Title: **Algebraic and Geometric Proof**

Key Knowledge/Prior Learning KS2/3 and Retrieval and Suggested Starters
• angle facts, triangle congruence, similarity and properties of quadrilaterals • use vectors • Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal

KS4 National Curriculum – what students will be practicing
• argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments to include proofs • apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs • use vectors to construct geometric arguments and proofs

Specific Ambitious Knowledge

Key Vocabulary/Literacy Opportunities		
• Congruent	Condition	Similar
• Corresponding	Prove	Common
• Term	Expression	Equation
• Identity	Solve/solution	Sequence
• Equivalent	Sum	Product
• Difference	Simplest form	Surd
•		

Key Formulae/Knowledge
Identities An identity is an **equation** that is **true no matter what values are inputted**. Examples include $4x + 2x = 6x$ and $y + y + y = 3y$. The equals sign, $=$, can be replaced with the 'identical to' sign, $\equiv$, when dealing with identities. In mathematics, a proof is a **sequence of true statements** that logically follow each other to **prove** a required result. An algebraic proof uses algebra instead of numbers, which means we can prove things are true for **all numbers at once**.

Proof by Exhaustion

To prove by exhaustion, you give an example of every possible scenario in which the statement is true. For example, consider the following question:

$$\textit{Prove that for every even number } n \textit{ where } n \leq 10, n^2 \textit{ is even.}$$

To prove this problem, you should list the even numbers 2, 4, 6, 8, 10 and list their squares 4, 16, 36, 64, 100. As the squares are all even, the statement is true. Write this in your conclusion and include the type of proof that you used.

You should only use **proof by exhaustion** when there are a small, **limited** number of **possibilities**.

Proof by Counterexample

To prove by **counterexample**, you give an example that directly **contradicts** the statement. You should prove by counterexample when there is an obvious contradiction. For example, consider the following question:

$$\textit{Prove that every multiple of 7 is odd.}$$

You should prove this statement by counterexample. For example, you know that $7 \times 10 = 70$, which is even. Write this in your conclusion and include the type of proof that you used.

Algebraic Proof

Proofs are most commonly completed algebraically. This involves showing, through a series of steps that use variables to represent values, that the statement is true beyond doubt.

Some algebraic substitutions are commonly included in proof.

Maths in Context (Historical, Real Life and Student Thinking Points)

Euclid of Alexandria (Εὐκλείδης, around 300 BCE) was a Greek mathematician and is often called the *father of geometry*. His book *The Elements* first introduced Euclidean geometry, defines its five axioms, and contains many important proofs in geometry and number theory – including that there are infinitely many prime numbers. It is one of the most influential books ever published, and was used as textbook in mathematics until the 19th century.

Euclid taught mathematics in Alexandria, but not much else is known about his life.

Projects/Enrichment/Investigations

[Sitting Pretty]

[Partly Circles]