Title: **Algebraic and Geometric Proof**

Key Knowledge/Prior Learning KS3 and Retrieval and Suggested Starters

- Work with coordinates in all four quadrants
- Plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form y = mx + c to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
- Plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form y = mx + c to identify parallel lines and perpendicular lines; find the equation of the line through two given points, or through one point with a given gradient
- Identify and interpret gradients and intercepts of linear functions graphically and algebraically

KS4 National Curriculum – what students will be practicing

- Identify and interpret roots, intercepts, and turning points of quadratic functions graphically; deduce roots algebraically
- Identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically and turning points by completing the square
- Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions and the reciprocal function, $y = \frac{1}{x}$ with x ≠ 0
- Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function, , $y = \frac{1}{x}$ with x ≠ 0, exponential functions $y = k^x$ for positive values of k, and the trigonometrical functions (with arguments in degrees) y = sin x, y = cos x and y = tan x for angles of any size
- Sketch translations and reflections of a given function
- Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts
- Recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point

Specific Ambitious Knowledge

Key Vocabulary/Literacy Opportunities

- Roots
- Intercepts
- Turning points
- Linear
- Quadratic

- Cubic
- Reciprocal
- Exponential
- Trigonometrical
- Tangent
- Reflection
- translation

Key Formulae/Knowledge

Linear graphs

A linear graph is a straight line, with a line equation of the form

$$y = mx + c,$$

where m is the gradient and c is the y-intercept.

Quadratic graphs

A quadratic graph is shaped like a parabola: ∪ or ∩.
Its equation is of the form

$$y = ax^2 + bx + c,$$

where $a \neq 0$.

- If $a < 0$ then the graph will have the shape ∩.
- If $a > 0$ then the graph will have the shape ∪.

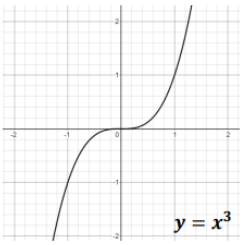
Cubic graphs

Cubic graphs can have more than one turning point and have equations of the form

$$y = ax^3 + bx^2 + cx + d,$$

where $a \neq 0$.

Cubic graphs have a shape similar to the curve on the right.



Sketching cubics can be done similarly to sketching a quadratic curve. Simply complete a table of values, plot the points and connect them with a smooth curve.

Reciprocal graphs

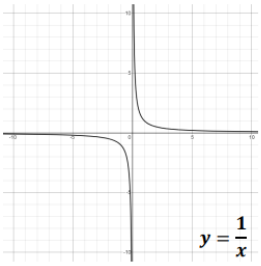
Reciprocal graphs are of the form

$$y = \frac{1}{x}$$

where $x \neq 0$.

The value of x cannot be 0 otherwise we would get $\frac{1}{0}$, which is undefined.

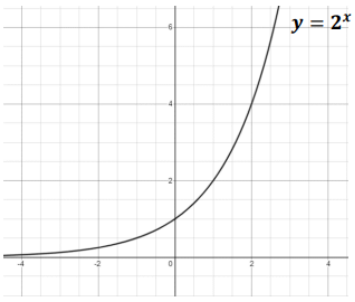
When sketching reciprocal graphs of this form you need to keep in mind there will be a discontinuity at $x = 0$. Apart from that, you proceed in the same way as the previous graphs: construct a table of values, plot the points and then connect them with a smooth curve.



Exponential functions
An exponential function is a function of the form

$$y = k^x$$

with $k > 0$.



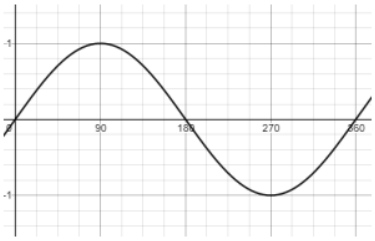
As the name would suggest these functions **increase 'exponentially'**. This means they increase at an increasing rate. This can be seen in the graph on the right where the curve gets steeper over time.

Exponential functions are **always positive** since k is positive and a positive number raised to any power is also positive.

- If $k > 1$, the graph will show an increasing curve like the one shown above.
- If $k = 1$, the graph will be constant and equal to 1.
- If $0 < k < 1$ the graph will be a decreasing curve which will get closer and closer to zero over time (but never reaches zero).

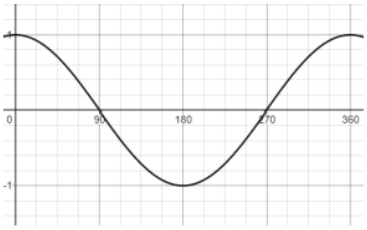
Sine

- $\sin x = 0$ at $x = 0^\circ$, 180° and 360°
- $\sin x = 1$ at $x = 90^\circ$
- $\sin x = -1$ at $x = 270^\circ$
- Repeats every 360°



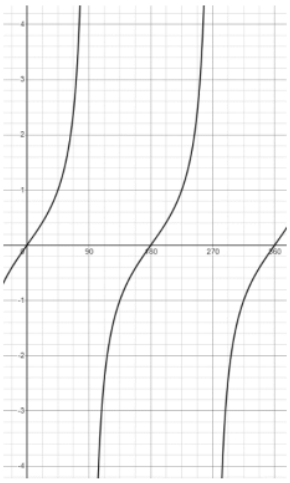
Cosine

- $\cos x = 0$ at $x = 90^\circ$ and $x = 270^\circ$
- $\cos x = 1$ at $x = 0^\circ$ and $x = 360^\circ$
- $\cos x = -1$ at $x = 180^\circ$
- Repeats every 360°



Tan

- $\tan x = 0$ at $x = 0^\circ$, 180° and 360°
- $\tan x$ approaches infinity at odd multiples of 90° and then 'starts again' from minus infinity.
- Repeats every 180°



Maths in Context (Historical, Real Life and Student Thinking Points)
• https://donsteward.blogspot.com/2012/06/weather-sines.html •

Projects/Enrichment/Investigations
• https://nrich.maths.org/4808?utm_source=secondary-map • https://nrich.maths.org/4809?utm_source=secondary-map • https://nrich.maths.org/4810?utm_source=secondary-map • https://nrich.maths.org/6424?utm_source=secondary-map • https://nrich.maths.org/7419?utm_source=secondary-map • https://nrich.maths.org/parabolicpatterns?utm_source=secondary-map • https://nrich.maths.org/802?utm_source=secondary-map • https://nrich.maths.org/6481?utm_source=secondary-map • https://nrich.maths.org/6427?utm_source=secondary-map