × Mid value
$60 \le £ < 100$	4		
$100 \le £ < 140$	19		
$140 \le £ < 170$	24		
$170 \le £ < 200$	11		
$200 \le £ < 220$	6		

2. The table below shows the heights of a number of rose trees at a garden centre. Copy and complete the table of results. Calculate the approximate mean height of the roses.

Height of plant, h, centimetres	Frequency		
$50 \le h < 70$	5		
$70 \le h < 90$	14		
$90 \le h < 100$	16		
$100 \le h < 110$	24		
$110 \le h < 120$	21		
$120 \le h < 140$	23		
$140 \le h < 160$	17		

3. The table below shows the speeds of 60 vehicles passing a certain point on a motorway.

27.6	58.5	80.5	64.8	54.8	46.6	77.9	84.1	54.9	59.6
64.1	45.8	43.6	30.6	73.9	28.5	43.1	43.9	39.5	49.6
40.4	76.0	24.7	48.6	45.8	75.6	22.5	58.9	45.5	60.8
37.4	42.8	54.8	35.9	45.2	32.6	83.5	43.9	39.4	42.4
51.6	47.9	33.7	57.8	33.6	57.2	54.9	64.5	61.0	73.6
32.1	67.9	57.8	75.7	23.6	52.0	38.6	54.2	27.3	55.8

Make a frequency table from the values and hence calculate the approximate mean speed of the traffic, in miles per hour.

125. Mean 2

Find the mean value for each of the following sets of frequencies, correct to 4 significant figures.

1. The number of children in the families of pupils in a class.

No. of children in a family	frequency
1	7
2	13
3	5
4	2
5	1
7	1

2. The number of absentees from a class during a period of 60 days.

No. of absentees	frequency (days absent)
0	9
1	12
2	15
3	11
4	6
5	4
6	2
7	1

3. The number of broken glasses found when 1500 boxes, each containing 6 glasses were opened.

No. of broken glasses	frequency
0	1337
1	76
2	49
3	21
4	10
5	6
6	1

4) Packets of sweets each contain 24 sweets. The number of red sweets were counted in a sample of 100 packets.

Number of red sweets (number of packets)	frequency
3	5
4	12
5	33
6	23
7	18
8	9

126. Mean, Median and Mode.

1. 30 pupils in a class were asked to keep a record of the number of pints of milk their family bought during one week. The results are given below.

18	21	15	21	22	14	14	28	21	14
14	17	21	22	14	15	21	21	16	24
15	13	14	21	25	21	14	18	12	22

   a) What was the modal number of pints bought per week?
   b) What was the mean amount of milk bought, correct to the nearest 0.1 pint?
   c) What was the median amount of milk bought?

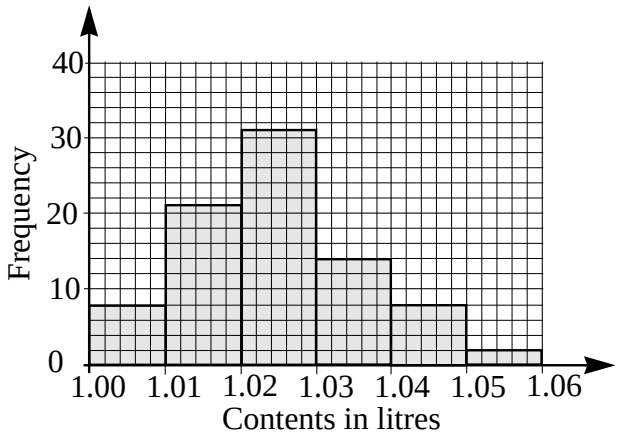
2. The length of the words in the first two sentences of Pride and Prejudice by Jane Austin are given in the table on the right.
   a) Calculate the mean length of the words (correct to 1 decimal place)
   b) Calculate the median number of letters per word.
   c) What is the modal number of letters in a word?
   d) Do you think that these values are a fair indication of the words in the whole book? Explain.

No. of letters	Frequency
1	6
2	21
3	9
4	10
5	9
6	2
7	2
8	5
9	1
10	2
11	2
12	1
13	1

3. The table on the right shows the weights of 100 packages brought into a post office during one day.
   a) Complete the table for the mid values.
   b) Calculate an estimate for the mean weight of a package brought in that day.
   c) In which class interval does the median value lie?
   d) What is the modal class interval?

Weight w	Frequency	Mid Value
$60g < w \leq 100g$	21	
$100g < w \leq 300g$	44	
$300g < w \leq 600g$	17	
$600g < w \leq 1kg$	9	
$1kg < w \leq 2kg$	5	
$2kg < w \leq 3kg$	4	

4. 84 packs of orange juice were sampled from a days production at a drinks factory. The contents of each pack were measured. The amounts are shown in the diagram on the right.
   From the graph:
   a) Write down the modal class.
   b) In which class interval does the median value lie?
   c) Calculate an estimate for the mean contents of a pack.



127. Mean, Median, Mode and Range

Exercise 1

In each of the following, put the data into a frequency table and write down the mode and range.

1. 2, 3, 3, 3, 4, 2, 6, 1, 5, 1, 1, 4, 4, 3, 5, 2, 3
   2, 1, 1, 1, 3, 1, 5, 4, 5, 1, 2, 2, 2, 3, 4, 7, 6
2. 1, 4, 7, 4, 3, 8, 0, 0, 1, 8, 2, 9, 2, 6, 0, 8
   2, 0, 6, 2, 8, 1, 9, 0, 3, 7, 1, 0, 7, 5, 1, 9
   8, 3, 2, 6, 2, 6, 0, 2, 5, 2, 8, 7, 3, 0, 1, 2
3. 13, 14, 17, 14, 15, 14, 17, 14, 13
   16, 15, 13, 16, 17, 15, 13, 16, 17
   17, 16, 14, 15, 13, 17, 14, 16, 16
   15, 15, 15, 14, 14, 15, 15, 15, 13
   15, 16, 13, 15, 14, 16, 14, 17, 15

Exercise 2.

Find the median and range of each of the following sets of data.

1. 8, 7, 4, 10, 1, 5, 6, 6, 5, 4, 3, 4, 8, 7 10, 4, 9, 5, 3, 2, 7
2. 9, 9, 7, 6, 7, 4, 3, 2, 3, 7, 7, 6, 5, 7, 5, 8
3. 70, 72, 30, 74, 80, 83, 36, 50, 38, 85
   92, 50, 70, 68, 17, 48, 77, 72, 60, 74
   14, 75, 83, 65, 33, 52, 46, 34, 32, 37

Exercise 3.

Calculate the mean of each of the following sets of data, giving your answer correct to four significant figures wherever necessary.

1. 4cm, 7cm, 8cm, 5cm, 4cm, 3cm, 2cm, 9cm, 8cm, 6cm
2. 21 grams, 40 grams, 8 grams, 73 grams, 68 grams
3. 6 metres, 4m, 3m, 8m, 5m, 6m, 4m, 7m, 2m, 5m
4. 13, 16, 20, 24, 27, 29, 33
5. 221, 352, 234, 421, 301, 383
6. 2.6, 1.9, 2.7, 2.1, 3.2, 3.0
7. $43\frac{1}{2}, 47\frac{1}{2}, 39\frac{1}{2}, 34\frac{1}{2}$
8. 179, 111, 152, 233, 244, 221
9. 141, 126, 117, 64, 72, 65, 85, 120, 141, 132
10. 41, 85, 72, 17, 41, 16, 54, 55, 10

Exercise 4.

In each of the following give the answer correct to four significant figures wherever necessary.

1. The mean of six numbers is 25.5 and the mean of a further seven numbers is 23. What is the mean of all thirteen numbers combined?
2. The mean weight of six people is 83kg. If three more people, weighing 93kg, 107kg and 78kg join them, what is their new mean weight?
3. The mean weight of six people in a lift is 90kg. If the maximum total weight allowable in the lift is 1 tonne, approximately how many more people will be allowed in?

128. Mean and Standard Deviation

1. Nigel weighs 10 bags of potatoes. His readings are:
   5.01kg, 5.03kg, 5.07kg, 5.05kg, 5.08kg, 5.04kg, 5.03kg, 5.01kg, 5.04kg, 5.08kg.
   a) Calculate the mean and standard deviation of the readings.
   b) Nigel notices a sign on the scales saying that all measurements are undersize by 0.02kg. Explain the effect this has on (i) the mean (ii) the standard deviation.
2. The heights of the children in year 10 were measured by the school nurse. Her results are shown below.

Height of child (cm)	Frequency f	Mid point x
$120 < h \leq 130$	3	
$130 < h \leq 140$	7	
$140 < h \leq 150$	20	
$150 < h \leq 160$	54	
$160 < h \leq 170$	32	
$170 < h \leq 180$	9	

   a) Complete the table for the mid point values.
   b) Use the mid point values to calculate (i) the mean (ii) the standard deviation, for the heights of the children, giving your answer correct to one decimal place.
3. The weekly wages of the employees at a supermarket are given in the table below.

Wage £	Frequency f	Mid Value x
$80 < £ \leq 120$	5	
$120 < £ \leq 160$	8	
$160 < £ \leq 200$	9	
$200 < £ \leq 240$	11	
$240 < £ \leq 280$	14	
$280 < £ \leq 320$	17	
$320 < £ \leq 360$	3	

   a) Complete the table for the mid values.
   b) Using the mid point values, calculate the mean and standard deviation of the distribution, giving your answers correct to two decimal places.
   c) It is decided to give each person in the store a bonus of £10 for one week. What effect will this have on the mean and the standard deviation for that week?
4. The list below shows the ages of the 30 members of a tennis club.

21	65	33	29	32	29	53	61	39	48
42	45	46	27	48	35	46	25	43	59
24	37	44	63	39	22	38	33	55	51

   a) Complete this grouped frequency table.

Age (years)	Frequency	Mid value
$20 \leq a < 30$		
etc		

   b) Use this table of values to calculate an estimate of the mean and standard deviation of the distribution.
   c) At the Badminton club, the mean age is 48 and the standard deviation is 8.4. Comment on the differences between these figures and those for the tennis club.

129. Moving Averages

The table below shows the value of the shares of a company over a period of 24 weeks. The value of each share is taken at the end of each week. A shareholder needs to know the trend in the share value of the company and does this by calculating an 8 week moving average.

Day	Value of a share in pence	8 week moving average
1	251	
2	263	
3	294	
4	330	
5	350	
6	345	
7	342	
8	340	
9	336	
10	331	
11	330	
12	321	
13	315	
14	302	
15	321	324.5
16	317	
17	320	
18	318	
19	307	
20	301	
21	331	
22	287	
23	305	
24	306	

a) Explain why there can be no moving average for week 5.

b) When can the first moving average be calculated?

c) Complete the table of moving averages.

d) By looking at the moving averages, what do you think the trend in his share values is?

130. Frequency Polygons 1

1) These tables show the average monthly rainfall and temperature at two holiday destinations.
Destination A

Month	J	F	M	A	M	J	J	A	S	O	N	D
Temp ºC	9	10	15	17	20	25	28	31	25	21	14	11
Rainfall cm	5.2	4.0	3.6	1.9	1.8	1.7	1.0	1.2	1.9	4.1	8.3	6.2

Destination B

Month	J	F	M	A	M	J	J	A	S	O	N	D
Temp ºC	27	30	32	26	23	17	13	11	12	17	20	23
Rainfall cm	3.2	4.4	5.7	6.3	7.2	8.0	6.4	6.2	5.0	4.1	3.5	3.1

a) Draw a frequency polygon showing the *temperatures* at both destinations. Show both polygons on the same diagram.
(i) Which destination is in Australia and which is in Europe?
(ii) Give reasons for your choice.
(iii) In which months of the year are the temperatures about the same?
b) Draw a frequency polygon to show the *rainfall* at both destinations.
Choose the best month to go on holiday, for each destination. Explain why you made your choices.

2) The following table shows the amount of profit made by a company during 1991 and 1992 (in millions of pounds)

Year	J	F	M	A	M	J	J	A	S	O	N	D
1991	5.4	5.3	4.9	4.4	4.5	4.0	3.2	3.1	2.9	3.5	4.0	3.9
1992	3.7	3.4	3.0	2.7	2.8	2.7	2.1	2.0	2.1	1.9	2.7	2.5

a) Draw a frequency polygon showing the profits for both years. Show them both on the same diagram. Plot the profit vertically and the twelve months of the year horizontally.
b) From the diagram, make comments on
(i) The profit in 1992 compared with 1991.
(ii) The trend at the end of 1992 (i.e. are profits still going down or are they picking up?) Explain your answer.
(iii) Predict what the profits will be for the first three months of 1993.

3) Devonham High School are allowed to enter one person for each event in the annual county games. The three best athletes in the 100 metres are Brian, Mike and John. At the last ten races in which they ran against each other, their times (in seconds) were

Brian	12.3	12.2	12.4	12.3	12.4	12.2	12.2	12.4	12.3	12.4
Mike	12.2	12.7	12.9	12.1	12.0	12.7	12.9	12.8	12.1	12.7
John	12.9	12.9	12.8	12.7	12.6	12.5	12.4	12.3	12.3	12.2

Draw the three frequency polygons on one diagram, using a different colour for each.
Use a scale of 1cm between each of the races on the horizontal axis and 1cm to represent 0.1sec on the vertical axis. Begin your vertical scale at 12 secs.
From the diagram decide who is to represent the school. Explain why you chose that person.

4) The table below shows the profits made by two companies during 1992.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Company A	2.3	2.5	2.6	2.4	2.8	2.5	3.1	2.9	3.4	3.0	3.7	3.9
Company B	4.2	4.1	3.5	3.2	2.7	2.5	2.2	2.7	2.8	2.9	2.9	3.4

Draw a frequency polygon showing the profits for both companies. Show both polygons on the same diagram. Compare the two graphs and make comments

131. Frequency Polygons 2

1. The data below shows the weights of 30 cats treated by a vet over a period of a week.

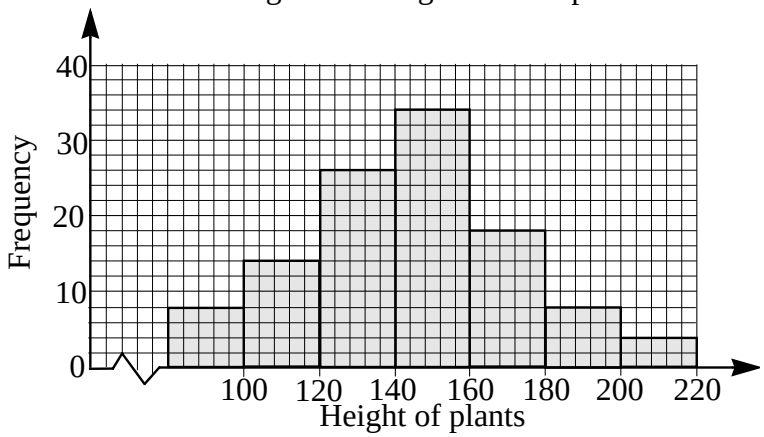
3.1	4.9	4.1	3.7	4.3	3.6	4.3	4.8	4.2	4.2
3.7	4.1	5.4	4.6	3.1	4.2	3.3	4.4	4.4	3.7
3.3	4.3	4.7	5.3	4.1	4.6	5.1	3.6	4.1	3.6

a) Use the data to complete this grouped frequency table.

Weight w kg	Frequency	Mid value
$3.0 < w \leq 3.5$		
$3.5 < w \leq 4.0$		
$4.0 < w \leq 4.5$		
$4.5 < w \leq 5.0$		
$5.0 < w \leq 5.5$		

b) From your table construct a frequency polygon for the data. Use a scale of 4cm to represent 1kg on the horizontal axis and 2cm to represent 2 cats on the vertical axis.

2. The histogram below shows the heights of 112 greenhouse plants



a) Use the histogram to complete this grouped frequency table.

Height h cm	Frequency	Mid value
$80 < h \leq 100$		
$100 < h \leq 120$		
$120 < h \leq 140$		
$140 < h \leq 160$		
etc.		

b) Use the frequency table to plot a frequency polygon for the data. Use a scale of 1cm to represent 10 cm on the horizontal axis and 2cm to represent 4 plants on the vertical axis.

132. Cumulative Frequency 1

1. The table below shows the lengths (l) of 100 engineering components taken at random from one days production in a light engineering works.

Length l	$l \leq 9.7$	$9.7 < l \leq 9.8$	$9.8 < l \leq 9.9$	$9.9 < l \leq 10.0$	$10.0 < l \leq 10.1$	$10.1 < l \leq 10.2$
Frequency	4	13	24	36	17	6

a) Copy and complete this cumulative frequency table for this data.

Length l	9.7	9.8	9.9	10.0	10.1	10.2
Cumulative Frequency	4	17				

b) Draw a cumulative frequency diagram. Use the scale of 2cm for 10 components on the vertical axis and 2cm for 0.1 unit on the horizontal axis.
From your diagram find:
c) The median length.
d) The upper and lower quartiles and hence the interquartile range.
e) The components measuring 9.75cm or less are under size and will be scrapped. Approximately what percentage will be scrapped?

2. The table shows the ages of the 250 employees at the headquarters of ‘MoneyBankPlc’.

Age	16-	20-	30-	40-	50-	60-65
Frequency	17	65	78	51	30	9

a) Draw up a cumulative frequency table for this data.
b) From the table draw a cumulative frequency diagram. Use a scale of 1cm to represent 10 employees on the vertical axis and 1cm to represent 5 years on the horizontal axis.
c) The company decide to offer early retirement to those who are 45 or over. From the diagram estimate approximately what percentage of employees this will involve.

3. The table shows the marks gained by 200 students taking a mathematics examination.

Mark	≤10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	3	7	22	31	34	37	28	19	13	6

a) Draw up a cumulative frequency table from this data.
b) From the table draw a cumulative frequency diagram. Use the scale of 1cm to represent 10 pupils on the vertical axis and 2cm to represent 20 marks on the horizontal axis.
c) If 60% of the students passed the examination, what was the approximate pass mark?
d) Those who get a mark of 75 or more are awarded a distinction. What percentage of the students are awarded a distinction?

133. Cumulative Frequency 2

1. ‘Shiny’ long life bulbs are guaranteed to last at least 7,500 hours before breaking down. In order to make this guarantee, ‘Shiny’ did trials on 300 bulbs. These are the results:

Life of bulb (hrs)	5000 to 5999	6000 to 6999	7000 to 7999	8000 to 8999	9000 to 9999	10,000 to 11,000
Frequency of breakdown	5	20	42	95	80	58

   a) Make a cumulative frequency table for this data.
   b) From the table, draw a cumulative frequency diagram. Use a scale of 2cm to 1000 hours on the horizontal axis and 2cm to 40 bulbs on the vertical axis.
   c) From the graph, estimate the percentage of bulbs that fulfil the guarantee.

2. A flying school gives 1 hour lessons. Some of that time is spent on the ground having instruction and some in the air. The time spent in the air for 100 lessons were as follows.

Time spent in the air	20-30 mins	30-35 mins	35-40 mins	40-45 mins	45-50 mins	50-55 mins
Frequency	6	10	12	34	23	15

   a) Make a cumulative frequency table for this data. From the table, draw a cumulative frequency diagram. Use a scale of 1cm to 2 mins horizontally and 1cm to 5 people vertically.
   c) From the diagram, estimate the median and interquartile range of the data.

3. The times that trains arrive at a station are recorded by the station staff. They record whether a train is early or late and by how much. The table below shows their data for one day.

Timing	10-5 mins early	5-0 mins early	0-5 mins late	5-10 mins late	10-15 mins late	15-20mins late
Frequency	33	66	22	17	10	2

   a) Make a cumulative frequency table for this data.
   b) From the table, draw a cumulative frequency diagram. Use a scale of 2cm to represent 5 minutes on the horizontal axis and 1cm to represent 10 trains on the vertical axis.
   c) The train company guarantees that its services will not be more than 7 minutes late. Use your graph to check what percentage were outside the guarantee.

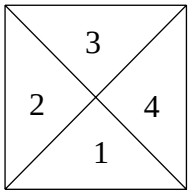
4. A large electrical retailer keeps stocks of television sets in its main warehouse for delivery to its shops. Over a period of 1 year (365 days) the number of TV’s in the warehouse are given in the table below.

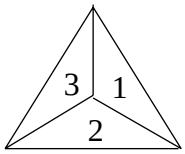
No. of TV’s in warehouse	50-100	101-150	151-200	201-250	251-300	301-350
No of days (frequency)	5	27	77	145	83	28

   a) Make a cumulative frequency table for this data.
   b) Draw a cumulative frequency diagram using a scale of 2cm to represent 40 TV’s on the horizontal axis, and 2cm to represent 40 days on the vertical axis.
   It is the policy of the company to keep a stock of between 130 and 260 TV’s.
   c) For approximately how many days was the stock within these limits?
   d) A stock of 110 TV’s is regarded as the minimum quantity that the warehouse should have. For approximately how many days was the stock below the minimum requirement?

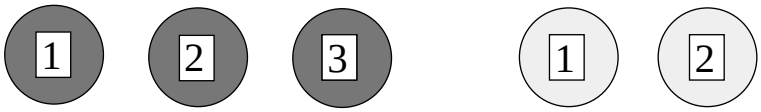
134. Probability 1

1) The diagram shows two spinners, one numbered 1 to 4, the other 1 to 3. The outcome 1+2=3 is shown
   a) Make a list of all the possible outcomes.
   b) What is the probability of getting numbers adding up to 5?
   c) What is the probability of getting a sum of more than 5?





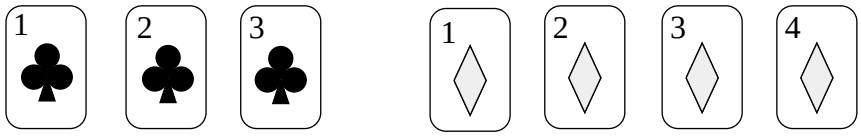
2)



Red discs Blue discs

Three red discs are numbered 1 to 3, and two blue discs are numbered 1 and 2. A red disc is chosen at random followed by a blue disc. List all the possible outcomes. What is the probability of getting
   a) a 2 followed by a 1?
   b) a 1 and a 2 in any order?
   c) a 3 and a 1 in any order?

3)



Three black cards are numbered 1 to 3 and 4 red cards are numbered 1 to 4.
A black card is chosen at random followed by a red card.
   a) List all the possible outcomes.
   b) In how many ways can the cards add up to 6?
   c) What is the probability of the two cards adding up to 6?

4) A bag contains 5 red discs with the numbers 1 to 5 on them. A second bag holds 6 discs, 2 white, 2 black, one green and one yellow. A disc is taken at random from both bags . Copy and complete this table of possible results.

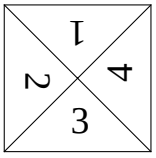
		First bag				
		1	2	3	4	5
Second bag	W	W1				
	W					
	B					
	B		B2			
	G					G5
	Y					

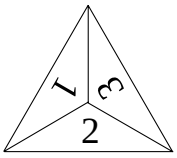
Use the table to find the probability of choosing
   a) a 1 followed by a black disc
   b) a black disc
   c) a black or white disc.

135. Probability 2

1. A bag contains 4 red discs and 5 green discs. A disc is selected at random from the bag, its colour noted and then replaced. This is carried out 3 times. Calculate the probability of getting:
   a) a green disc on the first draw
   b) a green disc followed by a red disc
   c) three green discs
   d) a green disc and two red discs in any order.

2. A game is played using a four sided spinner and a three sided spinner. The two spinners are spun together and their two values added together in the way shown in the diagram.





3+2 = 5

   Calculate the probability of getting an outcome of a) 3 b) 5 c) not 6

3. The probability that a parcel will be delivered the next day is 0.4. Two parcels are sent, independently of each other, on the same day. What is the probability that:
   a) both will be delivered the next day?
   b) only one will be delivered the next day?
   c) neither will be delivered the next day?

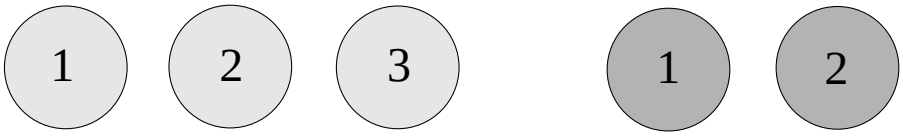
4. A biased dice has the numbers 1 to 6 on its faces. The probability of throwing a 1 is 0.1 and the probability of throwing a 2 is also 0.1. The probability of throwing each of the remaining 4 numbers is 0.2. Calculate the probability of throwing:
   a) a 1 followed by a 3
   b) a 2 followed by a 4
   c) two 2's.

5. The probability that the post arrives before 8.00 am is 0.2 and the probability that the daily newspaper arrives before 8.00 am is 0.7. Calculate the probability that:
   a) both newspaper and post arrive before 8.00
   b) the post arrives before 8.00 but the newspaper arrives after 8.00
   c) both arrive after 8.00.
   d) one or both of them don't arrive before 8.00.

6. A fair dice having the numbers 1 to 6 on it is rolled and its value noted. It is rolled a second time and its value added to the first one. For example, if 3 is rolled the first time and 2 the second time, the total after 2 rolls will be 5.
   a) What is the probability that the score will be greater than 12?
   b) What is the probability of scoring between 6 and 9 inclusive with two throws of the dice?

136. Probability 3

1. A bag contains 3 black discs and 7 white discs. A disc is taken at random from the bag and not replaced. This is carried out three times. Calculate the probability of getting:
   a) a black disc on the first draw
   b) a black disc followed by a white disc
   c) a black disc followed by two white discs.
2. It is known from past experience that of every 350 students entered for a typing examination 280 will pass the first time. Of those that fail, 80% of them pass on their second attempt. What is the probability that a student, chosen at random from a group of students sitting the examination for the first time, will pass on the first or second attempt?
3. The probability that there will be passengers waiting to be picked up at a bus stop is 0.85. A long distance bus has 3 stops to make. Calculate the probability that it will have to pick up passengers one or more times on its journey.
4. In a game of chance, two people each have a dice numbered 1, 1, 1, 2, 2, 2. They both throw the dice together. If both the outcomes are the same, then they have a second try. If they are the same again, then they have a third try, and so on until someone gets a 2 while the other gets a 1. The winner is the person who gets the 2.
   a) What is the probability that the game is over on the first pair of throws?
   b) What is the probability that someone wins on the second pair of throws?
5. In a class of 30 pupils there are 14 boys and 16 girls. Two names are chosen at random. Calculate the probability that:
   a) the first name selected is a girls'
   b) both the names selected are girls'.
6. In a bag there are 5 discs, 3 red and 2 blue. The red discs have the numbers 1, 2 and 3 written on them and the blue discs have the numbers 1 and 2 on them.



Red Discs Blue discs

   a) What is the probability of withdrawing a disc with a 2 on it from the bag?
   b) Two discs are drawn from the bag. What is the probability that one disc has a 2 on it and one disc is red?
7. An engineering works produces metal rods. The rods are 6.2cm long and 0.5mm in diameter, both dimensions to the nearest 0.1mm. When the rods are inspected there can be four different outcomes:
   either one or both dimensions are under size, in which case the rod is scrapped,
   or one dimension is oversize and one is correct, in which case it is reworked,
   or both dimensions are oversize, in which case the rod is reworked,
   or both dimensions are correct, in which case the rod is acceptable.
   It is known from past experience that 1.2% are under size in length and 1.5% are under size in diameter. Also 2.5% are oversize in length and 2.2% are oversize in diameter.
   A rod is selected at random. What is the probability that it is:
   a) scrapped b) reworked c) accepted?

137 Relative Frequency 1

1) Over a period of 10 minutes, 5 buses, 40 cars, 10 lorries and 15 vans travel down the high street.
   a) What is the probability that the next vehicle will be a van?
   b) Over the next hour approximately how many vehicles would you expect to travel down the street?
   c) How many of those vehicles would you expect to be buses?
   d) During another period, 50 lorries are observed. Approximately how many vans would you expect?

2) A fair dice having the numbers 1 to 6 is thrown 60 times.
   a) How many times would you expect the number 6 to occur?
   b) How many times would you expect the number to be greater than 4?

3) A bag contains red, white and yellow coloured discs. A disc is taken from the bag, its colour noted and then replaced. This is carried out 100 times. Red is chosen 52 times, white 29 times and yellow 19 times.
   a) If there are 10 discs in the bag, how many of each colour would you expect there to be?
   b) If the experiment is carried out 500 times, how many times would you expect to get a white disc?

4) A machine makes plastic cups. An inspector checks 100 cups and finds that 90 are acceptable and 10 are not.
   a) What is the probability that the next cup will be acceptable?
   During the day 100,000 cups are made.
   b) Approximately how many are likely to be unacceptable?
   c) If 12,000 cups are unacceptable the next day, approximately how many have been made altogether?

5) Raffle tickets are sold in aid of the local church. 500 are blue, 200 white and 50 pink. They are all put into a box and taken out at random.
   a) What is the probability that the first ticket is blue?
   b) If there are 12 prizes to be won, about how many prize-winners would you expect to have a blue ticket?

6) Ben has a biased coin. He tosses the coin 60 times and fills in the table shown below as he is doing it.

Number of tosses	10	20	30	40	50	60
Number of heads	7	10	18	22	29	34
Number of tails	3	10	12	18		
Probability of a head	0.7	0.5	0.6			
Probability of a tail	0.3	0.5				

   a) Copy and complete the table
   b) Plot a graph of 'Number of tosses' against 'Probability of a head'. Use the scale of 2cm to represent 10 tosses on the horizontal axis and 2cm to represent 0.1 on the vertical axis. From your graph estimate the probability of getting a head.

138. Relative Frequency 2

1. A biased coin is tossed 50 times. The number of times it comes down heads is 31 and the number of times it comes down tails is 19. If it is now tossed 1000 times, how many times would you expect it to show heads?
2. In an opinion poll for the general election, 500 people in a constituency were asked what party they would vote for. 220 said they would vote for the Left party, 170 for the Right party and 110 for the Centre party. The 500 people can be taken as representative of the population of the constituency.
   a) What is the probability that a person chosen at random will vote for:
      (i) the Left party (ii) the Right party (iii) the Centre party?
   b) It is expected that 45,000 people will vote in the election. Estimate the number of people who vote for each party.
3. A chocolate company make sugar coated chocolate sweets in red, yellow and green. It is known that more people prefer red sweets than any other colour so the company mix them in the ratio 5 red to 3 yellow to 2 green.
   a) In a bag of 50 sweets, how many of each colour would you expect?
   b) David takes a sweet from the bag without looking at it. What is the probability that it will be:
      (i) red (ii) yellow (iii) green?

   Sarah only likes the red sweets. She eats them all from her bag without eating any of the others. She gives the bag to David who now takes a sweet. What is the probability that it is:
      (i) green (ii) yellow?

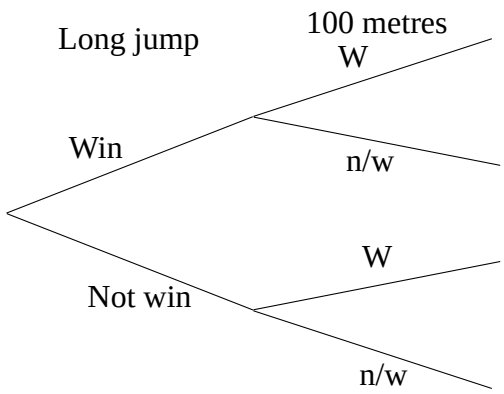
   During one days production of sweets at the factory, the company make 100,000 green sweets. How many red and yellow sweets do they make?
4. An unfair triangular spinner has the numbers 1, 2 and 3 on it. It is spun 100 times. Number 1 is obtained 25 times, number 2 occurs 30 times and number 3 occurs 45 times.
   a) Write down the probability of each number occurring.
   b) If it is spun 550 times, how often would you expect each number to occur?
5. A survey is carried out over a period of six days to determine whether the voters are for or against having a new by-pass outside their town. Each day 50 people were asked their opinion. The results were put into the following table.

Number of people	50	100	150	200	250	300
Number ‘for’	30	45	55	83	96	122
Number ‘against’	20	55				
Probability ‘for’	0.60	0.45				
Probability ‘against’	0.40	0.55				

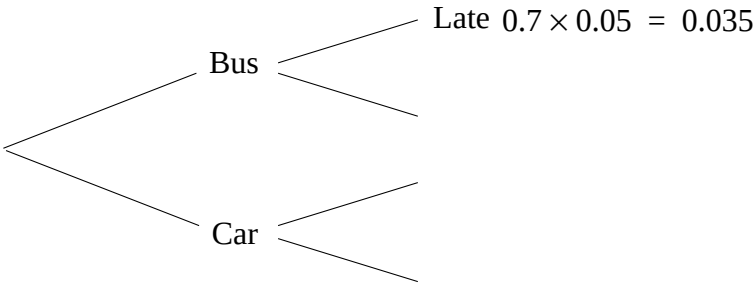
   a) Copy and complete the table.
   b) On graph paper plot ‘Number of people’ against ‘Probability for’. Use a scale of 2cm to represent 40 people on the horizontal axis and 2cm to represent 0.1 on the vertical axis.
   c) From your graph estimate the probability that a voter chosen at random would be for the by-pass.
   d) There are 70,000 voters in the town. If the people asked were a representative sample of the voting population, how many would you expect to be against the by-pass?

139. Tree Diagrams

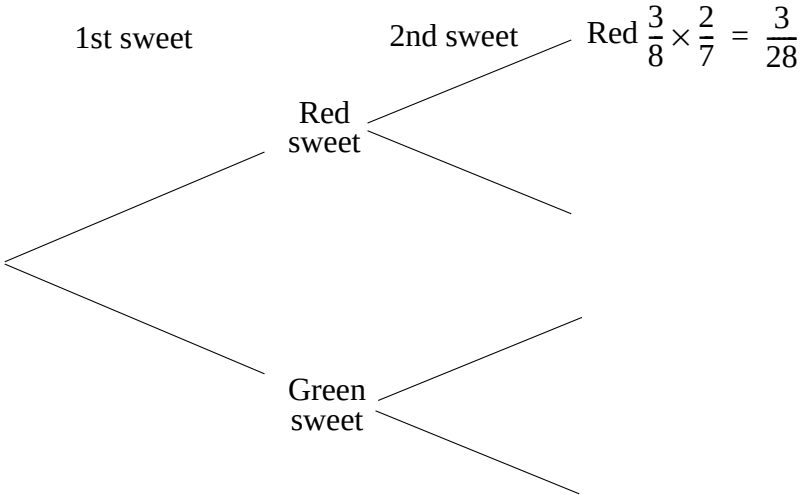
1. The probability that Sarah will win the long jump is 0.1 and, independently, the probability that she will win the 100 metres is 0.15.
   a) Complete the tree diagram to show all the possible outcomes.
   b) What is the probability that she wins one or both events?
   c) What is the probability that she wins neither event?



2. William waits at the bus stop each morning in order to catch the bus to work. Some mornings his friend arrives in his car and gives him a ride to work. From past experience the probabilities of William getting the bus is 0.7 and the probability that he gets a ride with his friend is 0.3. When he gets the bus, the chance of him being late for work is 0.05 and when he gets a ride the chance of him being late is 0.15.
   a) Complete the tree diagram to show all the possible outcomes.
   b) From the diagram calculate the probability that on a day chosen at random he will be late.



3. A bag contains 5 green sweets and 3 red sweets. Two sweets are withdrawn at random from the bag.
   a) Complete the diagram to show all the possible outcomes
   b) What is the probability of getting:
   (i) a sweet of each colour
   (ii) at least one red sweet?



4. A bag contains 8 discs, 4 black and 4 white. 2 discs are removed at random from the bag. By drawing a tree diagram calculate the probability that:
   a) both discs will be black b) at least one of the discs will be white.

5. In class 7B, two pupils are chosen at random from the class register to be the monitors. There are 12 boys and 16 girls in the class. By drawing a tree diagram calculate the probability that:
   a) the two people chosen will be girls b) the two people chosen will be boys
   c) one person of each gender will be chosen.

Answers

1. Estimations and Calculation

1) a) 1.857 b) $\frac{2\times5}{5}$ c) 2

2) a) 20.12 b) $\frac{10\times4}{2}$ c) 20

3) a) 18.02 b) $\frac{9\times9}{4}$ c) 20

4) a) 1.002 b) $\frac{0.5\times7}{3}$ c) 1

5) a) 0.1074 b) $\frac{3\times0.4}{9}$ c) 0.1

6) a) 0.3274 b) $\frac{8\times0.3}{8}$ c) 0.3

7) a) 0.1099 b) $\frac{0.7\times0.8}{5}$ c) 0.1

8) a) 0.01814 b) $\frac{0.2\times0.6}{6}$ c) 0.02

9) a) 0.06096 b) $\frac{0.6\times0.8}{8}$ c) 0.06

10) a) 1.035 b) $\frac{2\times0.2}{0.4}$ c) 1

11) a) 12.22 b) $\frac{8\times0.3}{0.2}$ c) 12

12) a) 4.332 b) $\frac{20\times0.2}{0.8}$ c) 5

13) a) 616.3 b) $\frac{10\times30}{0.6}$ c) 500

14) a) 6500 b) $\frac{30\times100}{0.4}$ c) 7500

15) a) 21080 b) $\frac{200\times30}{0.3}$ c) 20000

16) a) 0.9753 b) $\frac{20\times0.02}{0.3}$ c)1

17) a) 1445 b) $\frac{30\times0.8}{0.02}$ c) 1200

18) a) 1,411,000 b) $\frac{70\times30}{0.001}$ c) 2,100,000

19) a) 395.9 b) $\frac{(30)^2\times10}{30}$ c) 300

20) a) 5.303 b) $\frac{5\times(0.8)^2}{0.6}$ c) 5

21) a) 58.88 b) $\frac{20\times(20)^2}{100}$ c) 80

22) a) 246.9 b) $\frac{(20)^2\times2}{0.6\times3}$ c) 400

23) a) 0.5510 b) $\frac{100\times(0.3)^2}{0.8\times30}$ c) 0.4

24) a) 0.7783 b) $\frac{(0.3)^2\times30}{0.5\times7}$ c) 0.8

25) a) 13.36 b) $\frac{40\times7}{20}$ c) 14

26) a) 6.883 b) $\frac{10\times20}{50}$ c) 4

27) a) 2.099 b) $\frac{30\times0.9}{10}$ c) 2.7

28) a) 16.89 b) $\frac{30^2\times0.4}{20}$ c) 18

29) a) 1231 b) $\frac{40\times10}{0.4}$ c) 1000

30) a) 639.9 b) $\frac{30^2\times20}{3\times10}$ c) 600

31) a) 7009 b) $\frac{30\times20^2}{0.3\times4}$ c) 10,000

32) a) 1372 b) $\frac{30^2\times0.8^2}{0.4}$ c) 1440

33) a) 213.1 b) $\frac{40\times4^2}{0.3\times10}$ c) 160

34) a) 8.471 b) $\frac{10\times30}{40}$ c) 7.5

35) a) 2455 b) $\frac{40^2\times0.02}{0.8\times0.01}$ c) 4000

36) a) 0.1352 b) $\frac{0.5^2\times0.006}{0.8\times0.02}$ c) 0.09

37) a) 6784 b) $\frac{100}{0.02}$ c) 5000

38) a) 5.866 b) $\frac{27000\times0.008}{32}$ c) 7

2. Multiplication and Division

Exercise 1

1) 8r1 2) 13r5 3) 11r6 4) 26r2 5) 20r3
6) 66r5 7) 124 8) 123 9) 194r4 10) 57r6
11) 37r8 12) 96

Exercise 2

1) 5r2 2) 4r4 3) 7r7 4) 4r7 5) 6r20
6) 8r5 7) 7r33 8) 13r1 9) 12r4 10) 11r17
11) 13r14 12) 10r14 13) 12r21 14) 16r1
15) 16r23 16) 29r7 17) 40r1 18) 16r24
19) 20r5 20) 18r38

Exercise 3

1) 7.5 2) 6.25 3) 7.25 4) 6.8 5) 7.5
6) 15.5 7) 23.5 8) 11.25 9) 20.25
10) 45.5 11) 5.52 12) 26.25 13) 36.25
14) 29.75 15) 53.5 16) 19.5 17) 2.94
18) 10.08 19) 8.4 20) 145.75

Exercise 4

1) 864 2) 1596 3) 1222 4) 1122 5) 4644
6) 2057 7) 5832 8) 4862 9) 12032
10) 1062 11) 23562 12) 39566 13) 37066
14) 54592 15) 11529 16) 62926 17) 45663
18) 26904 19) 36501 20) 43808

3. Negative numbers

Exercise 1

1) 14 2) 2 3) –3 4) 2 5) –13 6) 5 7) –20
8) –2 9) –4 10) –11 11) –14 12) 14 13) –6
14) 0 15) 0 16) –23 17) –17 18) 9 19) –26
20) –4

Exercise 2

1) 4 2) 6 3) 9 4) 9 5) 3 6) 7 7) 3 8) 9
9) 8 10) 5 11) 4 12) 12 13) 5 14) 24
15) 6 16) 34 17) 46 18) 8 19) 16 20) 32

Exercise 3

1) 4 2) 0 3) 6 4) 16 5) 5 6) 6 7) 3 8) 7 9) 2 10) 7 11) 7 12) 8 13) –9 14) –13 15) 0 16) –7 17) –7 18) 16 19) –10 20) 10

Exercise 4

1) –5 2) 3 3) 10 and –6 4) 12 and –12

4. Use of the Calculator

Exercise 1

1) 6,3 2) 23,32 3) 12,48 4) 27,12 5) 8,2 6) 7,4 7) 26,36

Exercise 2

1) 2.195 2) 4.682 3) 7.2 4) 3.668
5) 0.5193 6) 0.2980 7) 4.1 8) 8.453
9) 0.7714 10) 1.960 11) 36.25 12) 1.74 13) 0.32
14) 2.031 15) 3.520 16) 3.890
17) 11.83 18) 3.344 19) 5.305 20) 14.82
21) –0.6382 22) 25.05 23) 2.720
24) 9.214 25) 4.922 26) 8.928
27) 4.801 28) 1.993

5. Standard Form

Exercise 1

1) 4.57×10^{2} 2) 1.427×10^{3} 3) 9.431×10^{3}
4) 1.56321×10^{5} 5) 1.7×10^{7} 6) 2.813×10^{-1}
7) 8.142×10^{-2} 8) 4.86×10^{-4} 9) 9.7×10^{-6}

Exercise 2

1) 280,000 2) 64,000,000 3) 93,000
4) 4,315,000 5) 8,614,000,000 6) 0.0431
7) 0.0000032 8) 0.000000684 9) 0.00000000438

Exercise 3

1) 2.432×10^{7} 2) 4.482×10^{9} 3) 1.564×10^{-4}
4) 3.784×10^{-1} 5) 5.312×10^{7} 6) 2.438×10^{-3}
7) 1.676×10^{-4} 8) 1.537×10^{-9} 9) 3.5×10^{8}
10) 1.25×10^{-5} 11) 1.37×10^{10} 12) 8.123×10^{4}
13) 1.316×10^{8} 14) 2.400×10^{-4} 15) 1.207×10^{4}
16) 2.093×10^{2} 17) 9.928×10^{5} 18) 1.963×10^{-14}
19) 2×10^{2} 20) 3×10^{-3}

Exercise 4

1) a) 9.4608×10^{12} km b) 8.456 years
2) a) 5.976×10^{21} and 7.35×10^{19} b) 81 times
3) a) 1,839 b) 1.098×10^{30} electrons

6. Rational & Irrational Numbers

Exercise 1

1) $\frac{23}{90}$ 2) $\frac{14}{45}$ 3) $\frac{49}{90}$
4) $\frac{28}{45}$ 5) $\frac{2}{45}$ 6) $\frac{11}{15}$
7) $\frac{7}{900}$ 8) $\frac{4}{225}$ 9) $\frac{5}{33}$
10) $\frac{32}{99}$ 11) $\frac{53}{99}$ 12) $\frac{4}{55}$

Exercise 2

1, 2, 7, 9, 10, 12, 13, 15

Exercise 3

2, 3, 4, 5, 6, 8, 9, 14, 15, 16

Exercise 4

1, 3, 4, 5, 6, 8, 11

Exercise 5

c, e

7. Surds 1

Exercise 1

1) $2\sqrt{3}$ 2) $2\sqrt{6}$ 3) $3\sqrt{3}$ 4) $4\sqrt{2}$ 5) $3\sqrt{5}$
6) $3\sqrt{7}$ 7) $4\sqrt{3}$ 8) $5\sqrt{2}$ 9) $6\sqrt{2}$ 10) $5\sqrt{3}$
11) $4\sqrt{5}$ 12) $5\sqrt{5}$ 13) $7\sqrt{3}$

Exercise 2

1) $2\sqrt{5}$ 2) $\frac{\sqrt{3}}{3}$ 3) $\frac{\sqrt{5}}{5}$ 4) $\frac{\sqrt{7}}{7}$ 5) $\frac{\sqrt{11}}{11}$
6) $\sqrt{2}$ 7) $\sqrt{3}$ 8) $2\sqrt{3}$ 9) $3\sqrt{2}$ 10) $3\sqrt{3}$
11) $\frac{7\sqrt{5}}{5}$ 12) $5\sqrt{2}$ 13) $3\sqrt{7}$ 14) $2\sqrt{7}$
15) $\frac{22\sqrt{3}}{3}$ 16) $7\sqrt{3}$ 17) $15\sqrt{2}$

Exercise 3

1) $2\sqrt{6}$ 2) 10 3) $2\sqrt{3}$ 4) 6 5) $3\sqrt{7}$
6) $5\sqrt{2}$ 7) $2\sqrt{6}$ 8) $3\sqrt{2}$ 9) $3\sqrt{6}$ 10) $2\sqrt{5}$
11) $5\sqrt{6}$ 12) $2\sqrt{7}$ 13) $4\sqrt{5}$ 14) $6\sqrt{6}$
15) $16\sqrt{6}$ 16) $4\sqrt{5}$

Exercise 4

a) $\sqrt{2}$ b) $\sqrt{10}$ c) $2\sqrt{3}$ d) $\sqrt{21}$ e) $2\sqrt{13}$

8. Surds 2

Exercise 1

1) $2\sqrt{2}$ 2) $2\sqrt{3}$ 3) $2\sqrt{6}$ 4) $2\sqrt{7}$
5) $6\sqrt{3}$ 6) $2\sqrt{10}$ 7) $5\sqrt{2}$ 8) $3\sqrt{2}$
9) $4\sqrt{3}$ 10) $4\sqrt{2}$ 11) $2\sqrt{5}$ 12) $5\sqrt{5}$
13) $10\sqrt{2}$ 14) $6\sqrt{6}$ 15) $8\sqrt{3}$ 16) $8\sqrt{5}$

Exercise 2

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) 2 4) $3\sqrt{2}$
5) $2\sqrt{7}$ 6) $4\sqrt{2}$ 7) $3\sqrt{3}$ 8) $4\sqrt{3}$
9) $7\sqrt{2}$ 10) $10\sqrt{2}$ 11) $10\sqrt{3}$ 12) $10\sqrt{5}$
13) $14\sqrt{5}$ 14) $13\sqrt{3}$ 15) $7\sqrt{7}$ 16) $3\sqrt{21}$

Exercise 3

1) $3\sqrt{2}$ 2) $4\sqrt{3}$ 3) $5\sqrt{2}$ 4) $3\sqrt{2}$
5) $\sqrt{2}$ 6) $\sqrt{3}$ 7) $\sqrt{5}$ 8) $2\sqrt{2}$
9) $\sqrt{5}$ 10) $\sqrt{5}$ 11) $2\sqrt{7}$ 12) $7\sqrt{5}$

Exercise 4

1) $\sqrt{2}$ 2) $\frac{\sqrt{2}}{2}$ 3) $3\sqrt{3}$ 4) $8\sqrt{2}$

5) $8\sqrt{3}$ 6) $\sqrt{7}$ 7) $7\sqrt{5}$ 8) $\sqrt{3}$

9) $4\sqrt{7}$ 10) $2\sqrt{5}$ 11) $9\sqrt{5}$ 12) $18\sqrt{2}$

Exercise 5

1) $3\sqrt{2}$ 2) $5\sqrt{2}$ 3) 6 4) $6\sqrt{2}$

5) 14 6) 24 7) $16\sqrt{6}$ 8) $60\sqrt{2}$

9) 48 10) $20\sqrt{5}$ 11) $42\sqrt{6}$ 12) $112\sqrt{6}$

9. Prime Factors

Exercise 1

1) $2^2 \times 3 \times 5^2$ 2) $2^2 \times 3^2 \times 5^2$ 3) $2 \times 3^2 \times 5 \times 7$

4) $2^2 \times 5^2 \times 7$ 5) $2^3 \times 3^2 \times 11$ 6) $3^3 \times 5 \times 7$

7) $2^3 \times 5 \times 7^2$ 8) $3 \times 5 \times 11^2$ 9) $2^3 \times 3^3 \times 7$

10) $2^2 \times 3 \times 5 \times 11 \times 13$ 11) $2^4 \times 3 \times 5 \times 11$

12) $2^2 \times 3 \times 5 \times 7 \times 13$ 13) $2^5 \times 3^2 \times 13$

14) $2^6 \times 3^2 \times 11$ 15) $2^3 \times 3 \times 5 \times 7 \times 11$

Exercise 2

1) $2^2 \times 3 \times 5 \times 11$ 165 2) $2^2 \times 3 \times 5^2$ 3

3) $2 \times 3^2 \times 5^2$ 2 4) $2^2 \times 5^2 \times 7$ 7

5) $3^2 \times 5^2 \times 7$ 7 6) $3^2 \times 5 \times 7^2$ 5

7) $2^3 \times 3 \times 5^2$ 6 8) $2^2 \times 3^2 \times 11$ 11

9) $2 \times 3^3 \times 5^2$ 6 10) $2^4 \times 3^2 \times 13$ 13

11) $2 \times 3^2 \times 5^2 \times 11$ 22 12) $2^8 \times 3 \times 5$ 15

13) $2^2 \times 3^2 \times 5 \times 7^2$ 5 14) $2^4 \times 3 \times 5 \times 7^2$ 15

15) $2^2 \times 3^4 \times 5 \times 7$ 35

Exercise 3

1) 15 2) 105 3) 27

4) 13 5) 55 6) 11

7) 143 8) 75 9) 115

10) 231 11) 77 12) 33

13) 735 14) 4725 15) 1155

Exercise 4

1) a) 21 b) $2.1cm \times 2.1cm$ c) 1260

2) a) 15 b) 15

3) a) 18 b) 180 boxes c) 55 and 42

10. Fractions

Exercise 1

1) $\frac{3}{5}$ 2) 1 3) $1\frac{2}{5}$

4) $\frac{8}{15}$ 5) $\frac{15}{56}$ 6) $\frac{1}{12}$

7) $1\frac{4}{15}$ 8) $\frac{18}{35}$ 9) $1\frac{13}{36}$ 10) $1\frac{3}{20}$

11) $1\frac{23}{24}$ 12) $3\frac{22}{35}$

13) $\frac{3a}{7}$ 14) $\frac{6a}{5}$ 15) a

16) $\frac{8x}{15}$ 17) $\frac{12x}{35}$ 18) $\frac{3b}{10}$

19) $\frac{x}{8}$ 20) $\frac{29x}{70}$ 21) $\frac{67x}{90}$

22) $\frac{9a}{4}$ 23) $\frac{37x}{14}$ 24) $\frac{117x}{44}$

Exercise 2

1) $\frac{5}{x}$ 2) $\frac{1}{a}$ 3) $\frac{15}{x}$

4) $\frac{5}{2x}$ 5) $\frac{4}{x}$ 6) $\frac{11}{20x}$

7) $\frac{13}{6x}$ 8) $\frac{9}{20a}$ 9) $\frac{a+b}{ab}$

10) $\frac{2y-x}{xy}$ 11) $\frac{3y+5x}{xy}$ 12) $\frac{b+5}{5b}$

13) $\frac{2b+15}{5b}$ 14) $\frac{4+x}{5}$ 15) $\frac{16-x^2}{4x}$

16) $\frac{x^2-12}{4x}$

Exercise 3

1) $\frac{3a+11}{4(a+1)}$ 2) $\frac{5x-3}{8(x+1)}$

3) $\frac{3x-23}{10(x-1)}$ 4) $\frac{7x-3}{12}$

5) $\frac{3a-4(x+1)}{12}$ 6) $\frac{8b+15a}{20}$

7) $\frac{53x+6}{18}$ 8) $\frac{33a+12}{20}$

9) $\frac{41x+32}{24}$ 10) $\frac{2-x}{6}$

11) $\frac{14y+15}{30}$ 12) $\frac{61a+20}{28}$

13) $\frac{9(x-3)}{(x-1)(x-4)}$ 14) $\frac{18x+1}{(x+2)(2x-1)}$

15) $\frac{14x+1}{(3x+2)(4x+1)}$ 16) $\frac{15+2xy}{6x}$

17) $\frac{19x}{5a}$ 18) $\frac{7x}{3a}$

19) $\frac{1}{30}(x-9)$ 20) $\frac{1}{40}(9x+73)$

21) $\frac{1}{35}(36a-57)$ 22) $\frac{7x+9}{4}$

23) $\frac{a+30}{4}$ 24) $\frac{3(x+11)}{4}$

25) $\frac{x^2+2}{x}$ 26) $\frac{13a+1}{3}$ 27) $\frac{17x-3}{4}$

11. Fractions, Decimals and Percentages 1

Exercise 1

1) 0.4 2) 0.375 3) 0.8 4) 0.875 5) 0.75 6) 0.15
7) 0.45 8) 0.35 9) 0.2667 10) 0.2333 11) 0.4857
12) 0.25 13) 0.8667 14) 0.2353 15) 0.4444
16) 0.6429 17) 0.3429 18) 0.5385

Exercise 2

1) 15% 2) 42% 3) 31% 4) 94% 5) 38% 6) 56%
7) 72% 8) 38.7% 9) 55.2% 10) 67.3% 11) 84.1%
12) 52.9% 13) 78.1% 14) 70% 15) 10% 16) 450%
17) 278% 18) 523%

Exercise 3

1) 37.5% 2) 46.67% 3) 45% 4) 40% 5) 19.05%
6) 58.33% 7) 28% 8) 56.67% 9) 40% 10) 28%
11) 65.71% 12) 36.36% 13) 64.15% 14) 73.81%

15) 60.23% 16) 71.11% 17) 13.82% 18) 75.79%

Exercise 4

1) 0.4, 48%, $\frac{1}{2}$ 2) 0.61, $\frac{5}{8}$, 67% 3) $\frac{7}{12}$, 0.6, 67%

4) 0.2, $\frac{7}{32}$, 23% 5) $\frac{3}{10}$, 0.32, 34% 6) $\frac{5}{16}$, 33.5%, 0.35

7) 0.27, $\frac{7}{25}$, 30% 8) 29.4%, $\frac{6}{17}$, 0.37

9) 0.24, $\frac{6}{24}$, 27% 10) 27.4%, $\frac{9}{30}$, 0.34

Exercise 5

1) 7.5 2) 144 3) 33 4) 72 5) £38.5 6) 8.125 7) 9.1
8) £8 9) 1.4 10) £0.35 11) 70 12) 2.94

12. Fractions, Decimals and Percentages 2

Exercise 1

1) 222 2) 12 3) 342 4) 205 5) 90p 6) 60p
7) £11.40 8) 1.36m 9) £2.16 10) £3.24
11) £8.58 12) 1116

Exercise 2

1) 48% 2) 63% 3) 68% 4) 46% 5) 62%
6) 65% 7) 22% 8) 90% 9) 70% 10) 62%
11) 85% 12) 69% 13) 57% 14) 61%
15) 91% 16) 60%

Exercise 3

1) 20% 2) 60% 3) 33% 4) 20% 5) 11%
6) 13% 7) 4% 8) 6% 9) 6% 10) 24%

Exercise 4

1) £117 2) £242 3) £180 4) £2300
5) £5544 6) £215 7) £87.50 8) £56350
9) £101.60 10) £549

13. Percentages

1) £2000
2) 15%
3) a) £282 b) £24 c) £850
4) a) £97.50 b) £21.45
5) a) £24,725 b) 10.4%
6) 61,252
7) 20.5%
8) £5222.25
9) a) 800ml b) Yes - 20% increase in cola but a 33% increase in price.
10) a) 39% b) 4096
11) 57%
12) 6 years
13) 546,250
14) 6 years
15) A by £315
16) £1116.25

14. Interest

Exercise 1

1) £4, £4.04 2) £36, £38.16
3) £135, £147.51 4) £400, £464.10
5) £315, £337.56 6) £240, £249.73
7) £1040, £1120.63 8) £16.80, £17.39
9) £132, £142.84 10) £504, £551.20

Exercise 2

1) a) 249.24 b) 644.79 c) 2307.53
2) a) 1.047 b) 6282, 6577.25, 6886.38, 7210.05, 7548.92, 7903.72, 8275.19, 8664.12, 9071.34, 9497.69
3) 7345.77 4) 920.66

15. Scale Drawing and Ratio

Exercise 1

1) 40cm 2) 8cm 3) 62cm 4) 7cm 5) 400cm
6) 16cm 7) 1:30 8) 1:20 9) 5cm 10) 2.5m
11) 1:200 12) 1:2.5 13) 5.5cm 14) 450cm
15) 1:30 16) 11.5cm 17) 9m 18) 1:5
19) 8cm 20) 104cm

Exercise 2

1) £400, £500 2) £300, £700 3) £75, £125
4) £280, £320 5) £250, £550 6) £250, £450
7) £245, £385 8) £495, £770 9) £840, £1365
10) £300, £400,£500 11) £125, £150, £175
12) £70, £105, £140 13) £294, £336, £378
14) £315, £405, £495 15) £125, £200, £225
16) £16.80, £22.40, £39.20 17) £36, £48, £66
18) £1.98, £4.62, £7.26 19) £144, £252, £468
20) £73.50, £98, £171.50

Exercise 3

1) £36, £16 2) £36, £15, 3) £63, £30
4) £165, £77 5) £288, £126 6) £10.45, £6.05
7) £10.05, £5.36 8) £24.64, £14.56
9) £48.51, £20.79 10) £142.80, £66.30

16. Conversion Graphs 1

1) a) $2.87 b) $63 c) £48.78
2) a) £17.20 b) 1m 60cm
3) b) 42cm c) £27

17. Conversion Graphs 2

1) a. 6.50 b. £3.30 (approx)
2) a. $114 b. £37

18. Conversion Graphs 3

1) a. 51 b. 34 2) a. 49.5 b. 7.1
3) a. £88 b. 9000 4) a. 105 secs b. 12cms
5) a. 3 b. 7 c. 382

19. Distance Time Diagrams 1

1) a. 220 miles b. 40 mins c. 30mph
d. 44mph 2) a. 45mph b. 4 mins
c. 8:10 d. 32.5mph e. 8:27 f. 1.5 miles
g. 8:58 3) a) 60mph b) 12:40 4 minutes
c) 80mph d) 72mph e) 13:04 67miles

20. Distance Time Diagrams 2

1) a. 60mph b. 12:40; 4 mins c) 80mph
d. 72 mph e. 13:04, 68 miles
2) a. Journey B. It goes further in a shorter time.
b. 15kph c. B d. 7.5kph
e. 2 hours 48 mins f. 11:12 and 15:54
g. 17:24 5) a. 144 miles b. 18 mph and 72 mph
c. 60 mph and 22.5 mph d. 36 mins

e. 05:12, 104 miles f. 30 miles
g. 04:00 and 06:24

21. Distance Time Diagrams 3

1) a) 3.2 miles per hour b) 1.61
   c) 1.7 miles per hour d) 2.6 miles per hour
2) a) 0.8 b) $2.3ms^{-1}$
3) a) $8.8ms^{-1}$ b) Speed is increasing (accelerating)
   c) Speed is approximately constant
   d) Speed is decreasing
4) a) 30 metres b) $6ms^{-1}$ c) 12m d) 37.5m

22. Velocity - Time Diagrams

1) a) Area = 363 b) 363 metres c) $2.4ms^{-2}$
2) a) 2880 metres b) $0.5ms^{-2}$
3) 995 metres
4) a)

0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	3	7	12	18	25	33	42	52	63	75

   b) 40.75 metres c) $17ms^{-2}$

23 Number Patterns and Sequences

1) a) 36 and 6 b) 48 and 8 c) 12(n – 1), 2(n – 1)
d) 228, 38 e) 7 and 12
2) a) 35 b) 43 c) $8n + 3$ d) 163 e) 12
3) a) 10, 26 b) 12, 32 c) 2n + 2, 2 + 6n d) 42

24. Number Sequences

Exercise 1

1) 20, 23	2) 25, 29	3) 25, 32
4) 17, 23	5) 44, 52	6) –1, –8
7) –2, –4	8) –21, –34	9) –3, –8
10) 128, 256	11) –22, –29	12) –21, –27
13) 29, 35	14) –6, –10	15) 127, 255
16) 49, 64	17) –5, –7	18) 343, 512
19) 335, 504	20) 46, 61	21) 64, 128
22) 146, 191	23) 8, 13	24) 86, 139
25) 28, 36	26) 49, 60	

Exercise 2

1) 22, 25	$3n + 1$	2) 27, 32	$5n - 8$
3) –26, –30	$2 - 4n$	4) –22, –27	$13 - 5n$
5) –9, –17	$47 - 8n$	6) 36, 41	$5n + 1$
7) 23, 30	$7n - 26$	8) 49, 64	n^2
9) 51, 66	$n^2 + 2$	10) 42, 57	$n^2 - 7$
11) 36, 49	$(n-1)^2$	12) 42, 56	$n^2 - n$
13) $\frac{14}{15}, \frac{16}{17}$	$\frac{2n}{2n+1}$	14) $\frac{7}{8}, \frac{8}{9}$	$\frac{n}{n+1}$
15) $\frac{49}{10}, \frac{64}{11}$	$\frac{n^2}{n+3}$	16) $\frac{21}{97}, \frac{24}{127}$	$\frac{3n}{2n^2-1}$

Exercise 3

1)a) $n + 1$
b) $(n + 1)(y + 1) = ny + n + y + 1 = (ny + n + y) + 1$
which is a term in the sequence
2) a) $(2n + 1)$
b) $(2n + 1)(2y + 1) = 4ny + 2n + 2y + 1$
$= 2(2ny + n + y) + 1$ which is a term.
3) a) n^2 b) $n^2y^2 = (ny)^2$ which is a term.
4) a) $4n + 1$ b) $4(4ny + n + y) + 1$ is a term.

25. Indices 1

Exercise 1

1) 9 2) 27 3) 81 4) 243 5) 100 6) 1000
7) 10,000 8) 100,000

Exercise 2

1) 7,776 2) 15,625 3) 16,384 4) 117,649
5) 59,049 6) 161,051 7) 4,826,809
8) 40,353,607

Exercise 3

1) 2^7, 128 2) 3^9, 19,683 3) 4^6, 4096
4) 10^7, 10,000,000 5) 7^8, 5,764,801
6) 8^5, 32,768 7) x^7 8) a^{13} 9) b^9 10) y^{25}

Exercise 4

1) 4^4 2) 5^5 3) 7^3 4) 10^3 5) 15^3 6) 10^2 7) 9^3
8) 12^3 9) 8^6 10) 20^3 11) a^3 12) y^{12} 13) x^5

Exercise 5

1) 2^8 2) 4^{10} 3) 7^9 4) 4^{12} 5) 5^6 6) 2^{15} 7) 3^{16}
8) 7^8 9) 3^{10} 10) 5^8 11) x^{10} 12) y^9

Exercise 6

1) 1296 2) 248832 3) 2744 4) 512
5) 3375 6) 100000 7) 2985984
8) 194481 9) 7776 10) 160000

Exercise 7

1) a) x^5	b) x^{11}	c) a^{12}	d) y^{13}
2) a) a^2	b) 1	c) x^2	d) 2^6
3) a) a^{24}	b) x^{18}	c) y^8	d) b^{18}
4) a) x^4y^2	b) a^5b^3	c) x^4y^6	d) a^3b^6
5) a) $9x^2$	b) $8x^3$	c) $27x^3$	d) $25a^2$

26. Indices 2

Exercise 1

1) $12x^2$	2) $5a^7$	3) $24y^5$	4) $35x^{11}$
5) a^6	6)c^{18}	7)x^{28}	8)x^{12}
9) x^3y^3	10) a^7b^6	11) x^4y^8	12) a^7b^7
13) $16x^2$	14) $9x^4$	15) $16y^6$	16) $16a^8$
17) $3a^2$	18) $18x^3$	19) $4y^3$	20) $4a^3$
21) $4a^2$	22) $6b^6$	23) $4a^2b$	24) $3xy$
25) x^4	26) $6a^8$	27) $14y^4$	28) $3x^8$

Exercise 2

1) x^0 or 1	2) a^{-2}	3) $2y^{-1}$	4) $\frac{1}{2b^2}$
5) $\frac{1}{y}$ or y^{-1}	6) $\frac{6}{x^3}$	7) $\frac{12}{a^3}$	8) $\frac{6a^3}{b}$
9) x^3	10) $\frac{1}{y^3}$	11) $6x^2$	12) 24
13) a	14) 1	15) $b^{\frac{1}{2}}$	16) y^{-1}
17) 1	18) $y^{\frac{1}{4}}$	19) $a^{\frac{3}{4}}$	20) $b^{\frac{1}{2}}$
21) $x^{\frac{3}{2}}$	22) $a^{\frac{1}{2}}$	23) $8b$	

24) $\sqrt{5}y^{\frac{3}{2}}$ or $\sqrt{5y^3}$

Exercise 3

1) 5 2) $\frac{1}{5}$ 3) 2 4) $\frac{1}{3}$

5) 4 6) 8 7) 8 8) 27

9) 27 10) 16 11) 8 12) 125

13) $\frac{1}{125}$ 14) $\frac{1}{32}$ 15) $\frac{5}{2}$ 16) $\frac{25}{4}$

Exercise 4

1) $x = 2$ 2) $x = 2$ 3) $x = 4$ 4) $x = 3$

5) $x = \frac{1}{5}$ 6) $x = -\frac{1}{2}$ 7) $x = 2$ 8) $x = 3$

9) $x = 5$ 10) $x = 81$ 11) 16 12) $x = -\frac{1}{2}$

27. Substitution

Exercise 1

1) 38 2) 0 3) –16 4) –8

5) 50 6) 116 7) 52 8) 105

9) 2116 10) 156 11) –100 12) –589

Exercise 2

1) a) 22 b) $x = 5$

2) a) –27 b) $b = 20$

3) a) –39 b) $x = -12$

4) a) 700 cm b) $R = 26$

5) a) $23.565cm^3$ b) $7cm$

6) a) £71.25 b) £476.19

7) a) –29° b) $F = 14°$

8) a) 55 minutes b) 1 hour 19 minutes

c) Decrease the numbers that D and S are divided by.

28. Simplifying 1

Exercise 1

1) 11 2) 5 3) 9 4) –1 5) –3 6) –3 7) 4

8) 3 9) 6 10) –8 11) –11 12) –15 13) 3

14) 0 15) 1 16) –6 17) –1 18) –1 19) –4

20) 0 21) 2 22) –15 23) –15 24) –4 25) 3

26) –16 27) –19 28) –11 29) 5 30) –15

Exercise 2

1) $11y$ 2) $8y$ 3) $3y$ 4) $8y$ 5) $-2y$

6) $8y$ 7) $-9y$ 8) $-6x$ 9) $-23a$

10) $-19w$ 11) $15b + 5a$ 12) $12x + 13y$

11) $7b + 8a$ 14) $2x + 7y$ 15) $4a + 7b$

16) $8p + 10q$ 17) $8a + 2b$ 18) $4x + 6y$

19) $6x - y$ 20) $4a + b$ 21) $-2x - 3y$

22) $6a - 3b$ 23) $3y - xy$ 24) $6xy + 4y$

25) $6b - 13ab$ 26) $5x - 10xy$ 27) $5bc - 5ab$

28) $11xy + 4yz$ 29) $14xy - 9x$ 30) $18ab + a$

31) $4x^2$ 32) $13y^2$ 33) $5x^2 + 7y^2$ 34) $4x^2$

35) $6xy + x^2$ 36) $6x^2 + 2x$ 37) $5xy^2 - 4x^2y$

38) $2x^2y - 9xy^2$ 39) $\frac{3}{4}x$ 40) $\frac{1}{2}y$

29. Simplifying 2

Exercise 1

1) $4x$ 2) $-2y$ 3) $-5y$

4) $-13x$ 5) $9x + 9y$ 6) $3x - 2y$

7) $5y - 10x$ 8) $-4x - 10y$ 9) $3b - 4a - 4ab$

10) $12b - 11a + 3ab$ 11) $2x^2 + 3x$ 12) $y^2 - 4y$

13) $4y^2$ 14) $3y^2 - 10x^2$ 15) $3x + 4y$

16) $10x - 3y$ 17) $7x + 4y$ 18) $4x - 6y$

19) $6y - 2x$ 20) $8y - 2x$ 21) $5x + y$

22) $7x + 10y$ 23) $8x - 14y$ 24) $-y$

25) $3x + 3$ 26) $-5x - 9y$ 27) $2x^2 - 24x$

28) $18y - 14x - 5xy$ 29) $8y^2 - 6xy - 2x^2$

30) $4xy + 12x^2 + 9y^2$ 31) $\frac{7x}{12}$

32) $\frac{x}{6}$ 33) $\frac{y}{6}$ 34) $\frac{y^2}{6}$

Exercise 2

1) $x^2 - 2x - 3$ 2) $x^2 - 2x - 8$

3) $2x^2 - 11x - 21$ 4) $6x^2 + 11x - 10$

5) $6x^2 - 17x - 14$ 6) $6x^2 - 7x - 20$

7) $35x^2 - x - 6$ 8) $12x^2 + 16x - 16$

9) $8x^2 - 4x - 4$ 10) $6x^2 + 13xy + 6y^2$

11) $4x^2 + 13xy + 3y^2$ 12) $2x^2 - 14x + 24$

13) $x^2 + 2x + 1$ 14) $9x^2 + 12x + 4$

15) $25x^2 - 20x + 4$ 16) $9x^2 + 24x + 16$

17) $25x^2 - 60x + 36$ 18) $49x^2 + 28x + 4$

19) $9x^2 - 12x + 4$ 20) $25x^2 + 70x + 49$

21) $16x^2 + 48x + 36$ 22) $x^2 + 2xy + y^2$

23) $4x^2 + 12xy + 9y^2$ 24) $16x^2 - 16xy + 4y^2$

30. Multiplying Brackets

Exercise 1

1) 24 2) 35 3) –24 4) –24 5) –6 6) –40

7) 20 8) 30 9) 21 10) –30 11) –12 12) 56

Exercise 2

1) $3x + 3y$ 2) $18x + 24$ 3) $-2x + 3$

4) $-3x - 2$ 5) $-8x - 20$ 6) $-21x + 28$

7) $-12x - 12$ 8) $10x - 15$ 9) $9x + 6$

10) $13x + 20y$ 11) $4x - 5y$ 12) $20y - 22x$

13) $2x - 5y$ 14) $8x + 5y$ 15) $10x$

16) $15x + 13y$ 17) $38x - 19y$ 18) $3x - 26y$

19) $5x - 15y$ 20) $6x^2 + 17x$ 21) $-3x^2 - 22x$

22) $27x^2 - 5x$ 23) $7x^2 + 3x$ 24) $15x^2 - 6x$

25) $16x^2 + 27x$ 26) $6x^2 - 33x$

Exercise 3

1) $x^2 + 5x + 6$ 2) $2x^2 + 5x + 2$

3) $3x^2 + 14x + 8$ 4) $30x^2 + 47x + 14$

5) $6x^2 - x - 12$ 6) $12x^2 - 5x - 25$

7) $24x^2 - 24x - 18$ 8) $15x^2 + x - 6$

9) $8x^2 - 2x - 3$ 10) $3x^2 + 2x - 8$

11) $24x^2 - 2x - 15$ 12) $6x^2 - 38x + 56$

13) $42x^2 - 58x + 20$ 14) $12x^2 - 39x + 30$

15) $72x^2 - 70x + 12$ 16) $15x^2 + 29x - 14$

17) $25x^2 + 30x + 9$ 18) $36x^2 - 24x + 4$

19) $16x^2 - 40x + 25$ 20) $16x^2 + 72x + 81$

31. Factorising

Exercise 1

1) $4(x+2)$ 2) $3(2y-3)$ 3) $7(b-2a)$
4) $x(y-1)$ 5) $x(3+y)$ 6) $2y(2+5x)$
7) $2(3x^2+1)$ 8) $x(5x-1)$ 9) $3x(3x-1)$
10) $ab(a+b)$ 11) $ab(4-a)$ 12) $2ab(4+3b)$
13) $a(1+b-a)$ 14) $a(3b-c+a)$ 15) $y(5x^2-4y-3x)$
16) $\frac{x^2}{4}(2-x)$ 17) $\frac{y}{6}(2y-x)$ 18) $\frac{x}{6}(5x-4)$

Exercise 2

1) $(m-n)(m+n)$ 2) $(a-2)(a+2)$
3) $(xy-z)(xy+z)$ 4) $(ab-3)(ab+3)$
5) $(xy-2)(xy+2)$ 6) $(vw-5)(vw+5)$
7) $(ab-3c)(ab+3c)$ 8) $(5a-3b)(5a+3b)$
9) $(b-1)(b+1)$ 10) $2(a-5)(a+5)$
11) $2(2a-5)(2a+5)$ 12) $3(2x-3y)(2x+3y)$
13) $x(y-2x)(y+2x)$ 14) $2x(y-2x)(y+2x)$
15) $x^2(2y-3)(2y+3)$ 16) $(x-y)(x+y)(x^2+y^2)$
17) $(2x-3y)(2x+3y)(4x^2+9y^2)$
18) $3(a^2-2b)(a^2+2b)$

Exercise 3

1) $(x+3)(x+1)$ 2) $(x+2)(x+2)$
3) $(x+7)(x+1)$ 4) $(x+5)(x+2)$
5) $(x+4)(x+3)$ 6) $(x+6)(x+5)$
7) $(x-1)(x+3)$ 8) $(x-3)(x+1)$
9) $(x+5)(x-1)$ 10) $(x+2)(x-4)$
11) $(x-5)(x+3)$ 12) $(x+3)(x-4)$
13) $(x-6)(x-4)$ 14) $(x-3)(x-5)$
15) $(x-4)(x-7)$

Exercise 4

1) $(2x+1)(x+1)$ 2) $(2x+1)(x+4)$
3) $(2x+1)(x+3)$ 4) $(2x+2)(x+3)$
5) $(2x-3)(x+2)$ 6) $(3x+2)(x-3)$
7) $(2x-1)(x-4)$ 8) $(3x-1)(x-3)$
9) $(3x-2)(x-4)$ 10) $(3x+7)(x-2)$
11) $(3x+4)(x+5)$ 12) $(3x-6)(x-2)$
13) $(2x+2)(2x+3)$ 14) $2(x-1)(2x-3)$
15) $(4x+1)(x+3)$ 16) $(x+5)(4x+1)$
17) $(5x-2)(x+3)$ 18) $(3x+2)(2x-3)$
19) $(3x+1)(2x+1)$ 20) $(3x+2)(3x+2)$
21) $(x+1)(8x+3)$ 22) $(x-5)(4x-3)$
23) $(5x+2)(x-3)$ 24) $(4x-3)(3x-1)$

32. Re-arranging Formulae

Exercise 1

1) $v-at$ 2) $\frac{v-u}{t}$ 3) $\frac{d+c}{3}$
4) $c-pd$ 5) $7y-x$ 6) $\frac{a-c}{3}$
7) $\frac{3w-u}{4}$ 8) $4x-5y$ 9) x
10) $\frac{8y}{3}$ 11) $2(p-3b)$ 12) $\frac{w}{2}-\frac{u}{8}$ or $\frac{1}{2}(w-\frac{1}{4}u)$
13) $\frac{a+b}{c}$ 14) $\frac{2r-q}{3p}$ 15) $\frac{x}{2}-y$
16) $\frac{1}{4}(\frac{a}{3}-3b)$ 17) $2x-z$ 18) $9a-2b$
19) $y-12x$ 20) $\frac{3v-15w}{2}$ 21) $\frac{14}{11}y$
22) $-\frac{3}{19}b$

Exercise 2

1) $\sqrt{ac}$ 2) $\sqrt{\frac{y}{x}}$
3) $\frac{\sqrt{bc}}{2}$ 4) $\sqrt{\frac{3}{v}}$
5) $y = -\frac{x}{6}$ 6) $y = -8x$
7) $x = \frac{y}{6}$ 8) $b = -\frac{27}{7}c$
9) $x = \frac{ab}{b+a}$ 10) $x = \frac{2yz}{3(2z+3y)}$
11) $x = \left(\frac{6ab}{3b+2a}\right)^2$ 12) $x = \left(\frac{4y}{3+by}\right)^2$
13) $z = \frac{6x}{3xy-4}$ 14) $z = \frac{xy}{3(2x-y)}$
15) $a = \sqrt{\frac{b}{xb-1}}$ 16) $a = \sqrt{\frac{2}{3(4y-3b)}}$
17) $b = 2c$ 18) $x = \frac{y}{3}$
19) $y = \frac{z}{3x-2}$ 20) $c = \frac{b}{4a+3}$
21) $x = 1-3y$ 22) $y = \frac{x-4}{3}$

33. Equations

Exercise 1

1) $x = 2$ 2) $x = 10$ 3) $y = 12$ 4) $x = 6$
5) $y = 18$ 6) $a = 27$ 7) $y = 2$ 8) $x = 10$
9) $x = 14$ 10) $a = 3$ 11) $x = 7$ 12) $y = 4.5$
13) $b = -5$ 14) $y = -6$ 15) $b = -2.5$ 16) $a = 2$
17) $a = 3$ 18) $x = 2$ 19) $x = 3$ 20) $x = 2$
21) $x = 7$ 22) $y = 2.5$ 23) $b = -2$ 24) $y = -5$

Exercise 2

1) $x = 3$ 2) $x = 5$ 3) $x = 6$ 4) $x = 5$
5) $x = 3$ 6) $x = 2$ 7) $x = 6$ 8) $x = 2$
9) $x = 3.5$ 10) $x = 3$ 11) $x = 2.5$
12) $x = 3.5$ 13) $x = -1$ 14) $x = -4$
15) $x = 4$ 16) $x = 9$ 17) $x = 8$ 18) $x = 2$
19) $x = 3$ 20) $x = -8$ 21) $x = -3$

Exercise 3

1) $x = 3$ 2) $x = 4$ 3) $x = 1$ 4) $x = 7$
5) $x = 5$ 6) $x = 6$ 7) $x = 5$ 8) $x = 2$
9) $x = 4$ 10) $x = 2$ 11) $x = 8$ 12) $x = 3$
13) $x = 1$ 14) $x = 5$ 15) $x = 2$ 16) $x = 4$
17) $x = 20$ 18) $x = 10$ 19) $x = 2$ 20) $x = 7$
21) $x = 8$ 22) $x = 3$ 23) $x = 5$ 24) $x = 7$

34. Solving Equations 1

Exercise 1

1) $x = 9$ 2) $x = 12$ 3) $x = 6$
4) $x = 4$ 5) $x = 120$ 6) $x = 150$
7) $x = 12$ 8) $x = 8$ 9) $x = 1$
10) $x = \frac{7}{5}$ 11) $x = 1$ 12) $x = 2$
13) $x = -2$ 14) $x = 3$ 15) $x = 5$
16) $x = 8$ 17) $x = 2$ 18) $x = 7$
19) $x = 24$ 20) $x = 32$ 21) $x = 11$
22) $x = 6$ 23) $x = -11$ 24) $x = \frac{4}{5}$

25) $x = -\frac{1}{3}$ 26) $x = \frac{1}{2}$

Exercise 2

1) $x = 5$ or -5 2) $x = 9$ or -9
3) 6 or -6 4) 3 or -3
5) -2 or 3 6) 6 or -5
7) $-\frac{4}{3}$ or $\frac{1}{2}$ 8) 0 or $\frac{2}{3}$
9) 0 or $-\frac{3}{4}$ 10) 0 or 4
11) 0 or $-\frac{3}{2}$ 12) 0 or $\frac{2}{3}$
13) 0 or $\frac{1}{4}$ 14) -2 or 3
15) $\frac{3}{2}$ or -1 16) -1 or $\frac{1}{3}$
17) 5 or -2 18) 2 or -4
19) $-\frac{5}{4}$ or $\frac{3}{2}$ 20) 2.5 or -2.5
21) -2 22) $-\frac{1}{3}$

35. Solving Equations 2

Exercise 2

1) 1.16 or -5.16 2) 2.62 or 0.38
3) 4.65 or -0.65 4) 0.61 or -6.61
5) 0.62 or -1.62 6) 2.20 or -0.45
7) -0.73 or 0.23 8) 0.74 or -0.94
9) 1.08 or -2.33 10) 1.72 or -0.39

11) 0.79 or -2.12 12) 0.19 or -2.69
13) 1 or 0.75 14) 0.23 or -0.73
15) 0.18 or -1.61 16) 3.24 or 0.12
17) 0.78 or -0.92 18) 1.92 or 0.46
19) 0.37 or -5.37 20) 0.52 or -9.52
21) 1.44 or -0.69 22) 2.77 or -1.27
23) 2.30 or -1.30 24) 1.81 or -1.47
25) 2.78 or 0.72 26) 10.18 or -1.18

Exercise 2

1) $a = 5$ $k = 25$ 2) $a = 2$ $k = 4$
3) $a = 3$ $k = 9$ 4) $a = 7$ $k = 49$
5) $= 1.5$ $k = 2.25$ 6) $a = 2$ $k = 4$
7) $a = 5$ $k = 25$ 8) $a = 8$ $k = 64$
9) $a = 6$ $k = 36$ 10) $a = 4$ $k = 16$

36. Using Simple Equations

1) a) $10n + 6m = 200$ b) 8
2) a) $x - 300$, $x + 300$
b) $x + 300 = 2(x - 300)$ c) £1800
3) a) $400 - x$ b) 175 c) 225
4) 28.8 5) 22, 23, 24 6) 120
7) 4 8) a) 50, 75, 55 b) 70, 95, 75, 120

37. Problems Involving Equations

1) a) $x = 34°$ 34°, 68°, 78°
b) $x = 30°$ 90°, 40°, 50°
2) a) 538.46 mph
b) $\frac{x}{538.46}$ hours c) $\frac{538.46y}{60}$ miles
3) a) $x = 4.5$ cm b) $30.25cm^2$
4) a) $x - 1$ and $x + 1$ b) $x = 52$
5) a) i) $x + \frac{y}{4}$ ii) $\frac{3y}{4}$
b) $y = 8x$ c) 18,000 bottles
6) a) $250 - x$ b) $x = 110$ c) 140
7) a) $x - 6$ and $x - 21$ b) $3x - 27$ c) $x = 40$
34, 40 and 19

38. Simultaneous Equations 1

1) 4,1 2) 5,3 3) 1,–1
4) 3,1 5) 3,–2 6) 3,5
7) 4,1 8) 2,3 9) 5,–3
10) 5,4 11) 2,–1 12) 3,–2
13) 5,6 14) 9,–1 15) 2.5,1
16) 4,–2
17) a) $2x + 2y = 14$ $x + 4y = 13.60$ b) $x = 4.80$
c) Adults £4.80 Children £2.20
18) a) $x + y = 39$ $x - y = 9$ b) 24,15
19) 25.8 by 2.7 20) 90g and 120g
21) £1.72 and £1.02 22) 17 and 6
23) 5, 3 24) £22, £12
25) 11, 7 26) 28p and 22p

39. Problems Involving Quadratic Equations

1) a) $x = 7$ b) $x = 6.5$ cm
2) a) $x = 10$ b) $105cm^2$
3) a) $\frac{20}{x} + \frac{10}{x-5} = 4$ $x = 2.5$ or 10
b) 10 kph and 5 kph
4) a) x^2 and $8x$ b) $x^2 + 8x = 105$
so $x^2 + 8x - 105 = 0$ c) 64 cm
5) a) $2x + 4$ and $x + 4$ d) $x = 4$ m 8m by 4m.
6) a) $(x + 4)(x - 1)$ and $3x(x - 2)$
b) $x = 4$ c) 8 km/hour

40. Completing the Square

1) b) 4 c) $\frac{9}{4}$ d) $\frac{49}{4}$ e) $\frac{1}{4}$ f) $\frac{81}{4}$ h) $\frac{9}{16}$ i) $\frac{4}{7}$ j) $\frac{1}{6}$ k) $\frac{1}{4}$ l) $\frac{25}{64}$
2) a) 10 or -2 b) 7 or -3 c) -2 or -9 d) -3 or -2
e) 2 or -1 f) 1 or -4 g) 5 or -1 h) 4 or -7
i) -1 or -3 j) $-\frac{1}{2}$ or -2 k) $-\frac{3}{2}$ or 2 l) $\frac{1}{2}$ or -3
m) $\frac{1}{4}$ or $-\frac{1}{2}$ n) $\frac{1}{2}$ or $-\frac{2}{3}$ o) $\frac{1}{3}$ or $-\frac{3}{5}$

41. Iteration

1) 3.73 2) 5.30 3) 3.79
4) 8.77 5) 0.62 6) -6.6
7) a) (i) -3.50, -2.80, -3.09, -2.96 (ii) -3
(iii) $9+12-21 = 0$
8) a) (i) 7.4, 6.946, 7.008 (ii) 7.0 b) $49-42-7 = 0$

42. Direct and Inverse Proportion

1) a) $y = \frac{2}{3}x$ b) $y = 5\frac{1}{3}$ c) $x = 22\frac{1}{2}$
2) $v = 81$
3) a) $y = \frac{2}{3}\sqrt{x}$ b) $y = \frac{4}{3}$ c) $x = 81$
4) $a = 256$
5) a) $y = \frac{3}{x^2}$ b) $y = \frac{1}{3}$ c) $x = 6$
6) $y = \frac{3}{4}$
7) a) $y = 5\sqrt[3]{x}$ b) i) $y = 20$ ii) $x = 125$
8)

x	1	2	3	4
y	3	0.75	$0.\dot{3}$	0.1875

9) $a = \frac{2}{3}$

10)

x	0.125	1	8	64
y	0.25	0.5	1	2

11) a) $n = 3$ b) $y = 0.125$

12)

x	0.5	0.75	1	1.5
y	6	$2.\dot{6}$	1.5	$0.\dot{6}$

43. 3 Dimensional Co-ordinates 1

1) (4,6,8) 2) a) (0,0,1) (1,0,1) (0,1,0) (1,1,0) (1,1,1) (0,1,1) b) (i) (2,2,2) (3,2,2) (3,2,3) (2,2,3) (2,3,3) (2,3,2) (3,3,2) (3,3,3) (ii) (2.5, 2.5, 2.5)

3) (0, 2, 0) (2, 0, 0) (2, 0, 2) (2, 2, 0)

4) a) (3, 3, 2) (5, 3, 4) (1, 3, 4) (3, 1, 4) (3, 5, 4) (3, 3, 6) b) (3, 3, 4)

5) a) (0, 0, 3) (0, 4, 0) (5, 0, 3) (5, 4, 3) b) (2.5, 0, 1.5) (5, 2, 1.5) (2.5, 4, 1.5) (0, 2, 1.5) (2.5, 2, 0) (2.5, 2, 3)

44. 3 Dimensional Co-ordinates 2

1) (1, 0, 2)

2) a) (1, –1, 2) (5, –1, 2) (5, 3, –2) (5, 3, 2) (1, 3, 2) (1, 3, –2)

b) (i) (3, 1, 0) (7, 1, 0) (3, 1, 4) (7, 1, 4) (7, 5, 0) (7, 5, 4) (3, 5, 4) (3, 5, 0)

(ii) (5, 3, 2)

3) (–3,0,–1) (–1,–2,1) (–1,0,–1) (–1,–2,–1)

4) a) 6 units b) (1, –3, 1) (–2, 0, 1) (1, 3, 1) (4, 0, 1) (1, 0, –2) (1, 0, 4) c) (1, 0, 1) d) (6, 6, 0)

5) a) (–7, –3, –3) (–3, –6, –3) (–3, –6, –5) (–3, –3, –5)

b) (–5, –6, –4) (–7, –4.5, –4) (5, –3, –4) (–3, –4.5, –4) (–5, –4.5, –3) (–5, –4.5, –5)

c) (–5, –4.5, –4) d) $2 \times 3 \times 4$ e) (4, 3, 2)

45. Recognising Graphs 1

1) c 2) a 3) c 4) a

46. Recognising Graphs 2

(i) f (ii) e (iii) k (iv) c (v) d (vi) j (vii) b (viii) l (ix) i (x) a (xi) h (xii) g

47. Graphs 1

1) a)

5	0	–3	–4	–3	0	5

d) –2.8 and 1.8

e) $x^2 + x - 5 = 0$

2) a)

–8.625	–2	3	2	1	6	12.625

b) $x = -2.25$, 0 and 2.25

3) a)

13	3	–3	–5	–3	3	13

c) $x = -0.6$ and 2.6

d) $x = -1.1$ and 3.1

4) a)

–1	0	1	2	3	4	5
–5	–4	–3	–2	–1	0	1
5	0	–3	–4	–3	0	5

d) –2.55 and 1.6 $x^2 + x - 4 = 0$

48. Graphs 2

1) a)

17	10	$8.\dot{3}$	8	8.2	$8.\dot{6}$	10	$13.\dot{3}$	15.1

d) 0.6 and 6.4 e) 0.8 and 4.0 f) 1.5

2) a)

–24	–14	–6	0	4	6	6	4	0	–6

c) $x = -0.25$ and 3.25

e) $x = -1$ and 4

f) $y = -4$. $x = -1.7$ or 4.7

3) a) $x = -3.25$ and 1 b) $x = -2.2$ and 0.7

49. Graphs 3

1) c) $c = 1.5$

2) b) $a = 3$, $b = 50$ c) 197

3) b) $p = 3$, $q = 2$ c) 14.3

4) b) $a = 1.5$, $b = -0.5$ c) 0.56

5) b) $a = 3$, $b = 2$ c) 6.2

50. Straight Line Graphs

Exercise 1

1) 1 2) 3 3) –5 4) –4

5) 2 6) 2 7) $-\frac{5}{3}$ 8) $-\frac{1}{4}$

9) 2 10) 2 11) –1 12) –1

13) –1 14) –1 15) $\frac{4}{3}$ 16) $\frac{1}{3}$

Exercise 2

1) $y = x - 3$ 2) $y = 2x - 10$ 3) $y = 3x$

4) $y = -x + 2$ 5) $y = -3x + 4$ 6) $y = 4x - 14$

Exercise 3

1) b) $m = 2$, $c = -6$ c) 124 d) 120

2) b) $m = 5$, $c = 20$ c) 95 d) 17

3) a) $C = 10H + 40$ b) £90 c) £105 d) 6 hours

51 Perpendicular Lines

1) a) 3 b) $-\frac{1}{3}$ c) (i), (iii), (iv), (vii).

2) a) $y = x$ b) $y = -x + 12$

3) a) $y = \frac{1}{2}x + 2$ b) –2 c) (0,12) d) $y = -2x + 12$

4) $y = -3x + 20$

5) $\frac{2}{3}$ b) $y = \frac{2}{3}x + \frac{2}{3}$ or $y = \frac{2}{3}(x + 1)$ c) AD is $-\frac{3}{2}$

BC is $-\frac{3}{2}$ DC is $\frac{2}{3}$ d) AD is $y = -\frac{3}{2}x + 5$

BC is $y = -\frac{3}{2}x + \frac{23}{2}$ DC is $y = \frac{2}{3}x + 5$

52. Growth and Decay

1) a)

0	1	2	3	4	5	6
1	0.667	0.444	0.296	0.198	0.132	0.088

c) i) 0.85 ii) 0.24

2) a)

0	1	2	3	4	5	6	7
10	11	12.1	13.31	14.64	16.11	17.72	19.49

c) 16.9 million d) $4\frac{1}{4}$ years

3) a)

0	0.5	1	1.5	2	2.5	3	3.5	4
100	57.7	33.3	19.3	11.1	6.4	3.7	2.1	1.2

c) i) 0.63 days (15 hours 7 minutes)
ii) 8.44% iii) 0.26 days

53 Equation of a Circle

1) 10 2) a) A b) 6 c) $y = x$ and $y = x - 6$ d) $(3\sqrt{2}, 3\sqrt{2})$ and $(3)\sqrt{2}, 3)\sqrt{2})$

3) (6,8) and $(-\frac{14}{5}, -\frac{48}{5})$ 4) $(\sqrt{51}, -7)$ and $(-\sqrt{51}, -7)$

b) $(\sqrt{51}, 7)$ $(-\sqrt{51}, 7)$

5) a) $\frac{\sqrt{181}}{2}$ b) (6.6, 1.3)

54. Simultaneous Equations 2

1) (2,3) (3,2) 2) (–5, –4) (4,5) 3) (2,5) (5,2)
4) (7.8, 0.4) (5,6) 5) (–4.8, 1.4) (–4, 3)

55. Trial and Improvement

Exercise 1

1) 3.8	2) 4.3	3) 4.7
4) 5.5	5) 4.3	6) 6.5
7) 5.2	8) 3.8	9) 2.4
10) 2.1	11) 4.8	12) 2.9
13) 3.5	14) 1.9	15) 1.9

Exercise 2

1) 6.7	2) 8.2	3) 9.4
4) 3.3	5) 3.5	6) 1.5
7) 2.4	8) 3.3	

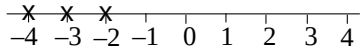
56. Inequalities

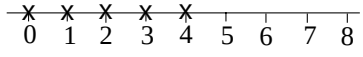
Exercise 1

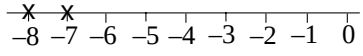
1) $x > 6$	2) $x > 2$	3) $x > 8$	4) $x > -3$
5) $x > -3$	6) $x > 2$	7) $x > 6$	8) $x > 4$
9) $x > 5$	10) $x < 1$	11) $x \leq 3$	12) $x \leq 5$
13) $x \geq 8$	14) $x < 3$	15) $x < 5$	16) $x > 2\frac{2}{5}$

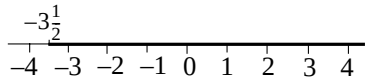
17) $x \leq 11$ 18) $x \leq -7\frac{2}{3}$ 19) $x \leq -1$ 20) $x \geq -26\frac{1}{2}$
21) $x \leq -2$

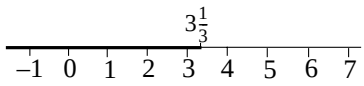
Exercise 2

1)


2)


3)


4)


5)


6) 2
7) 1
8) a) $20x < 10x + 70$
b) $x < 7$ Company A is cheaper when the number of hours is less than 7
Company A and B offer the same price when the number of hours is 7
Company B is cheaper when the number of hours is greater than 7

57. Linear Inequalities

1) (4,6) (5,5) (5,6) (6,4) (6,5) (6,6) (6,7)
2) (0,6) (1,4) (1,5) (1,6) (2,6)
3) (2,4) (3,3)
4) (3,4) (3,5)
5) (3,4) (3,5) (2,5)
6) (2,0) (3,0) (4,0) $(3, -1)$
7) $(1, -1)$ $(2, -1)$
8) (1,2) (1,3) (2,3)
9) (0,2) $(-1, 3)$ (0,3) (1,3) (0,4) (0,5)

58. Linear Programming

1) a) $y \geq 7$ $y \leq 2x$ $x + 2y \leq 20$ c) 360kg d) 6,7 and 4,8
2) a) $x + y \leq 50$ $y \leq 2x$ $x \leq 2y$
c) i) $33\frac{1}{3}$ tins of red, $16\frac{2}{3}$ tins of orange
ii) $33\frac{1}{3}$ tins of yellow, $16\frac{2}{3}$ tins of orange
d) 8.4 litres
3) a) i) $2x + y \leq 100$ ii) $5x + 8y \geq 400$ iii) $y \leq 2x$
c) 25 large, 50 small.
4) a) i) $x + y < 15$ ii) $x + y > 10$ iii) $y > x$ iv) $y < 2x$
c) (4,7) (5,6) (5,7) (5,8) (6,7) (6,8) (5,9)

59. Angles and Triangles

1) 49°, 131°, 49°	2) 79°, 84°, 17°
3) 120°, 60°, 30°	4) 42°, 45°, 93°
5) 60°, 60°, 60°, 30°	6) 57°, 57°, 33°
7) 45°, 27°, 45°	8) 44°, 69°, 67°
9) 150°, 150°, 30°	10) 40°, 70°, 30°

60. Regular Polygons 1

1) 120º, 60º 2) 140º, 40º 3) 150º, 30º
4) 162º, 18º 5) 90º, 72º, 54º

6) 135º, 67.5º, 22.5º, 45º, 45º
7) 51.4º, 64.3º, 51.4º, 128.6º, 90º
8) 8 9) The interior angle of a regular pentagon is not a factor of 360º but in a regular hexagon it is. 10) 9 11) 144º, 72º

61. Regular Polygons 2

1) a) 135° b) 67.5° c) 45° 2) 360°
3) 360° minus the two interior angles of the shapes
4) $x = 108°$ (Interior angle of a regular pentagon)
$y = 36°$ $(180° - 2 \times 72°)$
5) Regular triangle with side $3x$
6) a) 128.6°, 77.1° b) $\angle HCB = \angle HDA$
7) 24 sides

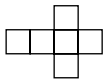
62. Congruent Triangles

1) All 2) a and b 3) a and c 4) b and c
5) Two sides and the included angle are equal on both triangles.
6) Two angles and a side are equal on both triangles.
7) (i) True - rest are false.

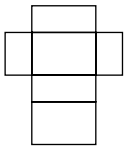
63. Nets and Isometric Drawing

(Diagrams are not to scale- use as a guide only)

1)

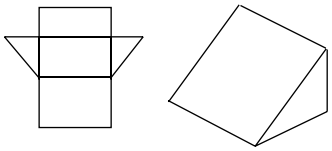


2)

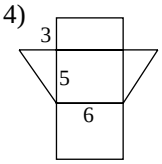


An example would be $4 \times 4 \times 3$ or $12 \times 1 \times 4$ etc.

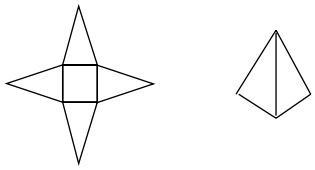
3)



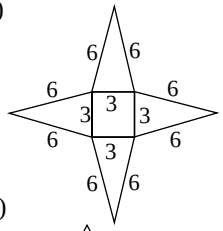
4)



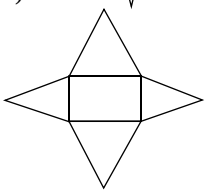
5)



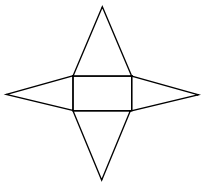
6)



7)



8)



64. Geometry of a Circle 1

1) 48°, 42°, 42° 2) 90°, 33°, 57°
3) 140°, 80°, 50° 4) 96°, 84°, 72°
5) 12°, 57°, 78° 6) 27.5°, 125°, 97.5°

65. Geometry of a Circle 2

1) 94°, 43°, 52°, 42° 2) 47°, 88°, 9°
3) 50°, 130°, 120°, 70° 4) 124°, 56°, 34°
5) 124°, 68°, 56° 6) 62°, 124°, 56°, 28°

66. Vectors 1

1) $\mathbf{b} + \mathbf{c}$, $\mathbf{b} + \mathbf{c} + \mathbf{d}$, $\mathbf{c} + \mathbf{d}$ 2) $\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ 3) $\overrightarrow{AD}$
4) a) $4\mathbf{c} - 4\mathbf{a}$ b) $2\mathbf{c} - 2\mathbf{a}$ c) $4\mathbf{a} + 2\mathbf{c} - 2\mathbf{a} = 2\mathbf{c} + 2\mathbf{a}$
d) $2\mathbf{c}$ e) PX is parallel to OC and half its length
5) a) $9\mathbf{b} - 6\mathbf{a}$ b) $3\mathbf{b} - 2\mathbf{a}$ c) $6\mathbf{a} + 3\mathbf{b} - 2\mathbf{a} = 4\mathbf{a} + 3\mathbf{b}$
6) a) $\mathbf{a} + \mathbf{b}$ b) $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ c) $\frac{1}{2}(\mathbf{b} - \mathbf{a})$
7) $-\mathbf{a}$, $-\mathbf{b}$, $2\mathbf{a}$, $-\mathbf{a}$, $\mathbf{b}-\mathbf{a}$, $2\mathbf{a}-2\mathbf{b}$
8) a) AB is parallel to CD, CD is twice as long as AB
b) Parallel. One is $\frac{2}{3}$ the length of the other
9) a) $2\mathbf{b} - 2\mathbf{a}$ b) $\mathbf{b} - \mathbf{a}$ c) $\overrightarrow{XY}$ is parallel to $\overrightarrow{AB}$ because $\overrightarrow{AB} = 2 \times \overrightarrow{XY}$
10) $\overrightarrow{AC} = \mathbf{a} + \mathbf{b}$ $\overrightarrow{BD} = \mathbf{b} - \mathbf{a}$

67. Vectors 2

1) a) $\frac{2}{3}\mathbf{b}$ b) $\frac{1}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}$ c) $\mathbf{b} + \frac{1}{2}\mathbf{a}$
2) a) $\mathbf{a} + \mathbf{b}$ b) $\frac{1}{3}(\mathbf{a} + \mathbf{b})$ c) $\frac{1}{3}\mathbf{b}$

d) $\frac{2}{3}$ **a** PQ is parallel to OC and $\frac{2}{3}$ of its length

3) a) (**a** + **b**) b) $\frac{1}{6}$ (**a** + **b**) c) **b** – **a** d) $\frac{1}{6}$ (**b** – **a**)
e) $\frac{2}{3}$ **a** f) They are parallel and XY is $\frac{2}{3}$ the length of CB

4) a) $\frac{1}{2}$ **a** b) **b** – $\frac{1}{2}$ **a** c) **a** + **b** d) $\frac{1}{3}$**b** – $\frac{1}{6}$ **a**
e) $\frac{1}{3}$ **a** + $\frac{1}{3}$ **b** f) $\frac{1}{3}$

5) a) 12**b** + 6**a** b) 5**a** – 2**b** c) 3**b** + 6**a**

68. Similar Shapes 1

1) a. FDE b. 3:2 or 1.5:1 c. 4.8cm d. 6.6cm
2) a. 4:5 or 1:1.25 b. 3m c. 2.6m
3) a. DBC b. 1.8cm c. 1.4cm
4) a. DCE b. 1:2.5 or 2:5 c. 7cm d. 3.6

69. Similar Shapes 2

1) a) 4:9 b) 8:27 c) 337.5 cm^3
2) a) 8:19 b) 450 cm^2
3) 122.88 grams 4) 390.625 grams
5) 11.9 cm 6) 8,000 cm^3
7) a) 112.5 cm^2 b) 14.81 cm^3
8) 350 grams

70. Similarity

1) a) i) $75°$ ii) Since DC and BE are parallel then angle CDE and BEA are equal
b) AE = 6 cm
2) a) i) $39°$ ii) Angle DCE = $116°$ (vertically opposite) Angle DEC = 180 – (116 + 25) = $39°$ (3 angles of a triangle add up to $180°$)
b) 7.5 cm
3) a) 7.5 cm b) 7.5cm 4) a) 6 cm 1.5cm b) 1:16

71. Reflections, Rotations and Translations 1

1) a. (1,–1) (4,–3.4) (4,–1)
b. (–1,1) (–4,3.4) (–4,1)
2) a. (0,4) (–3,4) (–3,2) (0,2)
b. (2,–2) (5,–2) (5,0) (2,0)
3) a. (3,–1) (0,–1) (0,–3) (1,–3) (3,–2)
b. (–5,2) (–2,2) (–2,4) (–3,4) (–5,3)
c. (1,2) (4,2) (4,0) (3,0) (1,1)

72. Reflections, Rotations and Translations 2

1) a. (3,–1) (3,1) (1,1) (1,–1)
b. (–3,1) (–3,–1) (–1,–1) (–1,1)
2) a. (2,–2) (5,1) (4,2) (1,–1)
b. (–2,2) (–5,–1) (–4,–2) (–1,1)
3) a. (0,–4) (2,–1) (–2,–1)
b. (0,4) (–2,1) (2,1)

73. Enlargements 1

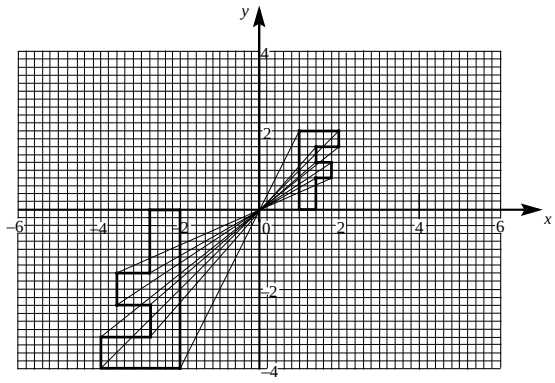
1) (4,8) (12,8) (12,4) (4,4)
2) (3,9) (12,3) (3,3)

74. Enlargements 2

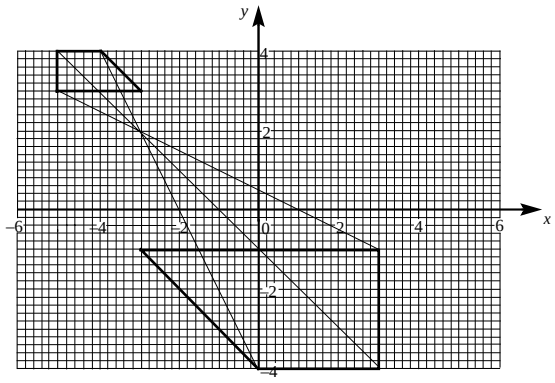
3) (1,2) (3.5,2) (3.5,–3) (1,–3)
4) (–1,1) (0,–1) (–2,–1)

75 Enlargement 3

1) $\frac{3}{2}$, $\frac{1}{2}$ and –1
2)

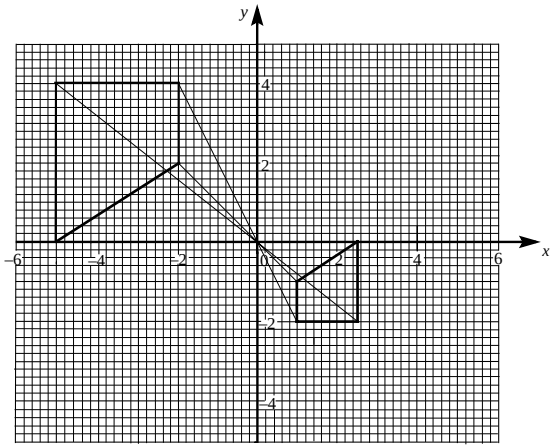


3)

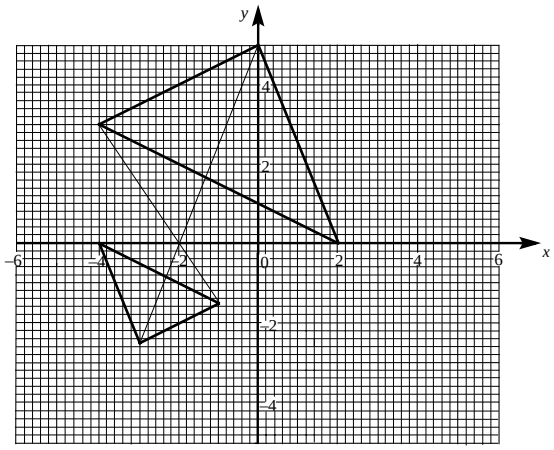


76 Enlargement 4

1)



2)



77. Transformations 1

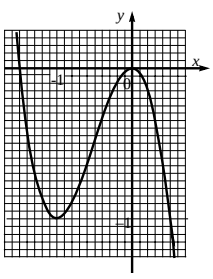
1) a) (–1, –2) (–3,–2) (–4,–5)
   b) (1,–2) (3,–2) (4,–5)
   c) Rotation of 180˚ about origin (0,0)
2) a) (2,2) (2,4) (4,2)
   b) (2,–2) (2,–4) (4,–2)
   c) (–1,1) (–2,1) (–1,2)
   d) An enlargement with a scale factor of 2 through the origin (0,0)
3) b) (1,2) (2,2) (2,1)
   c) An enlargement with a scale factor of $2\frac{1}{2}$ through the origin (0,0)

78. Transformations 2

1) a) (–1,–1) (–4,–4) (–3,–1)
   b) (–1,3) (–3,3) (–4,0)
   c) Rotation of 180˚ (clockwise or anticlockwise) about the point (0,2)
2) a) Rotation of 180˚ about the origin (0,0)
   b) Translation $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$ c) (–1,4) (–1,2) (1,2)
   d) Rotation of 180˚ about (–1,3)
3) a) (2,1) (5,1) (4,–2) b) (0,1) (0,–2) (–3,–1)
   c) Reflection in the line $y = 3 - x$

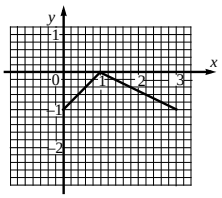
79. Transformations of Graphs

1) b) iii c) v d) iv e) ii f) i
2) b) Translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ c) Translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$
   d) *y* dimensions x 4 e) Translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
   f) Translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
3) a) $-2x^3 - 3x^2$
b)

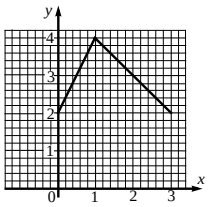


c) $-2x^3 + 3x^2$. Reflection in the *y* axis.

4) a)



b)



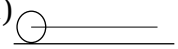
80. Matrix Transformations

1) a) Rotation of 180˚
   b) Enlargement of x 2 through origin (0,0)
   c) Reflection in the y axis
   d) Reflection in the line $y = x$
2) $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ (1,2)
3) a) Points are (–1,1) (–2,1) (–1,3) and (1,–1) (2,–1) (1,–3)
   b) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
   d) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Reflection in the *x* axis ($y = 0$)
4) a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ b) Rotation of 90˚ anticlockwise about (0,0)
   c) Rotation of 90˚ clockwise about (0,0) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
5) a) (1,–1) (2,–2) (1,–4) (4,–1)

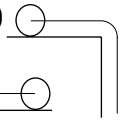
b) Reflection in the *x* axis ($y = 0$)
   d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ (5,3)

81. Loci 1

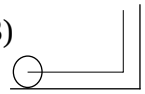
1)



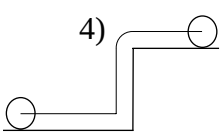
2)



3)

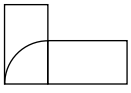


4)

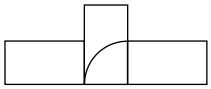


5)

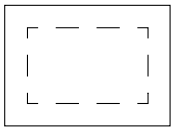
6)

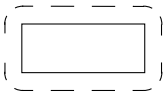


7) a) AB

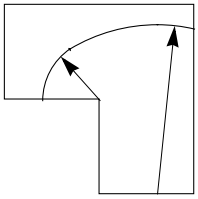
b) 

82. Loci 2

1) 

2) 

3)81metres

4) 

5) 700 metres

83. Construction

6) 21 metres 7) 4.5 cm 8) 1630 m

84. Ratios and Scales

1) 120, 150, 180 2) 150, 175

3) a) 2:4:1 b) 160, 320, 80

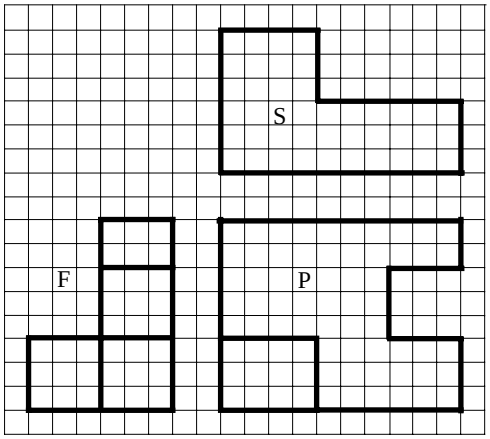
4) 450 ml 5) 50 cm, 2.2 m, 500 cm^2 , 40˚

6) 100 cm, 80˚, 120 cm^2 , 12.5 m

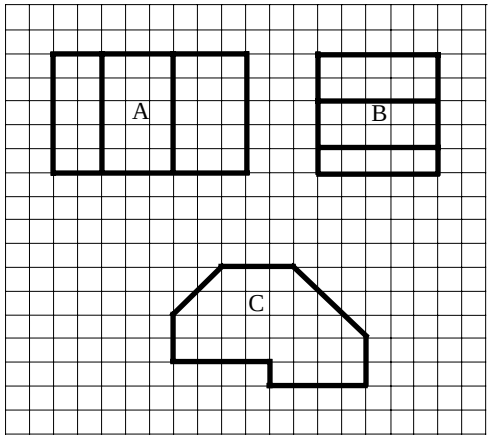
7) 150 cm, 7.68 m^2 , 93,750 cm^3 10 cm table

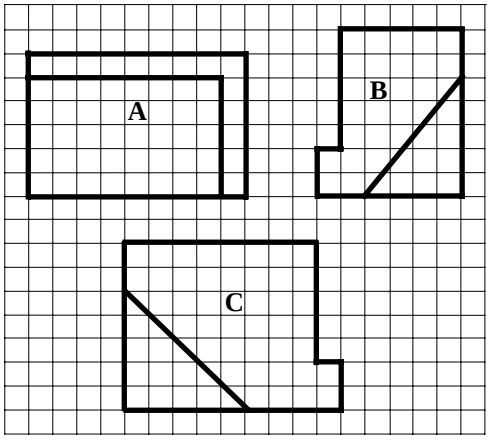
8) a) 50 cm b) 80 m^2 c) 3,750 cm^3

85. Plans and Elevations 1

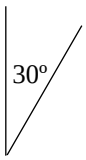


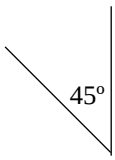
86. Plans and Elevations 2

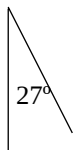
1) 

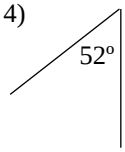
2**)** 

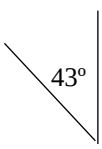
87 Bearing 1

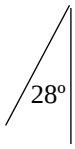
1) 

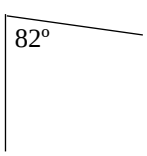
2) 

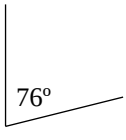
3) 

4) 

5) 

6) 

7) 

8) 

Exercise 2

1) N29ºE or 029º 2) S58ºW or 238º

3) N28ºW or 332º 4) S63ºW or 243º

5) N74ºE or 074º

Exercise 3

1) S36ºE or 144º 2) N54ºE or 054º

3) 9.4km S88ºE or 092º
4) 152km S33ºW or 213º

88. Bearings 2

1) S 28° W (208°) 2) N 73° E (073°)
3) a) N 30° E (030°) b) S 30° E (150°)
c) N 30° W (330°)
4) a) N 6° E (006°) b) N 55° W (305°)
c) S 55° E (125°)
5) a) N 60° E (060°) b) Due South (180°)
c) S 60° W (240°) d) S 60° W (240°)
6) a) N 51° W (309°) b) N 4° E (004°)
c) N 84° W (276°)

89. Degree of Accuracy 1

Exercise 1

1) a. 4000 b. 5200 c. 460 d. 800
e. 15000 f. 23000
2) a. 23300 b. 23300 c. 2300
d. 20000
3) a,b,f,g.
4) a. 1250 1349 b. 2450 2549
c. 4,150 4,249 d. 22,500 23,499
e. 65,000, 74,999 f. 6500 7499
g. 204,500 205,499 h. 235 244
i. 745 754 j. 1345 1354

Exercise 2

1) 121.5cm 2) ≤, < 3) 19.25, <
4) 7.815, 7.825 5) ≤, <

Exercise 3

1) 9.35, 9.45 2) 62.25, 62.35
3) 19.45, 19.55 4) 27.55, 27.65
5) 19.615, 19.625 6) 25.635, 25.645
7) 15.5, 16.5 8) 17.25, 17.35
9) 37.25, 37.35 10) 6.475, 6.485
11) 9.335, 9.345 12) 1.325, 1.335

90. Degree of Accuracy 2

1) 84, 86 2) a) 418.65 cm^3 b) 403.27 cm^3
3) 368 4) 0.35 cm, 0.05 cm
5) a) 9.79 cm^3 b) 8.24 cm^3 c) 15.8%
6) a) 89.22 litres b) 5.07 litres
7) a) 22 b) 20 c) 440 d) 20

91. Circumference of a Circle

Exercise 1

1) 25.136cm 2) 37.704cm 3) 62.84cm
4) 113.112m 5) 50.272m 6) 43.988m
7) 37.704cm 8) 50.272cm 9) 75.408cm
10) 7.2266m 11) 53.414m 12) 72.266m

Exercise 2

1) 6.365cm 2) 33.42cm 3) 73.20cm
4) 4.774m 5) 81.48cm 6) 56.02m

Exercise 3

1) 785.5cm 2) 786m 3) 26m
4) 579 turns 5) 21 turns 6) 979 turns
7) 55cm 8) 4m 8cm 9) 54m

92. Area and Perimeter 1

1) a) 9cm^2 and 12cm b) 48cm^2 and 28cm
c) 30.6cm^2 and 24.8cm
d) 78.12cm^2 and 35.4cm
e) 0.96m^2 and 4m f) 1.44cm^2 and 5m
2) a) 96cm^2 b) 31.92cm^2 c) 68cm^2
d) 67.5cm^2
3) a) 28.3cm^2 b) 176.7cm^2 c) 1134.3cm^2
d) 38.5cm^2 e) 10.2cm^2 f) 237.8cm^2
4) a) 48cm^2 b) 41cm^2 c) 21.994cm^2
d) 150cm^2 e) 117.45cm^2 f) 204cm^2
5) 150 6) 50.272cm^2, 28.568cm

93. Area and Perimeter 2

1) 60 cm^2 2) 35.3475 cm^2
3) a) 9.426 cm^2 3.142 cm
b) 175.952 cm^2 29.3 cm (to 1 d.p.)
c) 475.2 cm^2 (to 1 d.p.) 57.6 cm (to 1 d.p.)
d) 477.5 cm^2 (to 1 d.p.) 57.0 cm (to 1 d.p.)
4) a) 42.55 m^2 (to 2 d.p.) b) 27.02 m (to 2 d.p.)
5) a) 333.8 cm^2 (to 2 d.p.) b) 74.2 cm (to 1 d.p.)
6) a) 62.84 m^2 b) 14 tins 7) 402.2 m^2 (to 1 d.p.)
8) 509,500,000 km^2 (to 4 sig. figures)

94. Volume 1

Exercise 1

1) 2000 2) 3 3) 3,400,000 4) 15000
5) 550 6) 1.2m^3 7) 500 8) 53
9) 28,000 10) 3

Exercise 2

1) 1080cm^3 2) 25, 5 3) 16.8m^3
4) a) 384.895cm^3 b) 385ml
5) 90 6) 3142 litres 7) 577.8g
8) 11 times 9) 691cm^2 10) 100cm
11) 4.94mm 12) 350,000 litres

95. Volume 2

1) 960 cm^3 2) 7.5 cm
3) a) 752.8 cm^3 (to 4 sig. figures)
b) 564.6 grammes (to 4 sig. figures)
4) a) 134.06 cm^3 b) 18.1 kg
5) 4.63 cm (to 2 d.p.)
6) 16 litres 7) 5893 spheres
8) a) 98 cm^3 b) 20
9) a) 5832 cm^3 b) 52.4%
10) 3.4 cm (to 1 d.p.)
11) 255.55 cm^3 (to 2 d.p.) 12) 156 ml

96. Formula for Area, Volume and Perimeter

1) (i) b (ii) c 2) (i) d (ii)e 3) (i) e (ii) b 4) (i) b (ii)e

97. Formulae

Exercise 1

1) area 2) area 3) area
4) none 5) area 6) area

7) length 8) length 9) length
10) volume 11) volume 12) volume
13) none 14) none 15) length
16) length 17) area 18) volume
19) none 20) area 21) volume

Exercise 2

1) a) iii b) i 2) a) ii b) v

98. Compound Measure - Speed and Density

Exercise 1

1) a) 20, 80, 10 b) 15, 60, 7.5 c) 30, 120, 15
2) a) 160 miles b) 40 miles c) $2\frac{1}{2}$ hours
3) a) 90 miles b) 45mph
4) 2 hours 5) a) 15 miles b) 70 miles

Exercise 2

1) a) 3g b) 2.75g 2) 3 red 1 blue etc.
3) 1 red 3 blue etc.
4) multiples of 1 red and 3 blue.
5) a) 4g per cm^3 b) 3g per cm^3

99. Compound Measure - Best Buy and Mixed

Exercise 1

1) 125ml 2) 800ml 3) 700g 4) 4 litre
5) 150ml

Exercise 2

1) a) B b) D 2) 4 tins of 5 litres
3) 4 packets
4) 1 tin of 5 litres and 1 tin of 2 litres

100. Pythagoras Theorem

All to 4 significant figures

1) a) 7.211 b) 7.483 c) 4.8 d) 8.801
2) 45.5 cm 3) 5.657 cm, 2.828 cm
4) 6.403 cm 5) 2.721 metres 6) 18.81 metres

101. Sine, Cosine and Tangent Ratios 1

1) a) 15.28cm b) 6.576cm c) 5.503 d) 22.58
e) 24.04cm f) 11.21cm g) 17.78cm
2) a) 41.19º b) 46.90º c) 42.30º d) 42.22º e) 40.12º
f) 30º g) 18.43º

102. Sine, Cosine and Tangent Ratios 2

1) 25.72cm and 15.45cm 2) CD = 17.33cm
AC = 26.69 EB = 10.21 DEB = 43.31º
3) a) N24ºE or 024º b) 5.408km 4) a) 7.523m b) 28º

103. Sine and Cosine Rules

1) a) 72.97˚ b) 61.09˚ c) 4.74˚ d) 54.79˚
e) 54.80˚ f) 10.54 cm
2) 16.01 cm 3) 115.18˚ and 64.82˚
4) 4.05 km 5) 16.08 m
6) 58.75˚, 71.79˚, 49.46˚

104. Areas of Triangles

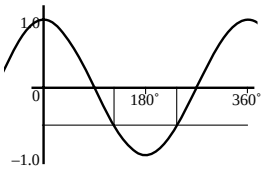
1) 20.99 cm^2 2) 53.83 cm^2 3) 22.36 cm^2
4) 27.39 cm^2 5) 131.95 cm^2
6) The perpendicular height from C to AD is 9 sin 20˚. The perpendicular height from B to DA produced is also 9 sin 20˚. Triangles ACD and ABD have the same base ie. AD. Therefore their areas are equal.
7) 49.85 cm^2 8) 2.3 m^2
9) 237.8 cm^2 10) 3.4 hectares

105. Trigonometry - Mixed Exercise

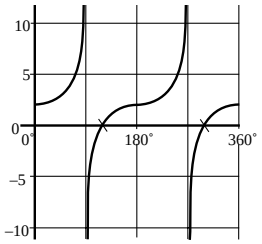
1) AC = 6.325 cm CD = 5.175cm
2) 10.91 cm 3) 9.226 m
4) 122.75˚, 114.5˚ 5) 14.62 cm 6) 13.09 m

106. Graphs of Sines, Cosines and Tangents

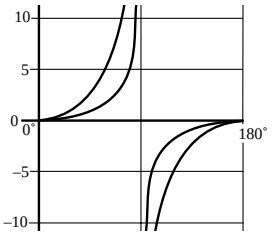
1) 210˚ and 330˚ 2) Approx 49˚ and 131˚
3)



4) Approx 71˚ and 289˚ 5) Approx 76˚ and 256˚
6) a)



b)



7) Approx 56˚ and 236˚

107. Three Dimensional Trigonometry

1) 33.31 m (to 4 sig. figures)
2) a) 28.28 cm b) 20.20 cm
3) a) 4.667 m b) 23.2˚ c) HC = 5.077 m
4) a) 3.830 m b) 21.0˚
5) 107.3 m

108. Questionnaires 1

These answers are examples only, there are many other acceptable answers

1) a) How often do you use a supermarket
(i) Once a week
(ii) Less than once a week
(iii) Never?
b) Do you think there is a need for a supermarket in the town?
(i) Yes
(ii) No
(iii) Don't know
2) The local supermarket– good cross-section

of ages- potential customers.
Residents of the high street- they will be most affected, so probably biased.
Residents of the local housing estate; cross-section of ages; potential customers

3) No; they will probably get a one-sided view of the problem (biased).

4) Would you be prepared to buy vegetarian food from the tuck shop? yes/no.
Which two of the following would you prefer to eat? a) Fruit b) Yoghurt c) Oatmeal biscuits d) Nuts e) Wholemeal sandwiches.

5) a) Biased- needs a group more representative of people in general.
b) (i) Biased- same age group
(ii) Probably the least biased group as they are more representative of people in general
(iii) Biased- no young people.
(iv) It is important to ask these people as they will be most affected but they will be biased.

6) Not a good questionnaire because a) it only gives one choice, b) it puts pressure on the staff to agree by saying the manager thinks it is a good idea. It would be better to give the two choices with no comment about whether the manager likes it. A third choice such as 'None of these' or a space for their own comment would make it less biased.

109. Questionnaires 2

The following answers are for guidance only. There are other solutions.

1) a) The survey could be biased for the following reasons:
i) It is done just in the month of July only.
ii) It is carried out at 12.00 only and not at other times of the day
iii) It is carried out in just one particular place. To get information about the whole of Britain she needs information from other towns.
iv) Town is of one particular type ie historical.
b) The survey could be improved by including the points raised in part a)
c) Are you a tourist ?
Which country do you come from ?
What is your gender m/f ?

2) a) How old are you ?
What is your gender, male or female ?
What other newspapers do you read?
b) i) It only asks those who already read the newspaper. It does not get to potential readers.

3) a) It tells her the age of the cars on the road now. She could, for example, calculate the average age of cars on the road but not the age at which they are taken off the road.
b) i) Was your last car sold to someone to use again or was it scrapped ?
ii) How old was the car when it was scrapped ?

110. Pie Charts 1

1) Angles are:
20^o, 100^o, 160^o, 20^o and 60^o
2) 187.5^o, 30^o, 120^o, 22.5^o
3) 160^o, 40^o, 60^o, 48^o, 20^o and 32^o
4) a. 6 b. 720, 540, 360, 270, 180, 90.
5) a. 12^o b. 84^o, 156^o, 96^o, 24^o

111. Pie Charts 2

1) Business stock £5972 Employee costs £14722
Premises £7361 General administration £10278
Advertising £3889 Other costs £7778
2) Angles 115.2˚, 43.65˚, 37.35˚, 59.4˚, 83.25˚, 21.15˚
3) a) 1008 b) 140˚ d) 126

112. Flow Charts 1

1)

n	S
40	1603
41	1684
42	1767
43	1852
44	1939
45	2028

There are 44 numbers less than 2000

2) W. Jones - Merit
J. Connah - Pass
C. Smith - Distinction
H. Patel - Pass

113. Flow Charts 2

1) 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181.
Each number is obtained by adding together the previous two numbers.
2) M = 8 It calculates the mean of a list of numbers.

114. Stem and Leaf Diagrams

1) a) 70 b) 65 c) 30 d) 76 and 50 e) 26

2)

16	1,1,8
17	0, 9, 3, 9, 9
18	6, 7, 5, 6, 5, 5
19	1, 2, 1, 2, 5
20	3, 5, 4
21	7, 0

3) a)

0	0, 3, 0
0	6, 7
1	3, 2, 4
1	8, 9, 6, 5, 8
2	0, 3, 3, 4, 2, 3, 2
2	6, 7, 9
3	2, 4, 1, 1, 0, 3
3	6

b)

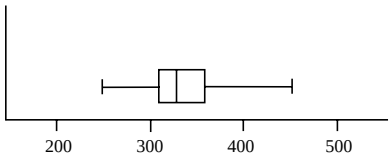
0	0, 0
0	
1	
1	

2 1, 2, 4
2 5, 6, 9, 9, 7
3 0, 4, 1, 4, 0
3 6, 6, 5, 5, 9
4 2, 3, 2, 2, 0, 1, 1, 1
4 7, 7

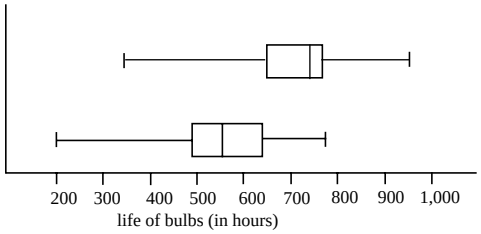
c) The rainfall in England averages about 2mm per day whereas the rainfall in Ireland averages about 3.5mm per day. In England the rainfall is spread between 0 and 3.6mm. In Ireland it is generally spread between 2.1 and 4.7mm although there are two days when it is zero. On days when it rains in Ireland it rains more than in England

115. Box Plots

1)



2)



The Super bulbs last longer in general than the Regular bulbs. However there is an overlap, with some Regular bulbs lasting longer than the Super ones.

3) Machine A. Both the upper and lower quartiles lie outside the acceptable range indicating that less than 50% of the rods are acceptable.

Machine B. Both the upper and lower quartiles lie within the acceptable range indicating that more than 50% of the rods are acceptable. Also, the largest diameter rod is smaller than the largest rod from machine A and the smallest diameter rod is larger than the smallest from machine A. All this indicates that machine B is the more accurate.

116. Sampling 1

1) a) 14, 12, 14, 12, 12, 4, 2

b) Do you watch TV ? Yes/no
What is your gender ? Male/female
Which channels do you watch mostly ? 1, 2, 3 etc.
Which types of programmes do you mainly watch ? Plays, sport, news etc.

2) a) 10, 14, 10, 4, 2

b) Are you happy with your working conditions ?
What areas of your working environment need to be improved?
What type of work do you do ?
How old are you ?
Gender, male/female.

c) Age range, type of work undertaken, gender etc.

3) a) 19, 37, 33, 11

b) i) What would you like to see improved in the centre ?
ii) Which facilities do you use most often?
iii) Would you use other facilities if they were introduced ? Yes/no
iv) What other facilities do you think ought to be introduced ? (give a choice)

117. Sampling 2

1) 6, 29, 14, 8, 33, 10 2) a) 369, 31, 623 b) 3000, 130
3) 17, 13, 9, 6, 3, 2

118. Sampling 3

1) a) To get two digits you would need two dice. Preferably of different colours but one distinguishable from the other. One would be designated for the first digit and the other the second digit. Throw the dice and write down the numbers.

b) Use the same method with three dice.

2) a) Allocate each of the numbers 1 to 77 to each of the 77 people. Now go through the table picking off the first 10 numbers with values of 77 or less. These numbers would indicate the 10 chosen males.

b) 11, 55, 59, 38, 33, 25, 34, 14, 74, 39

c) The largest 2 digit number would be 99. Since there are 265 people in this group you would need to use a table containing 3 digits.

d) A table of 3 digit numbers.

119. Scatter Diagrams 1

1) b) 235
2) a) 7:30 pm b) 3˚C c) 4:45 pm
3) a) 15.5 tonnes (approximately) b) 4.8 minutes

120. Scatter Diagrams 2

Answers are approximate
1) a. 29g b. 18.1cm
2) 64mph 3) 3 4) 142cm

121. Histograms 1

1) a)

Time	0–	5–	10–	15–	20–	25–	30–	35–	40–	45–50
Freq	7	12	22	28	33	24	14	8	6	3

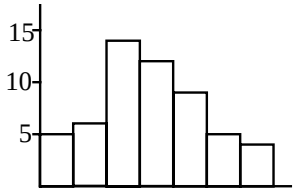
b) mean = 21.83 minutes

c) 35%

2) a)

Mass	40–	50–	60–	70–	80–	90–	100–110
Freq.	5	6	14	12	9	5	4

b)



c) 60–70

122. Histograms 2

1) a)

Age	15–	25–	30–	35–	45–	60–70
People	18	21	28	32	12	6

b) mean = 35.45

2) a)

Speed	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 100$
No.of cars	36	80	208	112	60

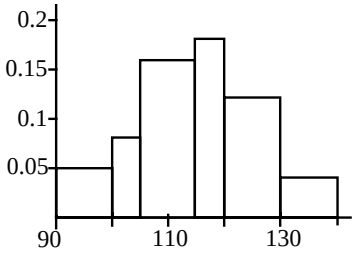
b) $40 < x \leq 60$ is the modal range

123. Histograms 3

1) a)

Height in cm	No. of plants	Frequency density
$90 < x \leq 100$	5	0.5
$100 < x \leq 105$	4	0.8
$105 < x \leq 115$	16	1.6
$115 < x \leq 120$	9	1.8
$120 < x \leq 130$	12	1.2
$130 < x \leq 140$	4	0.4

b)

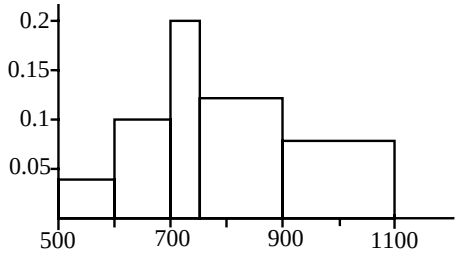


c) 50%

2) a)

Time	Frequency	Frequency density
$500 < x \leq 600$	4	0.04
$600 < x \leq 700$	10	0.10
$700 < x \leq 750$	10	0.20
$750 < x \leq 900$	18	0.12
$900 < x \leq 1100$	16	0.08

b)



c) 52%

124. Mean 1

1)

Wages, £	Frequency	Mid value	Frequency x mid value
60 - 100	4	80	320
100 - 140	19	120	2280
140 - 170	24	155	3720
170 - 200	11	185	2035
200 - 220	6	210	1260

mean = £150.23

2)

Weight	Frequency	Mid value	Frequency x mid value
50 - 70	5	60	300
70 - 90	14	80	1120
90 - 100	16	95	1520
100 - 110	24	105	2520
110 - 120	21	115	2415
120 - 140	23	130	2990
140 - 160	17	150	2550

mean = 111.8 cm

3)

Speed	Frequency	Mid value	Frequency x mid value
$20 \leq s < 30$	6	25	150
$30 \leq s < 40$	10	35	350
$40 \leq s < 50$	15	45	675
$50 \leq s < 60$	14	55	770
$60 \leq s < 70$	6	65	390
$70 \leq s < 80$	6	75	450
$80 \leq s < 90$	3	85	255
Totals	60		3040

mean = 50.67

125. Mean 2

1) 2.345 2) 2.3 3) 0.2087 4) 5.64

126. Mean, Median and Mode

1) a) 21 pints b) 18.3 pints c) 18 pints

2) a) 4.3 letters b) 3 c) 2

d) No - because the sample is very small compared with the whole book and it is only taken from one part of the book.

3) b) 428.3 kg c) 100 – 300 d) 100 – 300

4) a) 1.02 – 1.03 litres b) 1.02 – 1.03 litres
c) 1.025 ml (to 4 sig. figures)

127. Mean, Median, Mode and Range.

Exercise 1

1)

1	2	3	4	5	6	7
8	7	7	5	4	2	1

Mode=1, Range =6

2)

0	1	2	3	4	5	6	7	8	9
8	6	9	4	2	2	4	4	6	3

Mode=2, Range=9

3)

13	14	15	16	17
7	10	13	8	7

Mode=15, Range=4

Exercise 2

1)

1	2	3	4	5	6	7	8	9	10
1	1	2	4	3	2	3	2	1	2

Median=5, Range=9

2)

2	3	4	5	6	7	8	9
1	2	1	2	2	5	1	2

Median=6.5, Range=7

3) Median=62.5, Range=78

Exercise 3

1) 5.6cm 2) 42 3) 5.0 4) 23.14
5) 318.7 6) 2.583 7) 41.25 8) 190
9) 106.3 10) 43.44

Exercise 4

1) 24.15 2) 86.22kg 3) 5

128. Mean and Standard Deviation

1) a) 5.044, 0.0246
b) i) The mean is increased by 0.02 kg
ii) There is no change to the standard deviation

2) b) i) 155.6 ii) 10.7

3) b) £230.15, 67.48 c) Mean is increased by £10, standard deviation remains the same.

4) b) 41 and 12.54
c) The mean age at the badminton club is higher, indicating that the members tend to be older. The S.D. at the badminton club is lower, indicating that a greater proportion of the ages are closer to the mean than at the tennis club.

129. Moving Averages

a) There needs to be at least 8 weeks data to calculate an 8 week moving average.

b) At the end of week 8

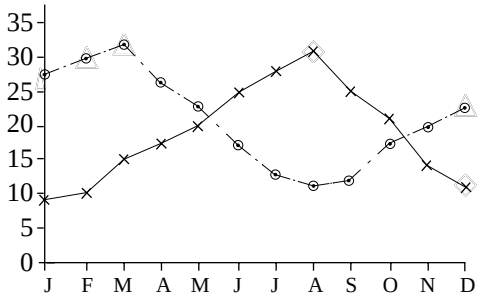
c)

Day	Value of a share in pence	8 week moving average
1	251	
2	263	
3	294	
4	330	
5	350	
6	345	
7	342	
8	340	314.4
9	336	325.0
10	331	333.5
11	330	338.0
12	321	336.9
13	315	332.5
14	302	327.1
15	321	324.5
16	317	321.6
17	320	319.6
18	318	318.0
19	307	315.1
20	301	312.6
21	331	314.6
22	287	312.8
23	305	310.8
24	306	309.4

d) The trend is down

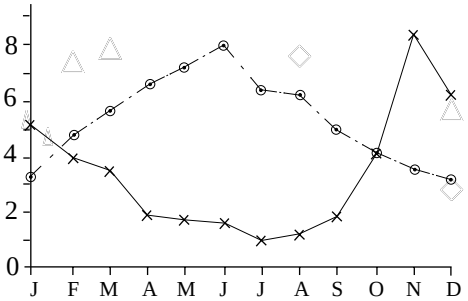
130. Frequency Polygons 1

1) a)



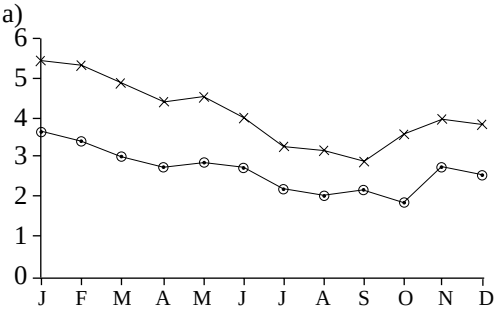
(i) A-Europe, B- Australia
(ii) Temperatures are high in August in Europe and high in February in Australia.
(iii) May and October.

b)



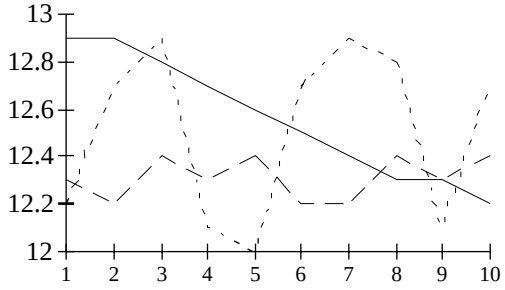
Personal choices with references made to
a) the temperatures b) the rainfall.

2) a)



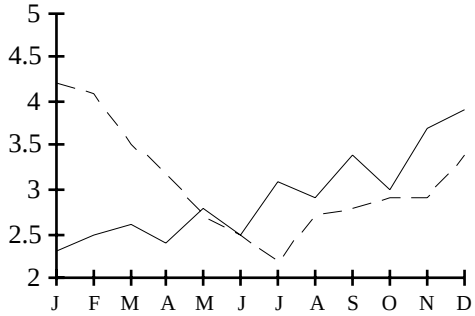
b) (i) Profits in 1991 were generally higher than in 1992. (ii) Profits are picking up as the graph is generally rising, but it did the same in 1991 and then fell in the new year. (iii) Just over 2 million per month

3)



Choose from a) Brian is most consistent b) John is getting worse c) Mike is not consistent but has the best times.

4)



Company A's profits are steadily rising but company B's profits went down substantially during the first half of the year, but rose again during the second half.

131. Frequency Polygons 2

1) a)

Weight	Frequency	Mid value
3.0 - 3.5	4	3.25
3.5 - 4.0	6	3.75
4.0 - 4.5	12	4.25
4.5 - 5.0	5	4.75
5.0 - 5.5	3	5.25

2)

Height	Frequency	Mid value
80 - 100	8	90
100 - 120	14	110
120 - 140	26	130
140 - 160	34	150
160 - 180	18	170
180 - 200	8	190
200 - 220	4	210

132. Cumulative Frequency 1

1) a)

Length l	9.7	9.8	9.9	10.0	10.1	10.2
Cumulative Frequency	4	17	41	77	94	100

c) 9.92 cm d) 9.99, 9.83, 0.16 e) 10%

2) a)

Age	20	30	40	50	60	65
Cumulative Frequency	17	82	160	211	241	250

c) 26%

3) a)

Mark	10	20	30	40	50	60	70	80	90	100
Cumulative Frequency	3	10	32	63	97	134	162	181	194	200

c) 44 d) 14%

133. Cumulative Frequency 2

1) a)

Life of bulb	6000	7000	8000	9000	10,000	11,000
Cumulative Frequency	5	25	67	162	242	300

c) 86%

2) a)

Time spent in air	30	35	40	45	50	55
Cumulative Frequency	6	16	28	62	85	100

c) 43 minutes, 10 minutes

3) a)

Timing (minutes)	5 early	0	5 late	10 late	15 late	20 late
Cumulative Frequency	33	99	121	138	148	150

c) 15%

4) a)

No. of T.V.'s	100	150	200	250	300	350
Cumulative Frequency	5	32	109	254	337	365

c) 246 days d) 10 days

134. Probability 1

1) b) $\frac{3}{12}$ or $\frac{1}{4}$ c) $\frac{3}{12}$ or $\frac{1}{4}$

2) a) $\frac{1}{6}$ b) $\frac{2}{6}$ or $\frac{1}{3}$ c) $\frac{1}{6}$

3) a) 1,1 1,2 1,3 1,4
2,1 2,2 2,3 2,4
3,1 3,2 3,3 3,4
b) 2 c) $\frac{2}{12}$ or $\frac{1}{6}$

4) a) $\frac{2}{30}$ or $\frac{1}{15}$ b) $\frac{10}{30}$ or $\frac{1}{3}$ c) $\frac{2}{3}$

135. Probability 2

1) a) $\frac{5}{9}$ b) $\frac{20}{81}$ c) $\frac{125}{729}$ d) $\frac{80}{243}$
2) a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{5}{6}$
3) a) 0.16 b) 0.48 c) 0.36
4) a) 0.02 b) 0.02 c) 0.01
5) a) 0.14 b) 0.06 c) 0.24 d) 0.86
6) a) 0 b) $\frac{5}{9}$

136. Probability 3

1) a) $\frac{3}{10}$ b) $\frac{7}{30}$ c) $\frac{7}{40}$ 2) 0.96 3) 0.996625
4) a) $\frac{1}{2}$ b) $\frac{1}{4}$ 5) a) $\frac{8}{15}$ b) $\frac{8}{29}$ 6) a) $\frac{2}{5}$ b) $\frac{3}{20}$
7) a) 0.02682 b) 0.045811 c) 0.927369

137. Relative Frequency 1

1) a) $\frac{3}{14}$ b) 420 c) 30 d) 75
2) a) 10 b) 20
3) a) 5, 3 and 2 b) 150
4) a) $\frac{9}{10}$ b) 10,000 c) 120,000
5) a) $\frac{2}{3}$ b) 8
6) b. 0.57

138. Relative Frequency 2

1) 620
2) a) i) 0.44 ii) 0.34 iii) 0.22
b) 19,800; 15,300; 9,900
3) a) 25, 15, 10 b) i) 0.5 ii) 0.3 iii) 0.2
c) i) 0.6 ii) 0.4 d) 250,000 and 150,000
4) a) 0.25, 0.3, 0.45 b) 137, 165, 247

5) a)

No. of people	50	100	150	200	250	300
No. 'for'	30	45	55	83	96	122
No. 'against'	20	55	95	117	154	178
Probability 'for'	0.60	0.45	0.37	0.415	0.384	0.407
Probability 'against'	0.40	0.55	0.63	0.585	0.616	0.593

c) 0.4 d) 42,000

139. Tree Diagrams

1) a) 0.015, 0.085, 0.135, 0.765
b) 0.235 c) 0.765
2) a) 0.035, 0.665, 0.045, 0.255
b) 0.08
3) a) $\frac{3}{28}$, $\frac{15}{56}$, $\frac{15}{56}$, $\frac{5}{14}$ b) i) $\frac{15}{28}$ ii) $\frac{9}{14}$
4) a) $\frac{3}{14}$ b) $\frac{11}{14}$ 5) a) $\frac{20}{63}$ b) $\frac{11}{63}$ c) $\frac{32}{63}$