A-level Mathematics – Year 1

Year 12

Year 13

Wk1	Wk2	Wk3	Wk4	Wk5	Wk6	Wk7	Wk8
Proof and mathematical communication		Indices and surds		Polynomials			Consolidate
Quadratic functions				Using Graphs			
Working with data							Consolidate

Wk9	Wk10	Wk11	Wk12	Wk13	Wk14	Wk15	Wk16
Holiday	Coordinate geometry					Binomial Expan.	Holiday
	Logarithms		Exponential Models			Triangle geometry	
	Probability						

Wk17	Wk18	Wk19	Wk20	Wk21	Wk22	Wk23	Wk24
Holiday		Binomial Expan.	Differentiation				Consolidate
		Triangle geometry	Trig functions and equations				
		Statistical Hypothesis Testing					

Wk25	Wk26	Wk27	Wk28	Wk29	Wk30	Wk31	Wk32
Holiday	Applications of differentiation			Integration		Holiday	
	Vectors			Introduction to kinematics	Consolidate		
	Introduction to kinematics						

Wk33	Wk34	Wk35	Wk36	Wk37	Wk38	Wk39	Wk40
Integration	Consolidate						Holiday
Motion with constant acceleration		Motion with constant acceleration	Objects in contact		Forces & Motion		

Wk41	Wk42	Wk43	Wk44	Wk45	Wk46	Wk47	Wk48
Consolidate		Proof and mathematical communication					
		Further Transformations of Graphs					

A-level Mathematics – Year 2
1 lesson per exercise + extra lesson per topic

Year 13

Year 12

Wk1	Wk2	Wk3	Wk4	Wk5	Wk6	Wk7	Wk8
Proof and mathematical communication		Functions			Further Transformations of Graphs		Consolidate and recap
Consolidate and recap	Sequences and series					Consolidate and recap	General Binomial Expansions
Applications of Vectors			Consolidate and recap	Projectiles			Forces in context
Wk9	**Wk10**	**Wk11**	**Wk12**	**Wk13**	**Wk14**	**Wk15**	**Wk16**
Holiday	Rational Functions and Partial Fractions			Radian Measure			Holiday
	Calculus of exp and trig		Further Differentiation			Consolidate and recap	
	Moments		Consolidate and recap		Conditional Probability		
Wk17	**Wk18**	**Wk19**	**Wk20**	**Wk21**	**Wk22**	**Wk23**	**Wk24**
Holiday		Further Trigonometry		Consolidate and recap	Numerical Methods		
		Further Integration techniques			Further Applications of Calculus		
		The Normal Distributions			Further Hypothesis Testing		
Wk25	**Wk26**	**Wk27**	**Wk28**	**Wk29**	**Wk30**	**Wk31**	**Wk32**
Holiday	Numerical Integration					Holiday	
	Differential Equations						
Wk33	**Wk34**	**Wk35**	**Wk36**	**Wk37**	**Wk38**	**Wk39**	**Wk40**
							Holiday
Wk41	**Wk42**	**Wk43**	**Wk44**	**Wk45**	**Wk46**	**Wk47**	**Wk48**

AS and A-level Mathematics Mathematics two-year route map

Algebra and proof

Geometry

Calculus

Trigonometry

Exponentials and logarithms

Sequences and series

Numerical methods

Statistics

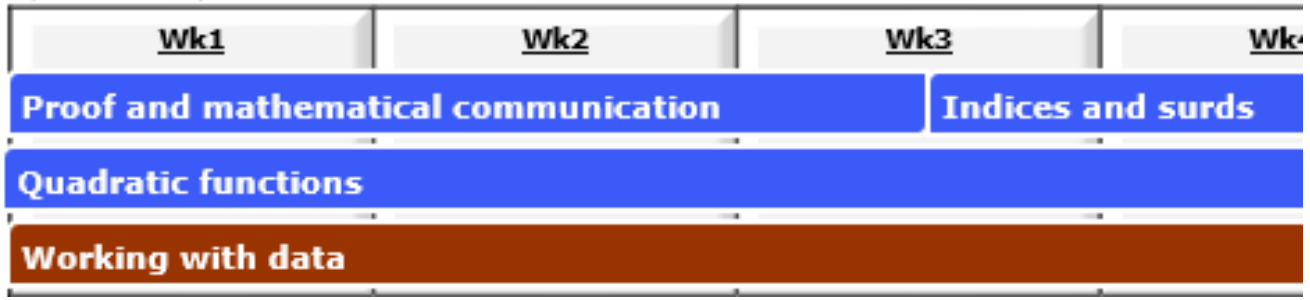
Mechanics

Year 12

Teacher 1 – 2 lessons pw

Teacher 2 – 2 lessons pw

Teacher 2 – 1 lesson pw

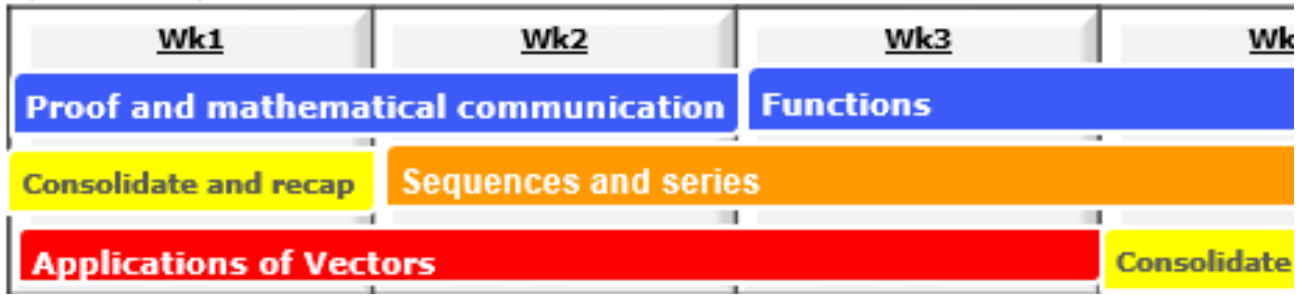


Year 13

Teacher 1 – 2 lessons pw

Teacher 2 – 2 lessons pw

Teacher 2 – 2 lessons pw



Year 12

Return to Routemap

Scheme of Learning Overview

1 Proof and mathematical communication

Key Objectives

- Use appropriate terms used to describe mathematical objects, such as identity and equation.
- Use a counter example to disprove a mathematical idea.
- Apply some techniques for proving a mathematical idea – deduction and exhaustion.

Lesson Breakdown

1. Mathematical structures and arguments
2. Inequality notation
3. Disproof by counter example
4. Proof by deduction
5. Proof by exhaustion

AQA specification references: A1

Rich Tasks

Quadratics – Discriminating

Circles – Teddy bear

Thinking About Numbers - The square root of 2 is irrational

Divisibility and Induction – The Fundamental Theorem of Arithmetic

Misconceptions

Initially it is common for students to have difficulty in setting out the steps in their reasoning logically and precisely, whether by using equals signs inappropriately, miscopying their own work from a previous line, failing to spot patterns or structure in their algebra or making avoidable errors in arithmetic and signs. Exercise 1A provides a selection of questions requiring clear notation and logical thought. Previous study may have left students with the impression that the answer to a question is all-important, whereas this Topic reinforces at the start of the AS or A Level course that it is in fact the whole solution that matters, the answer being only a small part of this.

Vocabulary

Counter example
Deduction
Exhaustion
Identity
Implication symbols
Interval notation
Set notation

Building Links

Mathematical structures and arguments: Links forward to Topic 3

Section 1: Links forward to Topic 7

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

1 Proof and mathematical communication (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should know the definition of the square root function.	1 Write down the value of $\sqrt{9}$.
GCSE	You should be able to manipulate algebraic expressions.	2 Factorise $4x^2 - 1$.
GCSE	You should know basic angle facts.	3 a What is the sum of the angles in a triangle? b What is the sum of the exterior angles of any polygon?
GCSE	You should know the definition of rational and irrational numbers.	4 Which of these numbers are irrational? π, $0.\dot{3}$, 0.5, $\sqrt{2}$
GCSE	You should be able to work with function notation.	5 If $\mathrm{f}(x) = 2x^2 - 3$ find $\mathrm{f}(3)$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

2 Indices and surds

Key Objectives

- Use laws of indices.
- Work with expressions involving square roots (surds).

Lesson Breakdown

1. Using the laws of indices
2. Working with surds

AQA specification reference: B1, B2

Rich Tasks

Thinking about Numbers – Ab-surd!

Thinking about Algebra – Nested Surds

Misconceptions

A common error is to multiply bases when combining indices, e.g. $2^3 \times 4^2 = 8^5$. This is addressed on p.17 and may be apparent in students' responses to Exercise 2A Q.2b. Exercise 2A Q. 2a gives an opportunity to discuss efficient strategies when dealing with fractional powers. In the surds topic, students may try to square root individual terms so it will be necessary to reinforce the idea that $\sqrt{(a^2 + b^2)} \neq a + b$, as in Work it out 2.1. When rationalising the denominator, drawing the parallel with factorising the difference of two squares will help students to decide by what to multiply, as in Example 2.11. Some students will try to multiply numerator and denominator by different expressions, and reminding them of equivalent fractions may illuminate the process.

Vocabulary

Index laws
Surds
Rationalise the denominator

Building Links

Using the laws of indices: Links forward to Topic 7, 12, 13 and 14

Working with surds: Links forward to Year 2, Topic 1

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

2 Indices and surds (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to evaluate expressions involving powers, including working with the order of operations.	1 Evaluate 3×2^3.
GCSE	You should be able to evaluate expressions involving roots.	2 Evaluate $\sqrt[3]{27}$.
GCSE	You should be able to work with the laws of indices.	3 a Evaluate $(x^2)^3$. b Simplify $x^2 \times x^5$. c Simplify $\frac{x^{10}}{x^5}$.
GCSE	You should be able to work with negative, fractional and zero indices.	4 Write each expression in the form ax^n. a $\frac{3}{x^2}$ b $\frac{1}{\sqrt[3]{x}}$ c $(4x)^0$
GCSE	You should be able to multiply out two brackets.	5 Expand $(1+x)(2-y)$.
GCSE	You should be able to recognise and use the 'difference of two squares' factorisation.	6 Expand and simplify $(2a+b)(2a-b)$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

3 Quadratic functions

Key Objectives

- Apply knowledge of factorisation and the quadratic formula for solving quadratic equations.
- Recognise the shape and main features of graphs of quadratic functions.
- Complete the square.
- Solve quadratic inequalities.
- Identify the number of solutions of a quadratic equation.
- Solve disguised quadratic equations.

Lesson Breakdown

1. Solving quadratic equations
2. Graphs of quadratic functions
3. Completing the square
4. Quadratic inequalities
5. The discriminant
6. Disguised quadratics

AQA specification references B3, B5

Rich Tasks

Quadratics – Name that Graph

Quadratics – Which Parabola?

Quadratics – Proving the Quadratic Formula

Quadratics – Discriminating

Misconceptions

Before solving a quadratic equation, students may forget to rearrange the terms into the correct form $ax^2 + bx + c = 0$. In Exercise 3A Q. 2, 3, 6 offer practice on this. Exact answers in terms of surds may need to be simplified using surd manipulation techniques from Topic 2. Students may not appreciate that the coefficient a in the completed square form $a(x + b)^2 + c$ does not affect the coordinates of the vertex. Many will attempt to solve quadratic inequalities without showing any method (e.g. graph or sign diagram) and this often leads to errors or guesswork. When using the discriminant for solving a quadratic equation formed using an unknown coefficient, students may be confused about the fact that this new quadratic equation is not the same as the original one (Exercise 3E Q. 3-10). Strategies for solving equations that students may not recognise as quadratics initially are suggested in Worked example 3.2, with practice questions in Exercise 3A Q. 4, 5 or, where a substitution is needed, Exercise 3F Q. 1.

Vocabulary

Completed square
Discriminant
Intercept
Quadratic inequalities
Real roots
Turning point

Building Links

Completing the square: Links forward to Topic 13

Quadratic inequalities: Links back to Topic 1

Disguised quadratics: Links forward to Topic 7

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

3 Quadratic functions (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to multiply out brackets.	1 Expand this expression. $(3x + 1)(2x - 3)$
GCSE	You should be able to solve quadratic equations by factorising.	2 Solve these equations. a $x^2 + x - 20 = 0$ b $2x^2 + 15x - 8 = 0$ c $5x^2 - 3x = 0$ d $4x^2 - 9 = 0$
GCSE	You should be able to use the formula to solve quadratic equations.	3 Solve these equations. a $x^2 - 4x + 2 = 0$ b $2x^2 - 10x - 5 = 0$
GCSE	You should be able to solve linear inequalities.	4 Solve this inequality. $5x - 1 > 2x + 5$

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

4 Polynomials

Key Objectives

- Define a polynomial.
- Find the product of two polynomials.
- Find the quotient of two polynomials.
- Quickly find factors of a polynomial.
- Sketch polynomials.

Lesson Breakdown

1. Working with polynomials
2. Polynomial division
3. The factor theorem
4. Sketching polynomial functions

AQA specification references: B6

Rich Tasks

Polynomials & Rational Functions – Divide it up

Polynomials & Rational Functions – Can you find … cubic edition

Polynomials & Rational Functions – Review question R6577 Can we factorise $f(x) = 6x^3 + 5x^2 - 17x - 6$ completely?

Misconceptions

Since many students will not have encountered long division, they may find this confusing. It is generally helpful to introduce long division using numbers before applying it to polynomials (Worked example 4.2), or instead to use a grid method (perhaps described as finding areas of rectangles to give a geometrical interpretation), an inspection method or to compare coefficients. Having mastered division, students may be inclined to use this at all times, so it is worth highlighting when the factor theorem might be appropriate instead. Students need to be aware of the effect that a repeated factor has on the shape of the graph. When finding the equation of a curve from its graph and the *x*-intercepts, for instance in Exercise 4D Q. 3 and 6, they must not forget to consider the *y*-intercept too.

Vocabulary

Factor theorem
Linear factor
Polynomial division

Building Links

Working with polynomials : Links back to Topic 1

Polynomial division: Links back to Topic 1

Sketching polynomial functions; Links forward to Topic 13

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

4 Polynomials (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to multiply out brackets.	1 Expand $(2x+1)(x-3)$.
GCSE	You should be able to factorise quadratic expressions.	2 Factorise $x^2-8x+15$.
GCSE	You should be able to use the quadratic formula to solve quadratic equations.	3 Solve $x^2+4x+2=0$.
Chapter 2	You should be able to work with indices.	4 Simplify $x^2 \times x^4$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

5 Using graphs

Key Objectives

- Link solving simultaneous equations and the intersection of graphs.
- Determine the number of intersections between a line and a curve.
- Use transformations of graphs.
- Use direct and inverse proportion.
- Illustrate two-variable inequalities on a graph.

Lesson Breakdown

1. Intersections of graphs
2. The discriminant revisited
3. Transforming graphs
4. Graphs of $\frac{a}{x}$ and $\frac{a}{x^2}$
5. Direct and inverse proportion
6. Sketching inequalities in two variables

AQA specification references: B4, B5, B7, B9

Rich Tasks

Combining Functions – It's a matter of perspective

Combining Functions – Transformers

Calculus meets Functions – Slippery slopes

Misconceptions

When finding all the possible solutions for a quadratic equation or inequality, students should ask themselves whether the graph matches the algebra. The errors in Work it out 5.1 provide a reminder to factorise correctly rather than divide so as to retain all possible solutions. Translations and stretches affecting the *x*-direction are a common source of confusion for students. Graphs and visualisation will help with understanding, as will Proof 2 which demonstrates this algebraically. Precise use of notation is particularly important when dealing with functions – f(x) or, for instance, f($x + a$) to describe a translation – as consistency will be needed for functions work in other parts of the course.

Vocabulary

Discriminant
Intersection
Transformation

Building Links

The discriminant revisited;; Links forward to Topic 6

Direct and inverse proportion; Links forward to Topic 20, Section 4

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

5 Using graphs (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to solve linear inequalities.	1 Solve $3x + 1 > 13$.
GCSE	You should be able to solve simple linear simultaneous equations by elimination.	2 Solve these simultaneous equations. $x + 2y = 5$ $3x + 4y = 11$
GCSE	You should be able to solve quadratic equations by factorising or by using the quadratic formula.	3 Solve $x^2 + x - 1 = 0$.
Chapter 3	You should be able to use the discriminant to determine the number of solutions of a quadratic equation.	4 How many solutions are there to the equation $x^2 + 4x + 4 = 0$?
Chapter 2	You should be able to solve equations involving indices.	5 Solve $2^x = 8$.
GCSE	You should be able to establish simple direct and inversely proportional relationships.	6 $m = 10$ when $n = 2$. Find an equation relating m and n if: a m is directly proportional to the square of n b m is inversely proportional to n.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

6 Coordinate geometry

Key Objectives

- Find the distance between two points and the midpoint of two points.
- Find the equation of a straight line using $y - y_1 = m(x - x_1)$.
- Determine whether two straight lines are parallel or perpendicular.
- Find the equation of a circle with a given centre and radius.
- Solve problems involving intersections of lines and circles.

Lesson Breakdown

1. Distance between two points and midpoint
2. Equation of a straight line
3. Parallel and perpendicular lines
4. Equation of a circle
5. Solving problems with lines and circles

AQA specification references: C1, C2

Rich Tasks

Geometry of Equations – Between the lines

Geometry of Equations – Simultaneous squares

Geometry of Equations – Straight line pairs

Circles – Teddy bear

Circles – Finding circles

Circles – Pairs of circles

Misconceptions

This is a topic where students benefit greatly from sketching a diagram, particularly when visualising spatial relationships in questions that are not purely routine practice, and students may need encouragement to do so. Exercise 6A Q. 4 and 5 and Exercise 6B Q. 5, 6 and 7 are good examples involving straight lines, as are Exercise 6E Q. 5-10 involving circles. It is not unknown for students to calculate gradients incorrectly from coordinates, e.g. as change in x/change in y, so additional practice may be needed. In questions concerning equations of straight lines, students should read the question carefully and note the form in which their answer should be given.

Vocabulary

Distance
Gradient
Midpoint
Normal
Tangent

Building Links

Distance between two points and midpoint: Links forward to Topic 15

Solving problems with lines and circles: Links back to Topic 5; Links forward to Topic 13

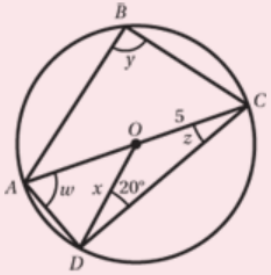
Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

6 Coordinate geometry (continued)

Prerequisite Knowledge
From Cambridge Textbook

GCSE	You should be able to find the equation of a straight line in the form $y = mx + c$.	1 Find the equation of a straight line: a with gradient 2 and y-intercept $(0, -1)$ b with gradient -2 and passing through the point with coordinates $(2, 5)$ c passing through the points with coordinates $(1, 3)$ and $(3, 9)$.
GCSE	You should be able to use the fact that parallel lines have the same gradient.	2 A straight lines passes through the points $(0, 1)$ and $(4, p)$ and is parallel to the line with equation $y = 3x + 4$. Find the value of p.
GCSE	You should be able to solve two linear simultaneous equations.	3 Solve these simultaneous equations. $3x - 2y = 13$ $x + 3y = -3$
GCSE	You should be able to use properties of tangents and chords of circles: • the angle in a semi-circle is a right angle • a tangent to the circle is perpendicular to the radius at the point of contact • the radius perpendicular to the chord bisects the chord.	4 Find the values of the angles and lengths marked with letters, giving reasons for your answers. a 
Chapter 3	You should be able to complete the square.	5 Write $x^2 - 4x - 3$ in the form $(x - p)^2 + q$.
Chapter 5	You should be able to solve linear and quadratic simultaneous equations, and interpret the solution as the intersection points between a line and a curve.	6 a Solve the simultaneous equations. $y = x^2$ $x + y = 6$ b Show that the line $y = 2x + 1$ is a tangent to the parabola $y = -x^2$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

7 Logarithms

Key Objectives

- Use logarithms to undo exponential functions.
- Use the laws of logarithms.
- Use logarithms to find exact solutions of some exponential equations.
- Use the number e.

Lesson Breakdown

1. Introducing logarithms
2. Laws of logarithms
3. Solving exponential equations

AQA specification references: F3, F4, F5

Rich Tasks

Exponentials & Logarithms – See the power

Exponentials & Logarithms – Logarithm lattice

Exponentials & Logarithms – Proving the laws of logarithms

Exponentials & Logarithms – Summing to one

Misconceptions

Students will readily work out logarithms of different bases on their calculator but, to develop understanding, it is important that they first work them out without a calculator. Exercise 7A Q. 1 and Q. 5 give plenty of practice at both evaluating log statements and solving simple equations without a calculator. Solving more complicated equations (Exercise 7A Q. 6-11) will necessitate showing all steps in the working and giving an exact answer so calculator shortcuts will not be sufficient. Work it out 7.1 addresses some typical misconceptions in using laws of logarithms while Exercise 7B Q. 2 onwards give plenty of practice at using the laws. Another common student error is to rewrite $\log a/\log b = \log a - \log b$ and this may occur in Exercise 7C Q. 2 or 7. This exercise covers a variety of exponential equations and Exercise 7D Q. 1a, 1b, 1d and 3 are equations which can be reduced to quadratics and require students to spot that not all the bases are the same initially.

Vocabulary

Base
e
Exponential
Logarithm
Natural logarithm

Building Links

Introducing logarithms: Links forward to Topic 8

Laws of logarithms: Links back to Topic 1

Solving exponential equations : Links back to Topic 3, Section 6

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

7 Logarithms (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to work with expressions involving exponents.	1 Answer 'true' or 'false'. a $2 \times 3^2 = 36$ b When $x = 25$, $4x^{\frac{1}{2}} = 10$ c $(2 \times 3)^7 = 2^7 \times 3^7$ d $\frac{1}{2x^3} = 2^{-3}x^{-3}$
GCSE	You should be able to evaluate fractional and negative exponents.	2 Evaluate, without using a calculator: a $27^{\frac{4}{3}}$ b $9^{-\frac{3}{2}}$
Chapter 2	You should be able to use laws of indices.	3 Write in the form x^p: a $x^2 \times \sqrt{x}$ b $\frac{x\sqrt{x}}{x^2}$
GCSE	You should be able to solve equations involving fractions.	4 Solve this equation. $\frac{x+1}{2x-3} = 2$
Chapter 3	You should be able to solve quadratic equations.	5 Solve these equations. a $(x-1)(x+3) = 5$ b $2x^2 - \frac{1}{x^2} = 1$

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

8 Exponential models

Key Objectives

- Recognise and use graphs of exponential functions.
- Use exponential functions in modelling.
- Use logarithms to transform curved graphs into straight lines.

Lesson Breakdown

1. Graphs of exponential functions
2. Graphs of logarithms
3. Exponential functions and mathematical modelling
4. Fitting models to data

AQA specification references: F1, F2, F6, F7

Rich Tasks

Exponentials & Logarithms – Reach for the stars

Exponentials & Logarithms – How fast does it grow?

Combining Functions – Picture the Process II

Misconceptions

Students may need reminding of the relationship between transformed graphs, for instance $\left(\frac{1}{2}\right)^x = 2^{-x}$ which appears on p.125. They will benefit from experimenting with a graphing program or a graphical calculator, predicting the shape and features of graphs before plotting them. This should help with interpreting exponential growth and decay models later. The importance of gradient as rate of change at a particular time, for example, can provide motivation for students to change the base and calculate the gradient of a function of e^x before encountering its derivative later in the course (Student Book 2). When fitting straight line models, students need to think carefully about how to interpret their gradient and intercept in relation to the given data.

Vocabulary

Asymptote
Exponential decay
Exponential growth

Building Links

Graphs of exponential functions: Links back to Topic 5, Section 3

Exponential functions and mathematical modelling: Links forward to Topic 20: Links back to Topic 6, Section 2

Return to Y12 Routemap

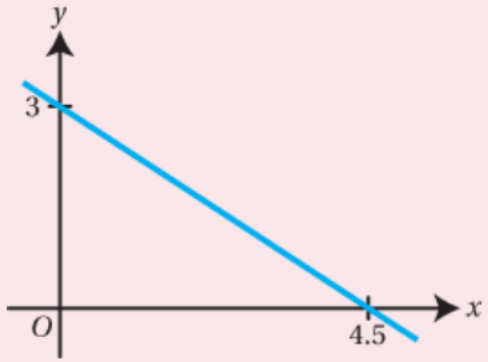
Continued on next page >

Scheme of Learning Overview

8 Exponential models (continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 7	You should be able to work with the number e and natural logarithms.	1 Simplify each expression. a $\ln(5e^2)$ b $e^{1+\ln(5)}$
Chapter 7	You should be able to use the laws of logarithms.	2 If $y = 100x^3$ write $\log y$ in the form $n + k \log x$.
Chapter 5	You should be able to transform graphs.	3 Describe the effect of changing $y = f(x)$ into $y = f(2x)$.
Chapter 6	You should be able to work with equations of straight lines.	4 a What is the gradient of $3y + 2x = 5$ b Find the equation of this line in the form $y = mx + c$. 

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

9 Binomial expansion

Key Objectives

- Expand an expression of the form $(a + b)^n$ for any positive integer n.
- Find individual terms in the expansion of $(a + b)^n$ for any positive integer n.
- Use partial expansions of $(a + bx)^n$ to find an approximate value for a number raised to a positive integer power.
- Understand and use the notations $n!$ and ${}^nC_r$.

Lesson Breakdown

1. The binomial theorem
2. Binomial coefficients
3. Applications of the binomial theorem

AQA specification references: D1

Rich Tasks

Binomial coefficients

Intro to the binomial theorem (video)

Misconceptions

Students need practice to become very familiar with using the binomial formula. An introduction to combinations could help students to understand ${}^nC_r$ or Pascal's triangle coefficients. In the expansion of $(a \pm bx)^n$, students will often calculate $\pm bx^n$ instead of $(\pm bx)^n$. Encouraging the use of brackets around the entire term helps them to arrive at correct coefficients in the expansion. Exercise 9A Q. 2, 4, 6, 7, 8 all provide practice with this aspect of binomial expansion, as do applications questions in Exercise 9C Q. 2, 3, 6, and 7. Once students are familiar with the structure and pattern of the terms, they can find coefficients of particular powers of x without needing to produce the whole expansion.

Vocabulary

Approximation
Binomial
Coefficient
Expansion
Factoral

Building Links

The binomial theorem: Links forward to Topic 21

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

9 Binomial expansion (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to evaluate expressions involving powers, including working with the order of operations.	1 Evaluate: a 2×3^2 b $3 - 4 \times (-2)^3$.
GCSE **Chapter 2**	You should be able to work with the rules of indices.	2 a Evaluate $(2x^3)^4$. b Simplify $x^4 \times x^7$. c Simplify $\frac{x^{12}}{x^3}$.
GCSE	You should be able to multiply out two brackets.	3 Expand $(2x + 3)^2$.
GCSE **Chapter 3**	You should be able to solve quadratic equations by using the formula or factorising.	4 Solve $x^2 + 5x + 4 = 0$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

10 Trigonometric functions and equations

Key Objectives

- Use the definitions of the sine, cosine and tangent functions, their basic properties and their graphs.
- Solve equations with trigonometric functions.
- Use the identities between different trigonometric functions.
- Use identities to solve more complicated equations.

Lesson Breakdown

1. Definitions and graphs of the sine and cosine functions
2. Definition and graph of the tangent function
3. Trigonometric identities
4. Introducing trigonometric equations
5. Transformations of trigonometric graphs
6. Harder trigonometric equations

AQA specification references: E3, E5, E7

Rich Tasks

Sine and cosine

Tangled trig graphs

Degree ceremony

Misconceptions

Proofs using trigonometric identities are often challenging for students, particularly if they are not fluent with algebra. They would like to be able to see their way through a proof from beginning to end, so this is an opportunity to build confidence by reinforcing the step-by-step nature of proof and helping them develop the ability to spot where an identity could be useful. Solving equations that involve transformations can be more difficult for students, so carrying out a substitution can remind them to find all possible values of the new variable then to find the solutions they need. Exercise 10E comprises questions of this type. Sketching the graphs also helps them to locate all the possible solutions and no extras within the interval required. Some students find it difficult to factorise quadratic equations involving trigonometric functions and may prefer to substitute another variable. Exercise 10F contains many questions where this method could be used. Worked example 10.14 and Exercise 10F Q. 2 and 3 highlight a particular type of equation where it is very tempting to divide instead of factorising. It is worth drawing the distinction between these and the equations in Exercise 10G Q. 1 in which division is desirable so as to use the identity $\tan x = \sin x / \cos x$.

Vocabulary

Amplitude
Cosine
Period
Identity
Sine
Tangent

Building Links

Definition and graph of the tangent function: Links back to Topic 5, Section 3

Trigonometric identities: Links back to Topic 1

Return to Y12 Routemap

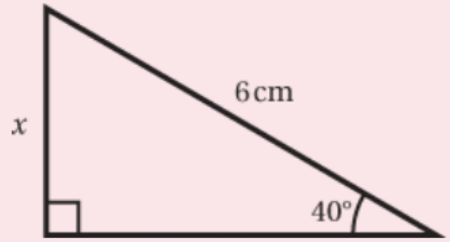
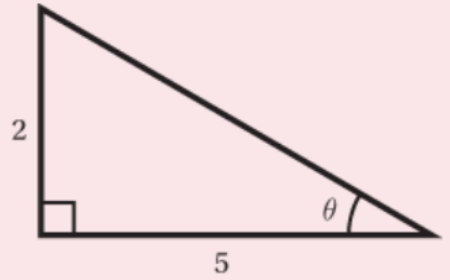
Continued on next page >

Scheme of Learning Overview

10 Trigonometric functions and equations (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to use trigonometry in right-angled triangles to find unknown lengths.	1 Find the value of x in the diagram. 
GCSE	You should be able to use trigonometry in right-angled triangles to find unknown angles.	2 Find the value of θ in the diagram. 
GCSE	You should be able to use Pythagoras' theorem in a right-angled triangle.	3 The lengths of the two shorter sides of a right-angled triangle are $5\,\text{cm}$ and $12\,\text{cm}$. Find the length of the hypotenuse.
Chapter 3	You should be able to solve quadratic equations, using the formula or factorising.	4 Solve the equation $x^2 - 2x + 1 = 0$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

11 Triangle geometry

Key Objectives

- Use the sine rule to find sides and angles of any triangle.
- Use the cosine rule to find sides and angles of any triangle.
- Use a formula for the area of a triangle when you don't know the perpendicular height.

Lesson Breakdown

1. The sine rule
2. The cosine rule
3. Area of a triangle

AQA specification references E1

Rich Tasks

Triangles to Functions – Sine-ing the way

Cosines Rule

Square World

Misconceptions

Drawing a correctly labelled diagram is generally the first step in thinking through a geometrical problem. Students who don't do so will often struggle to progress in solving the problem. Inaccuracies can creep in through premature rounding or through misunderstanding of the units given in the problem. Worked example 11.3 and Exercise 11A Q. 2b deal with the ambiguous case of the sine rule and this can be explained graphically: multiple solutions appear on the sine graph and it is important to check which are valid in the context of the question.

Vocabulary

Cosine rule
Perpendicular
Sine rue

Building Links

The sine rule: Links back to Topic 10, Section 4

Return to Y12 Routemap

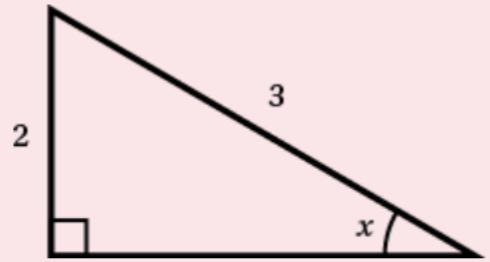
Continued on next page >

Scheme of Learning Overview

11 Triangle geometry (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to use trigonometry in right-angled triangles.	1 Find the size of the angle marked x in the diagram. 
GCSE	You should be able to use three-figure bearings.	2 Point A is a on a bearing of 290° from B. Find the bearing of B from A.
Chapter 3	You should be able to solve quadratic equations by factorising or using the formula.	3 Solve these equations. a. $x^2 + x - 12 = 0$ b. $x^2 - 4x - 1 = 0$
Chapter 10	You should be able to solve trigonometric equations.	4 Solve the equation $\sin x = 0.15$ for $0^\circ < x < 180^\circ$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

12 Differentiation

Key Objectives

- Sketch the gradient function for a given curve.
- Find the gradients of curves from first principles.
- Differentiate x^n.
- Use differentiation to decide whether a function is increasing or decreasing.

Lesson Breakdown

1. Sketching derivatives
2. Differentiation from first principles
3. Rules of differentiation
4. Simplifying into terms of the form ax^n
5. Interpreting derivatives and second derivatives

AQA specification references: G1, G2, G3

Rich Tasks

Introducing Calculus – Gradient match

Calculus meets Functions – Gradients of gradients

Introducing Calculus – A tangent is …

Misconceptions

Students initially find gradient graphs conceptually difficult. A card sort activity, such as 'Gradient match' from Underground Mathematics, could be useful for students to discuss their thinking in pairs or groups before attempting to sketch gradient graphs themselves. They may be helped by drawing a gradient graph directly below the graph of the original function, matching x-coordinates of the main features. When differentiating, the importance of using the correct notation and correct variables should be stressed. Students may need a lot of practice with rewriting expressions in the form ax^n without changing the coefficient, as for example in Exercise 12D Q. 1dii, 1fii and 7. Dealing with quotients such as those in Work it out 12.1 and Exercise 12D Q. 3, 7 – 11 may also be tricky for those whose algebra is not fluent.

Vocabulary

Calculus
Decreasing function
Derivative
Increasing fuction
Gradient
Second derivative

Building Links

Sketching derivatives: Links back to Topics 5 and 6

Sketching derivatives: Links forward to Topic 13

Simplifying into terms of the form ax^n : Links back to Topic 2

Interpreting derivatives and second derivatives: Links forward to Topic 13

Interpreting derivatives and second derivatives: Links back to Topic 3

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

12 Differentiation (continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 2	You should be able to work with indices.	1 Write each expression in the form $nx^a + mx^b$. a $\sqrt[3]{x^2} - \frac{5}{2x}$ b $\frac{2+x}{\sqrt{x}}$
Chapter 3	You should be able to solve linear and quadratic inequalities.	2 Solve these inequalities. a $3x - 4 \geqslant 5x + 2$ b $x^2 - 4x - 12 \geqslant 0$
Chapter 6	You should know how to find the gradient of a straight line.	3 Find the gradient of the line that passes through the points $(-1, 4)$ and $(7, -2)$.
Chapter 9	You should be familiar with the binomial expansion.	4 Expand $(2+x)^3$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

13 Applications of differentiation

Key Objectives

- Find the equations of tangents and normals to curves at given points.
- Find maximum and minimum points on curves.
- Solve problems which involve maximising or minimising quantities.

Lesson Breakdown

1. Tangents and normal
2. Stationary points
3. Optimisation

AQA specification references: G3

Rich Tasks

Introducing Calculus – Talking about curves

Calculus meets Functions – Can you find … curvy cubics edition

Calculus meets Functions – Two-way calculus

Calculus of Powers – Tangent or normal

Calculus of Powers – Floppy hair

Misconceptions

Worked examples 13.2 and 13.3 and Exercise 13A will test students' ability to understand the meaning of the derivative and its use in finding equations of tangents and normals. Having found both the derivative and second derivative, students may be confused about which to use in characterising stationary points; graphical illustrations will be beneficial in explaining this. Some students will need encouragement to memorise the outcome of the second derivative test and to explain their findings logically. When thinking about applied problems involving stationary points, students should appreciate that drawing diagrams is helpful for clarifying their thinking.

Vocabulary

Normal
Optimisation
Stationary point
Tangent

Building Links

Tangents and normal: Links back to Topic 6, Section 1

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

13 Applications of differentiation (continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 12	You should be able to differentiate functions involving x^n.	1 Differentiate each expression. a $\frac{3x^4-2}{5x^2}$ b $\frac{3}{2\sqrt{x}}$
Chapter 12	You should be able to evaluate second derivatives.	2 Given that $y=\frac{3}{2x^2}$, evaluate $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x=-2$.
Chapter 6	You should be able to find the equation of a straight line.	3 Find the equation of the line through the point $(2,1)$ with gradient 3.
Chapter 6	You should be able to find the equation of a perpendicular to a line.	4 Find the equation of the perpendicular to the line with gradient $\frac{3}{4}$ that passes through the point $(2,-3)$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

14 Integration

Key Objectives

- Reverse the process of differentiation (integrate).
- Find the equation of a curve, given its derivative and a point on the curve.
- Find the area between a curve and the *x*-axis.
- Find the area between a curve and a straight line.

Lesson Breakdown

1. Rules for integration
2. Simplifying into terms of the form ax^n
3. Finding the equation of a curve
4. Definite integration
5. Geometrical significance of definite integration

AQA specification references: H1, H2, H3

Rich Tasks

Introducing Calculus – Approximating areas

Calculus of Powers – Integral chasing

Calculus of Powers – Meaningful areas

Calculus meets Functions – Additional integrals

Misconceptions

A very common error is omitting the constant when obtaining indefinite integrals. Students may not appreciate this until they find the wrong equation for a curve. With more complicated expressions, students may forget to expand or rewrite terms algebraically before integrating, or they may correctly do the algebraic manipulation but forget to integrate. Practice at spotting the steps needed and working through them systematically is part of the discipline students should acquire for this and many other topics, including proof.

Vocabulary

Calculus
Constant of integration
Definite integral
Limit

Building Links

Finding the equation of a curve: Links forward to Topic 16

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

14 Integration (continued)

Prerequisite Knowledge
From Cambridge Textbook

Chapter 3	You should be able to solve quadratic and cubic equations by factorising.	1 Solve these equations. a $3x^2 - 4x = 0$ b $x^3 - 4x^2 = 0$ c $x^3 - 5x = 0$
Chapter 12	You should be able to differentiate expressions of the form ax^n.	2 Find $\frac{dy}{dx}$ for each function. a $y = 3x^2 - x + 2$ b $y = x^{\frac{1}{2}} - 3x^{-\frac{2}{3}} + 2$
Chapter 12	You should be able to convert an expression to the form ax^n in order to differentiate.	3 Find $\frac{dy}{dx}$ for each function. a $y = \frac{3}{2x^2}$ b $y = 3x\sqrt{x}$

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

15 Vectors

Key Objectives

- Represent two-dimensional vectors using the base vectors **i** and **j**.
- Find the magnitude and direction of a vector.
- Add and subtract vectors, and multiply vectors by a scalar.
- Recognise when two vectors are parallel.
- Find unit vectors.
- Work with positions and displacement of points in the plane.
- Use vectors to solve problems about geometrical figures.

Lesson Breakdown

1. Describing vectors
2. Operations with vectors
3. Position and displacement vectors
4. Using vectors to solve geometrical problems

AQA specification references: J1, J2, J3, J4, J5

Rich Tasks

Vector Geometry – Hit the spot

Vector walk

Misconceptions

Vector notation needs to be used correctly as it helps students to think and communicate clearly, for example they may forget to underline a vector, e.g. $\underline{a}$, or to add an arrow to a vector to distinguish it from a line segment, e.g. $\overrightarrow{AB}$. It is not uncommon for students to confuse the *x* and *y* components in column vectors. The direction of a vector needs to be given with reference to another direction such as the **i** or **j** direction. Students will find that diagrams help considerably with visualising the relationships between vectors in the plane and when solving geometrical problems by the application of vector methods.

Vocabulary

Components
Displacement
Magnitude
Position vector
Unit vector

Building Links

Describing vectors: Links forward to Student Book 2, Topic 18

Return to Y12 Routemap

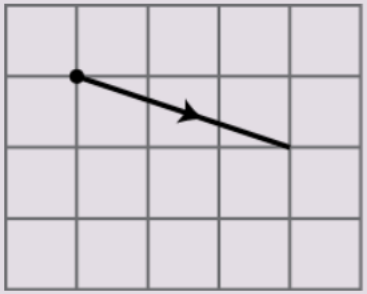
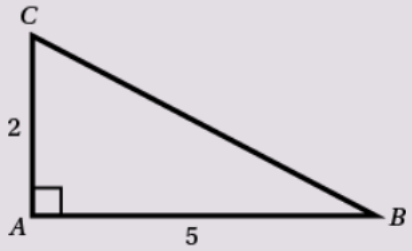
Continued on next page >

Scheme of Learning Overview

15 Vectors (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to represent vectors on a grid and write them as column vectors.	1 Write this as a column vector. 
GCSE	You should be able to use Pythagoras' theorem and trigonometry in right-angled triangles.	2 Find the length of the side BC and the size of the angle ABC. 
Chapter 3	You should be able to solve quadratic equations, and recognise when a quadratic equation has no solutions.	3 State the number of solutions of the equation $6x^2 + 9x + 1 = 0$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

16 Introduction to kinematics

Key Objectives

- Use mathematical models to simplify mechanical situations.
- Use the basic concepts in kinematics – displacement, distance, velocity, speed and acceleration.
- Use differentiation and integration to relate displacement, velocity and acceleration.
- Represent motion on a travel graph.
- Solve more complicated problems in kinematics, for example, involving two objects or several stages of motion.

Lesson Breakdown

1. Mathematical models in mechanics
2. Displacement, velocity and acceleration
3. Kinematics and calculus
4. Using travel graphs
5. Solving problems in kinematics

AQA specification references: P1, Q1, Q2, Q4

Rich Tasks

Introducing Calculus – Discussing distance

Introducing Calculus – Speed vs velocity

Introducing Calculus – Walk sorting

Introducing Calculus – Average speed

Calculus Meets Functions – Thinking constantly

Misconceptions

Students can get confused between speed and velocity, and between distance and displacement. Use of travel graphs, demonstrations or videos will help them to understand motion in a straight line, including the meaning of positive or negative velocity or acceleration and the difference between average speed and average velocity. Worked examples and Exercise 16D cover a variety of situations. Students may need to practise differentiation and integration, and the inclusion of calculus in this Topic provides a useful reminder early on that they cannot always assume acceleration is constant. Exercise 16B Q. 3 will help students to become familiar with units: dimensional analysis can help during calculations.

Vocabulary

Acceleration
Definite integral
Displacement
Gradient
Instantaneous
Velocity

Building Links

Mathematical models in mechanics: Links forward to Student Book 2, Topic 19

Mathematical models in mechanics: Links forward to Student Book 2, Topic 17

Displacement, velocity and acceleration: Links forward to Topics 18 and 19

Kinematics and calculus: Links back to Topic 12, Section 2

Kinematics and calculus: Links forward to Topic 18

Using travel graphs: Links forward to Topic 17

Using travel graphs: Links back to Topic 12, Section 1

Return to Y12 Routemap

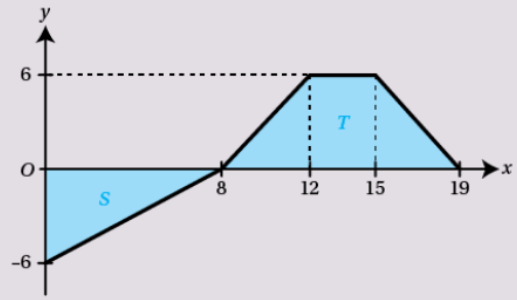
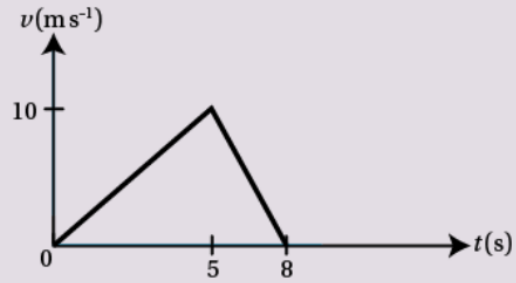
Continued on next page >

Scheme of Learning Overview

16 Introduction to kinematics (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to find the gradient of a straight line connecting two points.	1 Consider the points $A(2,5)$, $B(-1,3)$ and $C(7,-2)$. Find the gradient of the straight line connecting: a A and C b B and A.
GCSE	You should be able to find areas of triangles and trapeziums.	2 Find the areas of the shaded regions marked S and T. 
GCSE	You should be able to interpret displacement–time and velocity–time graphs.	3 Use this velocity–time graph to find: a the acceleration of the object during the first 5 seconds b the distance travelled during the whole 8 seconds. 
Chapter 12	You should be able to differentiate polynomials.	4 Given that $y = 3x^2 - 4x + \frac{5}{x}$, find: a $\frac{dy}{dx}$ b the gradient of the curve when $x = -1$.
Chapter 13	You should be able to find stationary points.	5 Find the coordinates of the maximum point on the graph of $y = -x^3 + 12x + 5$.
Chapter 14	You should be able to use integration to find an area under a graph.	6 Find the area enclosed by the graph of $y = 6x - 3x^2$ and the x-axis.
Chapter 14	You should be able to find the constant of integration.	7 A curve has gradient $\frac{dy}{dx} = 5 - 6x^2$ and passes through the point $(1,2)$. Find the equation of the curve.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

17 Motion with constant acceleration

Key Objectives

- Derive equations for motion with constant acceleration.
- Use constant acceleration equations for horizontal motion.
- Apply constant acceleration equations to vertical motion under gravity.
- Solve multi-stage problems.

Lesson Breakdown

1. Deriving the constant acceleration formulae
2. Using the constant acceleration formulae
3. Vertical motion under gravity
4. Multi-stage problems

AQA specification references: Q3

Rich Tasks

Suvat equations:

http://www.ocr.org.uk/Images/179302-suvat-equations-topic-exploration-pack.pdf

http://www.examsolutions.net/a-level-maths/ocr/m1-tutorials/

Misconceptions

Students need to learn the five equations and to become familiar with deciding which equation is appropriate in a given context. Worked examples 17.3, 17.4 and 17.5 and Exercise 17B all facilitate this. Drawing a diagram to indicate the direction of motion will assist students to think through the various stages of a problem and to use the appropriate signs for velocity and acceleration. In some cases, travel graphs will be helpful too.

Vocabulary

Acceleration
Displacement
Gravitaional constant
Resistance
suvat
Velocity

Building Links

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

17 Motion with constant acceleration (continued)

Prerequisite Knowledge
From Cambridge Textbook

Chapter 16	You should be able to to use integration to find velocity and displacement from acceleration.	1 A particle moves in a straight line. Its acceleration is given by $a = 2 - 3t^2$. a Given that its velocity at $t = 0$ is $4.2\,\text{m s}^{-1}$, find the equation for the velocity at time t. b Given that its initial displacement from point A is $14\,\text{m}$, find the equation for the displacement from A at time t.
Chapter 3	You should be able to solve quadratic equations.	2 Solve these quadratic equations. a $4.2t^2 - 11.5t + 2.6 = 0$ b $12t - 4.9t^2 = 5.2$
Chapter 3	You should be able to find the vertex of a parabola.	3 Find the coordinates of the vertex of the parabola with equation: a $y = 12.2x - 36.1x^2$ b $y = 2.1x^2 - 6.3x + 7.$

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

18 Force and motion

Key Objectives

- Understand what causes motion and the concept of a force (Newton's first law).
- Relate force to acceleration (Newton's second law).
- Work with situations where several forces act on an object.
- Work with different types of forces, including gravity.
- Determine whether a particle is in equilibrium.

Lesson Breakdown

1. Newton's laws of motion
2. Combining forces
3. Types of forces
4. Gravity and weight
5. Forces in equilibrium

AQA specification references: J5, R1, R2, R3

Rich Tasks

Vector Geometry – Make it stop

Misconceptions

A correct, clear force diagram will help students to understand the situation they are modelling, and to make good decisions about how to apply Newton's second law or the constant acceleration equations. In Exercise 18C, Q. 1-8 lend themselves to extracting information from the question in order to draw force diagrams as there are none provided. Students should appreciate the difference between mass and weight and be able to add the correct force due to gravity to their diagram. Exercise 18E provides a variety of questions which will help students to understand equilibrium and to practise resolving forces in two perpendicular directions.

Vocabulary

Equilibrium
Force
Mass
Newtons
Resolve
Resultant force
Weight

Building Links

Section 1: Links back to Topic 15, Section 1

Section 3: Links forward to Topic 19, Section 4

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

18 Force and motion (Continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 17	You should be able to use constant acceleration formulae.	1 A particle accelerates from $3\,\text{m s}^{-1}$ to $8\,\text{m s}^{-1}$ in 12 seconds. Find: a the acceleration b the distance the particle travels in this time.
Chapter 15	You should be able to work with vectors in component form.	2 a Add the vectors $\begin{pmatrix}1\\-3\end{pmatrix}$, $\begin{pmatrix}2\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\7\end{pmatrix}$. b Find vector $\mathbf{v}$ such that $3.5\mathbf{v} = -14\mathbf{i} + 7\mathbf{j}$.
Chapter 15	You should be able to find the magnitude and direction of a vector from its components.	3 Find the magnitude of each vector and the angle it makes with the horizontal direction: a $1.2\mathbf{i} + 2.5\mathbf{j}$ b $\begin{pmatrix}4\\-1\end{pmatrix}$

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

19 Objects in contact

Key Objectives

- Use Newton's third law: that two objects always exert equal and opposite forces on each other.
- Calculate the contact force between two objects.
- Find the tension in a string or rod connecting two objects.
- Analyse the motion of particles connected by a string passing over a pulley.

Lesson Breakdown

1. Newton's third law
2. Normal reaction force
3. Further equilibrium problems
4. Connected particles
5. Pulleys

AQA specification references: J5, R4

Rich Tasks

Newton's third law:

http://www.physicsclassroom.com/class/newtlaws/Lesson-4/Newton-s-Third-Law

Newton's third law:
http://teachertech.rice.edu/Participants/louviere/Newton/law3.html

Misconceptions

As in previous Topics, drawing clear force diagrams will help students to clarify their thinking when solving problems. Exercise 19B Q.1 provides students with practice at drawing force diagrams for each of two objects in contact. When considering connected particles, force diagrams are again essential and students could perhaps critique each other's force diagrams. Students need to note carefully when a system is said to be in equilibrium and realise what this means for the equations they are setting up. They should be careful to resolve forces in straight lines – for instance perpendicular directions – and not around corners.

Vocabulary

Inextensible
Light
Normal reaction
Rough
Smooth
Tension
Thrust

Building Links

Return to Y12 Routemap

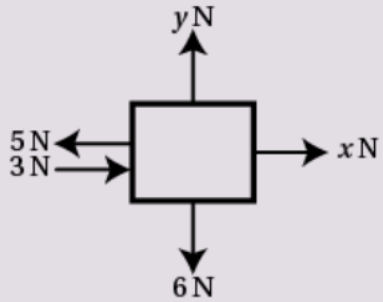
Continued on next page >

Scheme of Learning Overview

19 Objects in contact (continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 17	You should be able to use the constant acceleration formulae.	1 A particle accelerates uniformly from $2\,\text{m s}^{-1}$ to $8\,\text{m s}^{-1}$ while travelling $75\,\text{m}$ in a straight line. a Find the acceleration. b How long does the journey take?
Chapter 18	You should be able to find the resultant force and use it in Newton's second law.	2 A particle of mass $2.4\,\text{kg}$ is acted on by two horizontal forces, $26\,\text{N}$ to the left and $32\,\text{N}$ to the right. Find the acceleration of the particle.
Chapter 18	You should be able to calculate and use the weight of an object.	3 Find the weight of a box with mass $34\,\text{kg}$.
Chapter 18	You should know that if a particle is in equilibrium then the resultant force is zero.	4 The particle in the diagram is in equilibrium. Find the values of x and y. 

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

20 Working with data

Key Objectives

- Interpret statistical diagrams including histograms, scatter diagrams, cumulative frequency curves and box-and whisker plots.
- Calculate mean, median and mode and standard deviation for data.
- Understand correlation and use a regression line.
- Clean data to remove outliers.

Lesson Breakdown

1. Statistical diagrams
2. Standard deviation
3. Calculations from frequency tables
4. Scatter diagrams and correlation
5. Outliers and cleaning data

AQA specification references: L1, L2, L3, L4

Rich Tasks

Displaying and describing data

https://www.khanacademy.org/math/statistics-probability/displaying-describing-data

S1 Tutorials OCR A Level Maths http://www.examsolutions.net/a-level-maths/aqa/s1-tutorials/

Presenting numerical data http://www2.le.ac.uk/offices/ld/resources/numerical-data/numerical-data

Source of UK data sets

https://www.ons.gov.uk/

Misconceptions

Histograms are often challenging for students, primarily the concept of frequency density and variations in class width, and Worked example 20.1 deals with these aspects. Some students may need help finding the median, quartiles and percentiles, as shown in Worked examples 20.3 and 20.4. When interpreting statistical diagrams, they may have difficulty in writing precise statements to communicate their observations; good student answers could be shared as exemplars within a class. The progression in Exercise 20A is helpful: Q. 1 requires students to draw diagrams to help with understanding, Q. 2-5 to read off various statistical quantities, and Q. 6-9 to make comparisons and formulate comments. The fact that correlation does not imply causation is discussed on p.452 and could be supplemented with other examples.

Vocabulary

Bivariate data
Box-and-whisker plot
Correlation coefficient
Cumulative frequency
Frequency density
Histogram
Interquartile range
Mean
Median
Mode
Range
Regression line
Standard deviation
Variance

Building Links

Standard deviation: Links forward to Topic 22, Section 1

Calculations from frequency tables: Links back to Topic 1, Section 2

Scatter diagrams and correlation: Links forward to Student Book 2, Topic 22

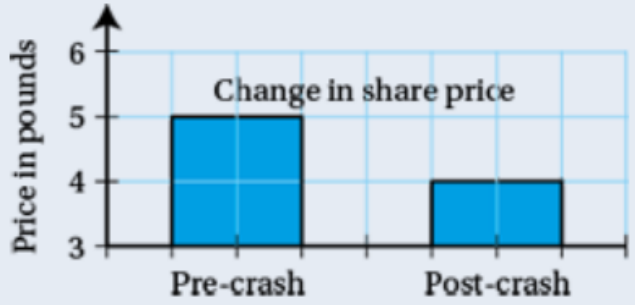
Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

20 Working with data (continued)

Prerequisite Knowledge
From Cambridge Textbook

GCSE	You should be able to interpret basic statistical diagrams such as pie charts and bar charts.	1 Find the percentage decrease in the stock price after the crash in this bar chart. 
GCSE	You should be able to calculate the mean, median and mode of a set of data.	2 Find the mean, median and mode of: $1, 1, 4, 5, 9, 10$.
GCSE	You should be able to calculate the range and interquartile range of a set of data.	3 Find the range and interquartile range of: $12, 15, 18, 18, 19, 16, 14, 20, 12$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

21 Probability

Key Objectives

- Work out combined probabilities when you are interested in more than one outcome.
- Work out the probability of a sequence of events occurring.
- Construct and use a table showing probabilities of all possible outcomes in a given situation (probability distribution).
- Calculate probabilities in a situation when an experiment is repeated several times (binomial distribution).

Lesson Breakdown

1. Combining probabilities
2. Probability distributions
3. The binomial distribution

AQA specification references: M1, N1

Rich Tasks

Thinking About Numbers – A difference of two fractions

Fixing the odds

Discussing risk and reward

Probability - https://www.khanacademy.org/math/statistics-probability/probability-library

Misconceptions

Students often become confused in calculating probabilities because they are not using notation appropriately. This is important in the earlier sections of this Topic as well as in the binomial distribution section where they should use conventional notation rather than calculator notation. They need to know their way around a graphical calculator, if they have one, being clear about the difference between exact binomial probabilities and cumulative ones. In dealing with inequalities, e.g. $P(6 < X < 12)$ or $P(X \geq 4)$, and deciding what to look up, you may need to remind them that this is discrete data. A number line (as on p.477) is often very helpful in arriving at the correct end points, for example in Work it out 21.1 as well as Exercise 21C Q. 2, 4, 7, 11 and 12. Exercise 21C Q. 9 will be useful in assessing whether students have grasped the concept of independent events.

Vocabulary

Binomial distribution
Complement
Cumulative probability
Independent
Mutually exclusive
Probability distribution

Building Links

The binomial distribution:
Links back to Topic 9

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

21 Probability (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to list all possible outcomes (sample space) of a single event or a combination of two events.	1 A five-sided spinner has the numbers 1 to 5 written on it, and a four-sided spinner has the letters A to D on it. What is the probability of getting an A and a 3 when the two spinners are spun together?
GCSE	You should be able to use tree diagrams to record probabilities of successive events, and to calculate probabilities of combined events.	2 A bag contains 7 red and 3 yellow sweets. A sweet is taken out of the bag and eaten. This is repeated twice more. Find the probability that three red sweets are picked.
Chapter 9	You should be able to calculate factorials and binomial coefficients.	3 Use your calculator to find: a $7!$ b $\frac{7!}{3!}$ c ${}^{7}C_{3}$ d $\binom{5}{5}$ e ${}^{10}C_{0}$.

<Back to previous page

Return to Y12 Routemap

Scheme of Learning Overview

22 Statistical hypothesis testing

Key Objectives

- Understand the difference between a sample and a population.
- Understand different types of sampling methods.
- Understand and use the vocabulary associated with hypothesis tests.
- Conduct a hypothesis test using the binomial distribution to test if a proportion has changed.

Lesson Breakdown

1. Populations and samples
2. Introduction to hypothesis testing
3. Critical region for a hypothesis test

AQA specification references: K1, O1, O2

Rich Tasks

Statistics and Probability – Designing studies

https://www.khanacademy.org/math/statistics-probability/designing-studies/sampling-and-surveys/v/statistics-sample-vs-population-mean

Hypothesis test for a proportion (binomial)

http://stattrek.com/hypothesis-test/proportion.aspx?Tutorial=AP

Misconceptions

Students need to understand the relationship between the proportion and the significance level of their test. Writing null and alternative hypotheses and deciding between one- and two-tail tests are steps which need to be correct for a meaningful hypothesis test. Work it out 22.2 and Exercise 22B Q. 1 and 2 provide practice with these while Exercise 22C Q. 1, 2 and 3 focus on finding critical regions. Students are liable to be too categorical when discussing the outcomes of hypothesis tests, for instance stating that the null hypothesis is correct (when they should state that there is insufficient evidence to reject it). Their conclusion should always refer to the context of the question rather than merely quoting figures and significance levels. Work it out 22.1 gives a selection of statements for students to critique, as a first step towards formulating their own conclusions with precision. Often hypothesis test questions are phrased in such a way that a theory or claim is to be tested, and so this theory or claim needs to feature in the conclusion too. Although the procedure in carrying out a hypothesis test is somewhat formulaic, building confidence at this stage will provide a firm foundation for other hypothesis tests later in the course.

Vocabulary

Bias
Conclusion
Hypothesis testing
Population parameter
Null hypothesis
Representative
P-value
Sampling procedures
Significance level
Statistic

Building Links

Return to Y12 Routemap

Continued on next page >

Scheme of Learning Overview

22 Statistical hypothesis testing

Prerequisite Knowledge
From Cambridge Textbook

Chapter 21	You should be able to calculate cumulative probabilities for a binomial distribution.	1 Given that $X \sim \mathrm{B}(25, 0.6)$ a Find: i $\mathrm{P}(X \leqslant 15)$ ii $\mathrm{P}(X \geqslant 17)$. b Find the smallest value of k such that $\mathrm{P}(X \geqslant k) \leqslant 0.05$.
Chapter 21	You should be able to deduce parameters of a binomial distribution in context.	2 It is known that on average, 35 out of 50 people like coffee. For a random sample of 12 people, let X be the number of those who like coffee. State the distribution of X, including any parameters.

<Back to previous page

Return to Y12 Routemap