A-level Mathematics – Year 1

Year 12

Year 13

Wk1	Wk2	Wk3	Wk4	Wk5	Wk6	Wk7	Wk8

- Proof and mathematical communication
- Indices and surds
- Polynomials
- Consolidate
- Quadratic functions
- Using Graphs
- Working with data
- Consolidate

Wk9	Wk10	Wk11	Wk12	Wk13	Wk14	Wk15	Wk16

- Holiday
- Coordinate geometrty
- Binomial Expan.
- Holiday
- Logarithms
- Exponential Models
- Triangle geometry
- Probability

Wk17	Wk18	Wk19	Wk20	Wk21	Wk22	Wk23	Wk24

- Holiday
- Binomial Expan.
- Differentiation
- Consolidate
- Triangle geometry
- Trig functions and equations
- Statistical Hypothesis Testing

Wk25	Wk26	Wk27	Wk28	Wk29	Wk30	Wk31	Wk32

- Holiday
- Applications of differentiation
- Integration
- Holiday
- Vectors
- Introduction to kinematics
- Consoli date
- Introduction to kinematics

Wk33	Wk34	Wk35	Wk36	Wk37	Wk38	Wk39	Wk40

- Integration
- Consolidate
- Holiday
- Motion with constant acceleration
- Motion with constant acceleration
- Objects in contact
- Forces & Motion

Wk41	Wk42	Wk43	Wk44	Wk45	Wk46	Wk47	Wk48

- Consolidate
- Proof and mathematical communication
- Further Transformations of Graphs

A-level Mathematics – Year 2

1 lesson per exercise + extra lesson per topic

Year 13

Year 12

Wk1 | Wk2 | Wk3 | Wk4 | Wk5 | Wk6 | Wk7 | Wk8

- Proof and mathematical communication
- Functions
- Further Transformations of Graphs
- Consolidate and recap
- Consolidate and recap
- Sequences and series
- Consolidate and recap
- General Binomial Expansions
- Applications of Vectors
- Consolidate and recap
- Projectiles
- Forces in context

Wk9 | Wk10 | Wk11 | Wk12 | Wk13 | Wk14 | Wk15 | Wk16

- Holiday
- Rational Functions and Partial Fractions
- Radian Measure
- Holiday
- Calculus of exp and trig
- Further Differentiation
- Consolidate and recap
- Moments
- Consolidate and recap
- Conditional Probability

Wk17 | Wk18 | Wk19 | Wk20 | Wk21 | Wk22 | Wk23 | Wk24

- Holiday
- Further Trigonometry
- Consolidate and recap
- Numerical Methods
- Further Integration techniques
- Further Applications of Calculus
- The Normal Distributions
- Further Hypothesis Testing

Wk25 | Wk26 | Wk27 | Wk28 | Wk29 | Wk30 | Wk31 | Wk32

- Holiday
- Numerical Integration
- Differential Equations
- Holiday

Wk33 | Wk34 | Wk35 | Wk36 | Wk37 | Wk38 | Wk39 | Wk40

- Holiday

Wk41 | Wk42 | Wk43 | Wk44 | Wk45 | Wk46 | Wk47 | Wk48

AS and A-level Mathematics Mathematics two-year route map

Algebra and proof

Geometry

Calculus

Trigonometry

Exponentials and logarithms

Sequences and series

Numerical methods

Statistics

Mechanics

Year 12

Teacher 1 – 2 lessons pw

Teacher 2 – 2 lessons pw

Teacher 2 – 1 lesson pw

Year 13

Teacher 1 – 2 lessons pw

Teacher 2 – 2 lessons pw

Teacher 2 – 2 lessons pw

Year 13

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Scheme of Learning Overview

1 Proof and mathematical communication

Key Objectives

- Review proof by deduction, proof by exhaustion and disproof by counterexample.
- Use a new method of proof called proof by contradiction.
- Criticise proofs.

Lesson Breakdown

1. A reminder of methods of proof
2. Proof by contradiction
3. Criticising proofs

AQA specification reference: A1

Rich Tasks

√2 is irrational

The Fundamental Theorem of Arithmetic

Euclid's algorithm

Proving Pythagoras

Line crossings

Notty Logic

Mind Your Ps and Qs

Misconceptions

In addition to criticising the proofs and examples of algebraic reasoning supplied in this Topic, students will benefit from criticising and discussing each other's proofs. Through this process, they should aim to become self-critical in order to see how they can develop their own proofs and logical argument. Common notation errors as well as algebraic and arithmetical errors can be carefully identified at this stage, so that students can track them in their working as they progress through the course.

Vocabulary

Contradicition
Counter example
Deduction
Exhaustion
Identity
Implication symbols
Interval notation
Set notation

Building Links

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Scheme of Learning Overview

1 Proof and mathematical communication (continued)

Prerequisite Knowledge
From Cambridge Textbook

Student Book 1, Chapter 1	You should be able to use logical connectors.	1 Insert $\Rightarrow$, $\Leftarrow$ or $\Leftrightarrow$ in the places marked a and b: $x^2 - 1 = 8$ a $\quad x^2 = 9$ b $\quad x = 3$
Student Book 1, Chapter 1	You should be able disprove a statement by counterexample.	2 Disprove the statement: 'Apart from 1 there are no other integers that can be written as both n^2 and n^3.'
Student Book 1, Chapter 1	You should be able to prove a statement by deduction.	3 Prove that the sum of any two odd numbers is always even.
Student Book 1, Chapter 1	You should be able to prove a statement by exhaustion.	4 Use proof by exhaustion to prove that 17 is a prime number.

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Scheme of Learning Overview

2 Functions

Key Objectives

- Distinguish between mappings and functions.
- Determine whether a function is one-to-one or many-to-one.
- Find the domain and range of a function.
- Find composite functions.
- Find the inverse of a function.

Lesson Breakdown

1. Mappings and functions
2. Domain and range
3. Composite functions
4. Inverse functions

AQA specification reference: B8, B11

Rich Tasks

Domain and range dominoes: https://undergroundmathematics.org/combining-functions/domain-and-range-dominoes

Compose!: https://undergroundmathematics.org/combining-functions/compose

Composing gets me nowhere: https://undergroundmathematics.org/combining-functions/composing-gets-me-nowhere

Misconceptions

Students may struggle to visualise the input and output for different types of function; a simple function machine can clarify their thinking and is especially helpful when composing functions and ensuring that the order is correct. Graphs will help them to distinguish between mappings and functions (using the quick visual tests on p.12 and 13), to classify functions, and to identify the domain and range of a function. Students will need to be proficient in completing the square, for instance in answering Exercise 2B Q. 4. When finding composite functions, a common error students make is to apply the functions the wrong way round. Work it out 2.1 and Exercise 2C Q.1-4 both address this point. Students may attempt to guess the inverse function (this may work perfectly well for simple functions) or errors in rearrangement may let them down when finding inverse functions algebraically. Exercise 2D Q.1 will enable them to practise the algebra, Q.2 to sketch the graphs and Q.3 to consider domain and range. Exercise 2E will assess their ability to combine all of the ideas when considering whether or not an inverse function exists and, if so, over what domain.

Vocabulary

Composite function
Domain
Function
Horizontal line test
Image
Inverse function
Many-to-one
Mapping
One-to-one
Range
Vertical line test

Building Links

Inverse functions: Links forward to Topic 7

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Scheme of Learning Overview

2 Functions (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to interpret function notation.	1 Given that $f(x) = 2 - x$, evaluate: a $f(3)$ b $f(-4)$.
Student Book 1, Chapter 1	You should be able to use interval notation to write inequalities.	2 Use interval notation to write these inequalities. a $x > 3$ and $x \leqslant 6$ b $x < 3$ or $x \geqslant 6$
Student Book 1, Chapter 3	You should be able to complete the square.	3 Express $f(x) = x^2 + 5x + 3$ in the form $(x + a)^2 + b$. Hence state the coordinates of the turning point of $f(x)$.
Student Book 1, Chapter 3	You should be able to solve quadratic inequalities.	4 Solve the inequality $x^2 - 4x - 5 > 0$.
Student Book 1, Chapter 7	You should be able to rearrange exponential and log expressions.	5 Make x the subject of each equation. a $y = e^{2x-1}$ b $y = \ln(3x + 4)$
Student Book 1, Chapter 13	You should be able to establish where a function is increasing/decreasing.	6 Find the range of x-values for which $f(x) = x^{\frac{3}{2}} - 2x$ is an increasing function.

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Scheme of Learning Overview

3 Further transformations of graphs

Key Objectives

- Draw a graph after two (or more) transformations.
- Find the equation of a graph after a combination of transformations.
- Sketch graphs of functions involving the modulus (absolute value).
- Use modulus graphs to solve equations and inequalities.

Lesson Breakdown

1. Combined transformations
2. The modulus function
3. Modulus equations and inequalities

AQA specification reference: B7, B9

Rich Tasks

Transformers: https://undergroundmathematics.org/combining-functions/transformers

Absolutely: https://undergroundmathematics.org/thinking-about-functions/absolutely

Piece it together: https://undergroundmathematics.org/thinking-about-functions/piece-it-together

Sine problem: http://nrich.maths.org/436

Misconceptions

When combining transformations, students need to be reminded of what happens to the algebra. Focusing on what is replaced will help them to find the new equation of a graph after a sequence of two transformations. It will also help them to diagnose which two transformations have happened, and in what order, to reach a particular equation. To reinforce this, it will be beneficial for them to work through algebraically two horizontal transformations or two vertical transformations that interfere with each other, considering carefully the order of operations. A useful discussion could centre on why the order matters when, for example, a stretch or translation in the x-direction is combined with a reflection in the y-axis. A similar situation – applied to the vertical direction – is illustrated in the graphs on p.36. Sketching the graphs may best be done in stages, showing the graph before and after the first transformation as well as after the second. Exercise 3A provides practice with the algebra and graphs and Exercise 3B focuses on sketching modulus graphs. Work it out 3.2 highlights common errors that students make when solving modulus equations. It may help for students to look at all four possible equations you can generate and see how those reduce to two. In Exercise 3C, graph-sketching should be encouraged even when not mentioned in the question, both for finding solutions and for checking whether the solutions obtained algebraically actually make sense on the graph.

Vocabulary

Absolute value
Modulus

Building Links

The modulus function: Links forward to Topics 4 and 6

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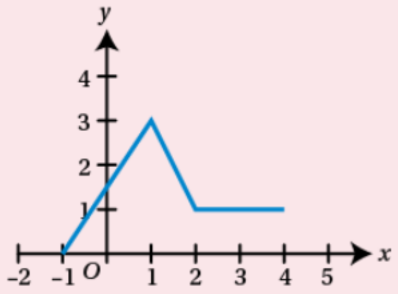
Scheme of Learning Overview

3 Further transformations of graphs (continued)

Prerequisite Knowledge

From Cambridge Textbook

Student Book 1, Chapter 5	You should be able to recognise a graph transformation from the equation.	1 The graph of $y = f(x)$ is shown in the diagram.  Sketch the graph of: a $y = f(x+2)$ b $y = -f(x)$.
Student Book 1, Chapter 5	You should be able to change the equation of a graph to achieve a given transformation.	2 A graph has equation $y = x^2 - 3x$. Find the equation of the graph after: a a translation of 5 units in the positive y-direction, followed by b a horizontal stretch with scale factor 2.
Student Book 1, Chapter 1	You should be able to use interval notation to express solutions of inequalities.	3 Solve each inequality and write the solution using interval notation. a $3x - 2 \geqslant 7$ b $2x + 5 < 1$ and $3 - 2x < 9$

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Scheme of Learning Overview

4 Sequences and series

Key Objectives

- Determine the behaviour of some sequences.
- Use a sigma notation for series.
- Work with sequences with a constant difference between terms.
- Work with finite series with a constant difference between terms.
- Work with sequences with a constant ratio between terms.
- Work with finite and infinite series with a constant ratio between terms.
- Apply sequences to real-life problems.

Lesson Breakdown

1. General sequences
2. General series and sigma notation
3. Arithmetic sequences
4. Arithmetic series
5. Geometric sequences
6. Geometric series
7. Infinite geometric series
8. Mixed arithmetic and geometric questions

AQA specification reference: D2-D6

Rich Tasks

When does the sum of this series first exceed 2999/4000?: https://undergroundmathematics.org/sequences/r7487

Can we sum from 1000 to 2000 excluding multiples of 5?: https://undergroundmathematics.org/sequences/r7424

Proof sorter – geometric sequence: http://nrich.maths.org/1398

Golden fibs: http://nrich.maths.org/2336

Clickety click and all the sixes: http://nrich.maths.org/1952

Misconceptions

Students need to be familiar with inductive definitions for sequences and sigma notation for series; writing out some of the terms will help them to visualise the sequence or series when it is not obvious to them from the shorthand notation. They also need to be able to carry out repeated calculations (iterations) using their calculator and to draw conclusions about the type of sequence they are dealing with (Exercise 4A and Exercise 4B). Students often tend to confuse the limit of a sequence as *n* tends to infinity and the sum to infinity of a convergent geometric series. It is worth spending time on questions to help distinguish between them (see Worked example 4.3, Worked example 4.18, Exercise 4A Q.5, 6 and 9 and Exercise 4G). Students need to develop confidence at picking out information from a question, deciding what they know and what they need to find, and hence which formula might help, employing other methods such as logarithms where appropriate (Exercise 4H and Mixed practice 4).

Vocabulary

Arithmetic sequence
Converge
Decreasing
Geometric sequence
Increasing
Periodic
Series
Sum to infinity

Building Links

General sequences: Links forward to Topic 14

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Scheme of Learning Overview

4 Sequences and series (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to find the formula for the nth term of a linear sequence.	1 Find the formula for the nth term of each sequence. a $2, 5, 8, 11, \ldots$ b $15, 11, 7, 3, \ldots$
GCSE	You should be able to use term-to-term rules to generate sequences.	2 Find the second and third terms of the sequence defined by: $u_{n+1} = 3u_n - 2, u_1 = 4$
GCSE	You should be able to solve linear simultaneous equations.	3 Solve the simultaneous equations: $a + 4b = 8$ $3a + 5b = 3$
Student Book 1, Chapter 3	You should be able to solve quadratic equations and inequalities.	4 Find the smallest positive integer that satisfies the inequality $3x^2 + 7x > 163$.
Student Book 1, Chapter 7	You should be able to solve exponential equations and inequalities.	5 Find the smallest integer value of n such that $3.5 \times 1.2^n > 75$.
Chapter 3	You should be able to use modulus notation.	6 List all integers r that satisfy $\left\lvert\frac{3r}{5}\right\rvert < 2$.

Scheme of Learning Overview

5 Rational functions and partial fractions

Key Objectives

- Manipulate rational functions, including by using polynomial division with remainders.
- Use the factor theorem to find factors of the form (ax + b).
- Decompose rational functions into a sum of algebraic fractions when the denominator contains distinct linear factors.
- Decompose rational functions into a sum of algebraic fractions when the denominator contains repeated linear factors.

Lesson Breakdown

1. An extension of the factor theorem
2. Simplifying rational expressions
3. Partial fractions with distinct factors
4. Partial fractions with a repeated factor

AQA specification reference: B6, B10

Rich Tasks

Divide it up: https://undergroundmathematics.org/polynomials/divide-it-up

Can we factorise f(x) = 6x3 +5x2 – 17x – 6 completely?: https://undergroundmathematics.org/polynomials/r6577

Translating or not?: https://undergroundmathematics.org/combining-functions/translating-or-not

Misconceptions

Students may be tempted to cancel expressions inappropriately (e.g. as in Work it out 5.2) so the emphasis needs to be on finding legitimate ways to factorise wherever possible. Exercises 5A and 5B give plenty of questions for practice. When finding partial fractions, the structure of the denominator in the original function should help students to decide the format of the partial fractions and also which method of finding coefficients is likely to be more efficient: the 'cover-up' (substitution) method or equating coefficients. Setting out work clearly and logically will help with accuracy in all of these methods.

Vocabulary

Degree (of the numerator)
Factor theorem
Partial fraction
Rational function

Building Links

Partial fractions with distinct factors: Links forward to Topics 6 and 11

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Scheme of Learning Overview

5 Rational functions and partial fractions (continued)

Prerequisite Knowledge
From Cambridge Textbook

GCSE	You should be able to add algebraic fractions.	1 Simplify into one fraction $\frac{1}{x} + \frac{1}{2+x}$.
Student Book 1, Chapter 3	You should be able to factorise quadratic expressions.	2 Factorise $6x^2 + 7x + 2$.
Student Book 1, Chapter 4	You should be able to carry out polynomial division.	3 Simplify $(x^3 - x^2 - 3x + 6) \div (x + 2)$.
Student Book 1, Chapter 4	You should be able to use the factor theorem.	4 Find a linear factor of $x^3 + x^2 + 5x - 7$.

Scheme of Learning Overview

6 General binomial expansion

Key Objectives

- Expand $(a + bx)^n$ where n is any rational power.
- Decide when a binomial expansion will converge.
- Use partial fractions to write expressions in the form required for the binomial expansion.
- Use binomial expansions to approximate functions.

Lesson Breakdown

1. The general binomial theorem
2. Binomial expansions of compound expressions

AQA specification reference: D1

Rich Tasks

Given the binomial expansion of $(1 + x)^n$, can we find x and n? :
https://undergroundmathematics.org/counting-and-binomials/r6295

Can we find a good rational approximation for √5?:
https://undergroundmathematics.org/counting-and-binomials/r6340

Misconceptions

It is helpful to reinforce the idea of the infinite series produced when using the binomial expansion with negative and fractional powers, unlike with positive integer powers. Students could compare the graph of, for example, $y = (1 + x)^{-1}$ with its truncated expansions $y = 1 – x + x^2$ and $y = 1 – x + x^2 – x^3 + x^4 – x^5$. They will see that the approximations are closest to the reciprocal function for small values of x, and that the more terms the closer the approximation. As before, use of brackets in the expansion will help students to generate correct coefficients and deal appropriately with signs. They will need to spot when to factorise an expression such as $(2 + x)^{1/2}$, reducing it to $\left[2\left(1+\frac{x}{2}\right)\right]^{\frac{1}{2}}$ before applying the formula. Work it out 6.1 addresses some possible misconceptions students may have with this step.

Vocabulary

Binomial
Rational
Series expansion

Building Links

The general binomial theorem: Links back to Topic 4

Binomial expansions of compound expressions: Links back to Topic 5

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Scheme of Learning Overview

6 General binomial expansion (continued)

Prerequisite Knowledge
From Cambridge Textbook

GCSE	You should be able to simplify expressions with exponents.	1 Simplify $(8x^6)^{\frac{1}{3}}$.
Student Book 1, Chapter 9	You should be able to use binomial expansions for positive integers.	2 Expand $(3-2x)^4$.
Chapter 3	You should be able to write inequalities, using the modulus function.	3 Write $\|x-2\|<3$ in the form $a<x<b$
Chapter 5	You should be able to write an expression in partial fractions.	4 Write $\frac{4-x}{x(x-2)}$ in the form $\frac{A}{x-2}+\frac{B}{x}$.

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Scheme of Learning Overview

7 Radian measure

Key Objectives

- Learn about different units for measuring angles called radians.
- Calculate certain special values of trigonometric functions in radians.
- Use trigonometric functions in modelling real-life situations.
- Solve geometric problems involving circles.
- Learn that trigonometric functions can be approximated by polynomials.
- Revise solving trigonometric equations.

Lesson Breakdown

1. Introducing radian measure
2. Inverse trigonometric functions and solving trigonometric equations
3. Modelling with trigonometric functions
4. Arcs and sectors
5. Triangles and circles
6. Small angle approximations

AQA specification reference: E1-E3

Rich Tasks

Triangles to Functions – General solutions: https://undergroundmathematics.org/trigonometry-triangles-to-functions/general-solutions

Equation or identity (I): https://undergroundmathematics.org/trigonometry-triangles-to-functions/equation-or-identity-i

Can you find … trigonometry edition: https://undergroundmathematics.org/trigonometry-triangles-to-functions/can-you-find-trigonometry-edition

Misconceptions

Students will at first need to develop familiarity with angles in radians and their equivalents in degrees (Exercise 7A Q.1-4) but their aim should quickly be to work in radians, not to convert from degrees. Being able to sketching the trigonometric graphs with radians on the horizontal axis is crucial, as is understanding the period of graphs in radians (Work it out 7.1 and Exercise 7A Q.5-7 will provide practice). When solving equations, students may obtain strange answers because their calculator is in the wrong mode; they may need reminding that they should check the set up for each trigonometry question. It is worth pointing out the possible notations for inverse trigonometric functions, e.g. $\sin^{-1}$ and arcsin; both are used in Work it out 7.2 for instance. Students need to learn to draw the inverse trigonometric graphs, with correct end points, perhaps by considering reflection in the line $y = x$ and the relationship between the domain and range of the original and inverse functions. They also need to learn the formulae for length of arc and area of sector and the small angle approximations, and it will be helpful for them to see where these come from.

Vocabulary

arcsin/arccos/arctan
Amplitude
Domain
Inverse function
Radian
Range

Building Links

Inverse trigonometric functions and solving trigonometric equations: Links back to Topic 2

Small angle approximations: Links back to Topic 6, Topic 3

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Scheme of Learning Overview

7 Radian measure (continued)

Prerequisite Knowledge

From Cambridge Textbook

Student Book 1, Chapter 10	You should be able to define trigonometric functions beyond acute angles, including exact values.	1 What is the exact value of $\sin 120^\circ$?
Student Book 1, Chapter 10	You should be able to solve trigonometric equations.	2 Solve $\cos 2x = \frac{1}{2}$ for $0^\circ < x < 360^\circ$.
Student Book 1, Chapter 11	You should be able to use the sine and cosine rules.	3 Find the smallest angle in a triangle with sides $3\,\text{cm}, 4\,\text{cm}$ and $6\,\text{cm}$.
Chapter 3	You should be able to identify transformations of graphs.	4 The graph of $y = \cos x$ is translated 30° in the positive x direction and stretched vertically with the scale factor of 2. Find the equation of the new graph.
Chapter 6	You should be able to use the binomial expansion for negative and fractional powers.	5 Find the first three non-zero terms in the expansion of $\frac{2}{1-3x^2}$, in ascending powers of x.

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Scheme of Learning Overview

8 Further trigonometry

Key Objectives

- Work with trigonometric functions of sums and differences of two angles, e.g. sin (A + B).
- Work with trigonometric functions of double angles, e.g. sin 2A.
- Work with sums of trigonometric functions, e.g. sin A + sin B.
- Work with reciprocal trigonometric functions, e.g. $\frac{1}{sinx}$

Lesson Breakdown

1. Compound angle identities
2. Double angle identities
3. Functions of the form a sin x + b cos x
4. Reciprocal trigonometric functions

AQA specification reference: E4-6, E8, E9

Rich Tasks

Triangles to Functions – Going round in circles: https://undergroundmathematics.org/trigonometry-triangles-to-functions/going-round-in-circles

Triangles to Functions – Muddled trig: https://undergroundmathematics.org/trigonometry-triangles-to-functions/muddled-trig

Triangles to Functions – Trig tables: https://undergroundmathematics.org/trigonometry-triangles-to-functions/trig-tables

Triangles to Functions – Equation or identity (II): https://undergroundmathematics.org/trigonometry-compound-angles/equation-or-identity-ii

The Compound Angle Formulae lesson, worksheet (RISP 26): https://www.tes.com/teaching-resource/the-compound-angle-formulae-lesson-worksheet-6056103

Misconceptions

Students may make errors even once they have met the compound angle formulae, for example expanding sin(A + B) as sin A + sin B. The questions in Exercise 8A will enable them to practise using the various formulae by spotting expressions matching one side or the other and rewriting them. When working with double angles, some students will try to divide by two to obtain the angle they need instead of using the double angle formulae. As an introduction to the compound angle formulae, a graphical treatment considering permutations of sinx, cosx, siny and cosy is an interesting investigation for students (see RISP below). It connects with the transformations work that students have done already in Topic 3. When students are finding R and α, algebraic errors may occur in equating coefficients, or in calculating tanα as cosα /sinα , (giving rise to an angle that is the complement of the correct one). Exercise 8C provides practice questions. Students need to be clear about the difference in notation between inverse functions, e.g. $\sin^{-1}x$, and reciprocal functions, e.g. $(\sin x)^{-1}$. They also need to be confident with sketching the graphs of sec, cosec and cot, and these are best done starting from the graphs of cos, sin and tan respectively.

Vocabulary

Compound angle identities
Cosecant
Cotangent
Double angle identities
Reciprocal functions
Secant

Building Links

Compound angle identities: Links back to Topic 7

Double angle identities: Links forward to Topic 11

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Scheme of Learning Overview

8 Further trigonometry (continued)

Prerequisite Knowledge
From Cambridge Textbook

Student Book 1, Chapter 10	You should be able to use the identities $\sin^2 x + \cos^2 x \equiv 1$ and $\tan x \equiv \frac{\sin x}{\cos x}$.	1 Given that x is an acute angle with $\cos x = \frac{1}{3}$, find the exact value of: a $\sin x$ b $\tan x$.
Chapter 7; Student Book 1, Chapter 10	You should know and be able to use graphs of trigonometric functions, in degrees and radians.	2 State the coordinates of the minimum point on the graph of $y = 1 - 3\sin 2x$, for $x \in \left[0, \frac{\pi}{2}\right]$.
Chapter 7; Student Book 1, Chapter 10	You should be able to solve trigonometric equations in degrees and radians.	3 Solve each equation. a $\sin 3x = 2\cos 3x$, for $0^\circ \leqslant x \leqslant 90^\circ$. b $2\cos^2\theta - \sin\theta - 1$, for $\theta \in [-\pi, \pi]$.

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Scheme of Learning Overview

9 Calculus of exponential and trigonometric functions

Key Objectives

- Differentiate e^x, $\ln x$, $\sin x$, $\cos x$ and $\tan x$.
- Integrate e^x, $\frac{1}{x}$, $\sin x$ and $\cos x$.
- Review applications of differentiation to find tangents, normals and stationary points.
- Review applications of integration to find the equation of a curve and areas.

Lesson Breakdown

1. Differentiation
2. Integration

AQA specification reference: G1, G2, H2

Rich Tasks

Estimating gradients: https://undergroundmathematics.org/calculus-trig-log/estimating-gradients

Inverse integrals: https://undergroundmathematics.org/calculus-trig-log/inverse-integrals

Stretching an integral: https://undergroundmathematics.org/calculus-trig-log/stretching-an-integral

Two for one: https://undergroundmathematics.org/calculus-trig-log/two-for-one

Trigsy integrals: https://undergroundmathematics.org/calculus-trig-log/trigsy-integrals

Misconceptions

A series of very common integration errors is shown in Work it out 9.1, and these examples could generate useful class discussion. In Statement 1, the expression is integrated rather than differentiated; in Statement 2, a non-linear expression is wrongly integrated to a ln function; in Statement 3, the chain rule is omitted. Since students will not encounter the chain rule until Topic 10, another way to consider Statements 3 and 4 at this stage would be to split $\ln(3x)$ using laws of logs into $\ln 3 + \ln x$ which will clearly differentiate to $\frac{1}{x}$. Students may also differentiate e^x erroneously as xe^{x-1}; practice questions will help them to be clear about the difference between powers of x and powers of e.

Vocabulary

Calculus
Derivatives
Differentiation from first principles
Rates of change
Stationary points

Building Links

Differentiation: Links back to Topic 5, 7 and 8

Differentiation: Links forward to Topic 10

Integration: Links forward to Topic 12

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9 Calculus of exponential and trigonometric functions (continued)

Prerequisite Knowledge
From Cambridge Textbook

Student Book 1, Chapter 7	You should be able to use rules of indices and logarithms.	1 Write $\ln(3x^4)$ in the form $A + B\ln x$.
Student Book 1, Chapter 12	You should be able to differentiate x^n.	2 Differentiate $y = \left(3x - \frac{1}{x}\right)\left(x + \frac{2}{3x}\right)$.
Student Book 1, Chapter 14	You should be able to integrate x^n for $n \neq -1$.	3 Find the exact value of $\int_1^2 \frac{x^3+3}{2x^2}\,dx$.
Chapters 7, 8	You should be able to use compound angle formulae and small angle approximations.	4 Use small angle approximations to find the approximate value of $\cos\left(\frac{\pi}{3} + \frac{\pi}{100}\right)$.

Scheme of Learning Overview

10 Further differentiation

Key Objectives

- Use the chain rule to differentiate composite functions.
- Differentiate products and quotients of functions.
- Work with implicit functions and their derivatives.
- Differentiate inverse functions.

Lesson Breakdown

1. The chain rule
2. The product rule
3. Quotient rule
4. Implicit differentiation
5. Differentiating inverse functions

AQA specification reference: G4, G5

Rich Tasks

Implicit circles: https://undergroundmathematics.org/chain-rule/implicit-circles

Differentiating exponentials: https://undergroundmathematics.org/chain-rule/differentiating-exponentials

Slippery slopes … another derivative: https://undergroundmathematics.org/chain-rule/slippery-slopes-another-derivative

The chain rule (proof): http://nrich.maths.org/10079

The product rule (proof): http://nrich.maths.org/10086

Integration and Differentiation Practice Questions: http://nrich.maths.org/10451

Misconceptions

Students may take some time to feel confident with distinguishing products from composite functions (Work it out 10.1) and therefore deciding when to use the chain rule or product rule. If they choose to substitute for the 'inner' function in a composite, they may run into problems with keeping track of all their variables. Consolidation and practice with a wide variety of different functions is beneficial (Exercise 10A and other questions which are not obviously chain rule questions later in the topic). With all of these differentiation methods, students will be greatly helped by setting out their work clearly, stating u, v, $\frac{du}{dx}$ and $\frac{dv}{dx}$ and so on as appropriate. Some students may try to use the quotient rule with the terms in the numerator reversed, giving incorrect results. When differentiating implicitly, students often fail to identify functions which are products and therefore forget to use the product rule. An extra lesson devoted to deciding which differentiation method to use for a wide selection of functions is time well spent once students have practised all of the methods separately (Work it out 10.2 and Mixed Practice 10).

Vocabulary

Chain rule
Implicit functions
Inverse functions
Product rule
Quotient rule

Building Links

The chain rule: Links back to Topic 9

The chain rule: Links back to Student Book 1, Topic 16

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Scheme of Learning Overview

10 Further differentiation (continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 9; Student Book 1, Chapter 12	You should be able to differentiate the functions $x^n, \sin x, \cos x, \tan x,$ $e^x, \ln x$.	1 Differentiate these expressions. a $2x^3 - 3\sqrt{x}$ b $5\ln x + \frac{1}{3x^3}$ c $5\,e^x$ d $4\sin x - 3\cos x + 2\tan x$
Chapter 9; Student Book 1, Chapter 13	You should be able to use differentiation to find the equations of tangents, normals and stationary points.	2 A curve has equation $y = x - 2\ln x$. a Find the equations of the tangent and the normal at the point where $x = 1$. b Find the coordinates of the stationary point and show that it is a minimum point.
Student Book 1, Chapter 10	You should know basic trigonometric identities.	3 Simplify these expressions. a $3\sin^2 x + 3\cos^2 x$ b $\frac{3\sin x}{4\cos x}$
Chapter 8	You should know the definitions of reciprocal trigonometric functions.	4 Write these in terms of $\sin x$ and $\cos x$. a $\sec x \tan x$ b $\frac{\operatorname{cosec} x}{\cot x}$
Chapter 5	You should be able to simplify expressions involving fractions and surds.	5 Simplify each expression. a $\dfrac{x - \frac{1}{x+1}}{\frac{2}{x+1} - 3}$ b $\dfrac{\sqrt{x-1} + \frac{1}{\sqrt{x-1}}}{x-1}$
GCSE; Student Book 1, Chapter 7	You should be able to change the subject of a formula.	6 Make y the subject of each formula. a $x = e^{2y-1}$ b $2x + 3xy = (x-2)y$

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Scheme of Learning Overview

11 Further integration techniques

Key Objectives

- Integrate using known derivatives.
- Integrate using the chain rule in reverse.
- Integrate using a change of variable (substitution).
- Integrate using the product rule in reverse (integration by parts).
- Integrate using trigonometric identities.
- Integrate using the separation of a fraction into two fractions.

Lesson Breakdown

1. Reversing standard derivatives
2. Integration by substitution
3. Integration by parts
4. Using trigonometric identities in integration
5. Integrating rational functions

AQA specification reference: H5, H6

Rich Tasks

Integral sorting: https://undergroundmathematics.org/chain-rule/integral-sorting

Which substitution?: https://undergroundmathematics.org/chain-rule/which-substitution

Slippery areas: https://undergroundmathematics.org/chain-rule/slippery-areas

Can we find all three integrals?: https://undergroundmathematics.org/product-rule/r5126

How could we integrate $e^{-x} \sin^n x$?: https://undergroundmathematics.org/product-rule/r8134

Integration and Differentiation Practice Questions: http://nrich.maths.org/10451

Calculus Countdown: http://nrich.maths.org/6552

Misconceptions

Work it out 11.1 deals with the intriguing case where integrating, for example, $\frac{1}{3x}$ can give $\frac{1}{3}\ln(3x) + c$ or $\frac{1}{3}lnx + c$, and students could be asked to investigate into how two apparently different answers could both be correct. Similarly, Work it out 11.2 presents three different ways of integrating cos*x*sin*x* and could lead to an interesting discussion about how we can prove that the answers are all equal. Students often tend to plunge into a complicated method when a simpler one might be more appropriate; it is worth encouraging them to stop and reflect first. For example, when integrating quotients, Exercise 11G reinforces the idea that students should choose an efficient strategy, examining a function carefully to decide whether they can integrate it as the f'(*x*)/f(*x*) type, or simplify it algebraically, or whether they first need to split the function into partial fractions. Errors often creep in when students use integration by substitution, for instance forgetting to change their limits or trying unsuccessfully to integrate a mixture of two variables. When integrating by parts, students need to keep track of negative signs, especially when using the formula twice. Also students often fail to spot when to use the chain rule in reverse. Practice questions will help to reinforce the methods, and these could be revisited at intervals during the course. As with differentiation in Topic 10, an extra lesson on choosing which method to use for integrating a variety of functions is a good idea once students have practised all of the methods separately. Exercise 11F and Mixed Practice 11 provide practice at rearranging functions into forms that can be integrated as well as deciding how best to integrate them.

Vocabulary

Integration by parts
Integration by substitution
Partial fractions
Trigonometric identity

Building Links

Using trigonometric identities in integration: Links back to Topic 8, Section 2

Integrating rational functions: Links back to Topic 2 and 5

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Scheme of Learning Overview

11 Further integration techniques (continued)

Prerequisite Knowledge

From Cambridge Textbook

Chapter 9	You should be able to differentiate and integrate polynomial, exponential and trigonometric functions.	1 Find: a $\int \left(4x^2 + \frac{3}{x}\right) \mathrm{d}x$ b $\int 5 \sin x \,\mathrm{d}\,x$. 2 Given that $y = 4\,\mathrm{e}^x$, find: a $\frac{\mathrm{d}\,y}{\mathrm{d}x}$ b $\int_0^1 y \,\mathrm{d}x$.
Chapter 10	You should be able to use the chain rule for differentiation.	3 Differentiate: a $\sin 4x$ b $\ln(x^2 + 1)$.
Chapter 8	You should be able to use double angle formulae.	4 Given that $\cos 2A = 0.28$, find the possible values of $\cos A$.
Chapter 2, Chapter 5	You should be able to split an expression into partial fractions.	5 Write $\frac{36}{(x-1)(x+2)^2}$ in partial fractions.

Scheme of Learning Overview

12 Further applications of calculus

Key Objectives

- Use the second derivative to determine the shape of a curve.
- Use a parameter to describe curves.
- Calculate rates of change of related quantities.
- Find the area between two curves, or between a curve and the *y*-axis.

Lesson Breakdown

1. Properties of curves
2. Parametric equations
3. Connected rates of change
4. More complicated areas

AQA specification reference: G1, C3, C4, G5, H3

Rich Tasks

Parametric preliminaries: https://undergroundmathematics.org/chain-rule/parametric-preliminaries

Parametric points: https://undergroundmathematics.org/chain-rule/parametric-points

Parametric paths: https://undergroundmathematics.org/chain-rule/parametric-paths

What else do you know?: https://undergroundmathematics.org/calculus-meets-functions/what-else-do-you-know

Inverse integrals: https://undergroundmathematics.org/calculus-trig-log/inverse-integrals

Can we track this constantly growing patch of fluid?: https://undergroundmathematics.org/calculus-of-powers/r6959

Misconceptions

Students often get confused when analysing first and second derivatives of curves, especially around points of inflection. Work it out 12.1 neatly encapsulates the calculations they can do and the language they can use to explain their findings. Exercise 12A Q.1-3 will help them to understand and describe sections of curves in various ways. When differentiating, students may initially try to convert parametric equations into Cartesian equations. It is a good idea to encourage them to use the chain rule by introducing curves where this is not possible (Worked examples 12.9 and 12.10 and Exercise 12C Q.4 and 8). In parametric integration, the formula could be introduced as a type of integration by substitution. When finding areas between curves, students need to be sure which curve is on top to avoid errors in subtracting. They may need convincing that the areas below the *x*-axis are taken care of in the algebra. The area between a curve and line can be found using a subtraction method or alternatively by finding the area of a geometrical figure such as a triangle or trapezium. Work it out 12.3 demonstrates common errors in strategy; students should be encouraged to think through possible strategies before deciding which one to implement.

Vocabulary

Concave
Convex
Parameter
Parametric equations
Point of inflection
Second derivative

Building Links

Parametric equations: Links back to Topic 10 and 11

Parametric equations: Links forward to Topic 17

Connected rates of change: Links back to Student Book 1, Topic 12

More complicated areas: Links back to Topic 5

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Scheme of Learning Overview

12 Further applications of calculus (continued)

Prerequisite Knowledge
From Cambridge Textbook

Chapter 10	You should be able to find the first and second derivatives of various functions including using the chain, product and quotient rules.	1 Differentiate each function. a $y = e^{2x} \sin 3x$ b $y = \frac{\ln(x^2+1)}{x^2+1}$
Chapter 8	You should be able to use trigonometric identities to simplify expressions and solve equations.	2 Solve the equation $\cos 2x = 3\sin x$ for $0 \leqslant x \leqslant 2\pi$.
Student Book 1, Chapter 14	You should be able to find the area between a curve and the x-axis.	3 Find the area between the x-axis and the graph of $y = \cos 2x$, between $x = 0$ and $x = \frac{\pi}{6}$.
Chapter 11	You should be able to integrate various functions, use substitution and integration by parts.	4 Integrate these expressions. a $\int \sin^2 x \, dx$ b $\int \frac{x}{x^2-2} dx$ c $\int x \cos 3x \, dx$

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Scheme of Learning Overview

13 Differential equations

Key Objectives

- Solve differential equations of the form $\frac{dy}{dx} = f(x)g(y)$
- Write differential equations in a variety of contexts.
- Interpret a solution of a differential equation and decide whether it is realistic in the given context.

Lesson Breakdown

1. Introduction to differential equations
2. Separable differential equations
3. Modelling with differential equations

AQA specification reference: G6, H7, H8

Rich Tasks

Differential Equation Matcher: http://nrich.maths.org/5875

It's Only a Minus Sign: http://nrich.maths.org/5874

Modelling with Differential Equations: http://nrich.maths.org/11052

Misconceptions

Work it out 13.1 highlights some common errors in separating variables and dealing with constants. Students may benefit from building confidence with supplementary questions focusing on individual aspects such as setting up differential equations, separating variables and recognising which method they should use to integrate different functions. It may also be useful for them to revise the rearrangement of expressions involving exponentials and logarithms. With this solid foundation, they will be better equipped to find general and particular solutions to differential equations (Exercise 13B) and to appreciate the consequences of omitting a constant.

Vocabulary

Differential Equation
Initial conditions
Separation of variables

Building Links

Separable differential equations: Links back to Topic 2

Modelling with differential equations: Links back to Topic 12

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Scheme of Learning Overview

13 Differential equations (continued)

Prerequisite Knowledge

From Cambridge Textbook

Student Book 1, Chapter 7	You should be able to rearrange expressions involving exponents and logarithms.	1 Given that $\ln(v-3) = t + \ln 5$, write v in terms of t.
Student Book 1, Chapter 18	You should be able to draw force diagrams and find net forces.	2 An object of weight $35\,\mathrm{N}$ falls under gravity. The magnitude of the air resistance is $8\,\mathrm{N}$. Find the net force on the object.
Chapter 10	You should be able to write equations involving related rates of change.	3 The rate of change of the radius, r, of a sphere is $5\sqrt{r}$. Find an expression for the rate of change of volume.
Chapter 11	You should be able to integrate, using partial fractions, and simplify the answer, using laws of logs.	4 Integrate and simplify $\int \frac{8}{4-x^2}\,\mathrm{d}x$
Chapter 11	You should be able to use integration by substitution and by parts.	5 Integrate a $\int \frac{4x}{x^2+3}\,\mathrm{d}x$ b $\int x^2 \ln x\,\mathrm{d}x$
Chapter 11	You should be able to integrate, using trigonometric identities.	6 Find $\int \tan^2 2x\,\mathrm{d}x$.

Scheme of Learning Overview

14 Numerical solutions of equations

Key Objectives

- Work with equations that cannot be solved by algebraic rearrangement.
- Find an interval that contains a root of an equation, and how to check that a given solution is correct to a specified degree of accuracy (the sign change method).
- Approximate a part of the curve by a tangent, and use this to find an improved guess for a solution (the Newton–Raphson method).
- Create a sequence which converges to a root of an equation (fixed point iteration).
- Identify situations in which the above methods fail to find a solution.

Lesson Breakdown

1. Locating roots of a function
2. The Newton–Raphson method
3. Limitations of the Newton–Raphson method
4. Fixed point iteration
5. Limitations of fixed point iteration and alternative rearrangements

AQA specification reference: I1, I2, I4

Rich Tasks

A cubic has one real root – can we find an approximation to it?:
https://undergroundmathematics.org/calculus-of-powers/r8231

What is the area under the curve $y = \cos x + \sin x - 2$?:
https://undergroundmathematics.org/calculus-trig-log/r8679

Misconceptions

Through understanding the behaviour of a graph, students will be able to check that their calculations make sense. As this topic is about numerical approximations, students need to be confident in using their calculators for iteration. They also need to ensure that they give their final answer to the correct degree of accuracy, retaining a greater degree of accuracy in their working so as to avoid rounding errors at the end.

Vocabulary

Approximation
Cobweb diagram
Converge
Diverge
Fixed point iteration
Newton-Rhapson method
Roots
Staircase diagram

Building Links

Fixed point iteration: Links back to Topic 4

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Scheme of Learning Overview

14 Numerical solutions of equations (continued)

Prerequisite Knowledge
From Cambridge Textbook

Chapter 4	You should be able to use the term-to-term rule to generate a sequence.	1 Find the first four terms of the sequence defined by $x_{n+1} = 5x_n - 2x_n^2,\ x_1 = 1$.
Chapter 2; Chapter 8	You should be able to rearrange equations involving polynomials, fractions, exponentials, logarithms and trigonometric functions.	2 Rearrange each equation into the required form. a $x = 3\ln(x+2)$ into $x = e^{kx} - C$ b $x = 2\sqrt{x^2-3}$ into $x = \frac{1}{2}\sqrt{x^2+k}$ c $x\cos x - 3\sin x = 0$ into $x = \arctan\left(\frac{x}{a}\right)$
Chapter 9	You should be able to differentiate a variety of functions.	3 Differentiate each equation. a $y = 3x^2\tan x$ b $y = \frac{\ln x}{x^2}$ c $y = e^{3x^2} - \ln(2x)$

Scheme of Learning Overview

15 Numerical integration

Key Objectives

- Understand why definite integration is connected to area under a curve.
- Approximate integrals that cannot be found exactly.
- Establish whether these approximations are overestimates of underestimates.

Lesson Breakdown

1. Integration as the limit of a sum
2. The trapezium rule

AQA specification reference: H4, I3

Rich Tasks

Approximating areas: https://undergroundmathematics.org/introducing-calculus/approximating-areas

Problem areas: https://undergroundmathematics.org/introducing-calculus/problem-areas

Is the Serpentine Lake really 40 acres?: https://undergroundmathematics.org/introducing-calculus/serpentine-lake

Underneath the arches: https://undergroundmathematics.org/introducing-calculus/underneath-the-arches

When does the trapezium rule give the exact answer here?: https://undergroundmathematics.org/introducing-calculus/r5642

Misconceptions

Students will find a sketch of the graph very helpful in checking whether they have the correct *x* and *y* values for their calculations in this topic. Work it out 15.1 and 15.2 deal with errors of this nature while Work it out 15.2 also points to common errors in using the trapezium rule such as using the wrong number of strips (ordinates) or incorrectly adapting the general formula. As with the other numerical approximation methods in Topic 14, students should be retain sufficient decimal places in their calculations to avoid rounding errors in their final answer.

Vocabulary

Limit
Lower bound
Trapezium rule
Upper bound

Building Links

Integration as the limit of a sum: Links forward to Topic 21

The trapezium rule: Links back to Topic 13

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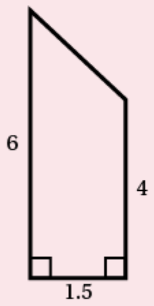
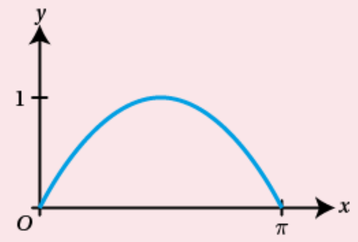
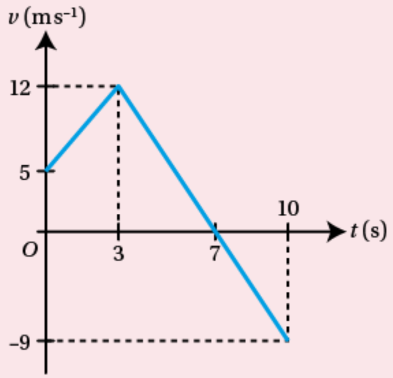
Scheme of Learning Overview

15 Numerical integration (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to calculate the area of a trapezium.	1 Find the area of this shape. 
Student Book 1, Chapter 14	You should know that a definite integral represents the area between the curve and the x-axis.	2 Shade the area given by $\int_1^2 \sin x \, dx$. 
Student Book 1, Chapter 16	You should know how to find distance travelled from a velocity–time graph.	3 Find the total distance travelled in the first 10 seconds. 

Scheme of Learning Overview

16 Applications of vectors

Key Objectives

- Use displacement, velocity and acceleration vectors to describe motion in two dimensions.
- Use some of the constant acceleration formulae with vectors.
- Use calculus to relate displacement, velocity and acceleration vectors in two dimensions when acceleration varies with time.
- Represent vectors in three dimensions using the base vectors **i**, **j** and **k**.
- Use vectors to solve geometrical problems in three dimensions.

Lesson Breakdown

1. Describing motion in two dimensions
2. Constant acceleration equations
3. Calculus with vectors
4. Vectors in three dimensions
5. Solving geometrical problems

AQA specification reference: J1-5, Q3, Q4

Rich Tasks

Bike: http://nrich.maths.org/7680

Motion of a projectile using SUVAT equations and vectors: https://www.geogebra.org/m/MfMyE7nx

Misconceptions

Students may benefit from tackling review questions on handling two- and three-dimensional vectors and on the equations for constant acceleration (Topics 15, 16 and 17 of Student Book 1) before they move on to the applications in this Topic. They need to be aware that each component of a vector must be equal on both sides of an equation and the constant of integration will also be a vector (Worked example 16.9). Students may need to develop confidence in deciding when to integrate or differentiate, and they could benefit from additional practice (Worked examples 16.9 and 16.10 and Exercise 16C). Drawing a diagram is always a good starting point for solving geometrical problems, and even three-dimensional problems do not require artistic ability as they can easily be reduced to a two-dimensional sketch (Exercise 16E Q. 2, 3, 4, 6, 7, 10 and 11).

Vocabulary

Base vector
Components
Magnitude
Unit vector

Building Links

Describing motion in two dimensions: Links back to Topic 12, Section 3

Describing motion in two dimensions: Links back to Topic 1

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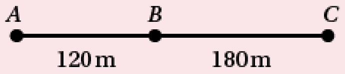
Scheme of Learning Overview

16 Applications of vectors (continued)

Prerequisite Knowledge
From Cambridge Textbook

Student Book 1, Chapter 15	You should be able to link displacement vectors to coordinates and perform operations with vectors.	1 Consider the points $A(2,5), B(-1,3)$ and $C(7,-2)$. Let $\mathbf{p}=\overrightarrow{AB}$ and $\mathbf{q}=\overrightarrow{BC}$. Write in column vector form: a $\mathbf{p}$ b $\mathbf{q}-\mathbf{p}$ c $4\mathbf{q}$ d $\overrightarrow{AC}$.
Student Book 1, Chapter 15	You should be able to find the magnitude and direction of a vector.	2 Find the magnitude and direction of the vector $\begin{pmatrix}-3\\2\end{pmatrix}$.
Student Book 1, Chapter 16	You should understand the concepts of displacement and distance, instantaneous and average velocity and speed and acceleration.	3 In the diagram, positive displacement is measured to the right.  A particle takes 3 seconds to travel from B to C and another 7 seconds to travel from C to A. Find: a the average velocity b the average speed for the whole journey.
Student Book 1, Chapter 16	You should be able to use calculus to work with displacement, velocity and acceleration in one dimension.	4 A particle moves in a straight line with velocity $v=2e^t-t^2$. Find: a the acceleration when $t=3$. b an expression for the displacement from the starting position.
Student Book 1, Chapter 17	You should be able to use constant acceleration formulae in one dimension.	5 A particle accelerates uniformly from $3\,\mathrm{m\,s^{-1}}$ to $7\,\mathrm{m\,s^{-1}}$ while covering a distance of $60\,\mathrm{m}$ in a straight line. Find the acceleration.
Chapter 12	You should be able to work with curves defined parametrically.	6 Find the Cartesian equation of the curve with parametric equations $x=1-2t^2, y=1+t$.

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Scheme of Learning Overview

17 Projectiles

Key Objectives

- Model projectile motion in two dimensions.
- Find the Cartesian equation of the trajectory of a projectile.

Lesson Breakdown

1. Modelling projectile motion
2. The trajectory of a projectile

AQA specification reference: J5, Q5

Rich Tasks

One windy day: https://undergroundmathematics.org/vector-geometry/one-windy-day

Where did it land?: https://undergroundmathematics.org/vector-geometry/where-did-it-land

Angle of shot: http://nrich.maths.org/7361

Two-dimensional projectile motion https://www.khanacademy.org/science/physics/two-dimensional-motion

Motion of a projectile using SUVAT equations and vectors:

https://www.geogebra.org/m/MfMyE7nx

Misconceptions

Students may benefit from practising review questions on the constant acceleration equations before they move on to the applications in this Topic. In solving real life problems, students will find it invaluable to draw a sketch of the projectile's motion when extracting information from the question and considering, for instance, any vertical displacement from the initial position. Work it out 17.1 address some common errors that students make in analysing a projectile's trajectory. Worked example 17.7, Exercise 17B and Mixed Practice 17 will provide practice at tackling various scenarios.

Vocabulary

Horizontal
Projectile
Trajectory
Vertical

Building Links

Modelling projectile motion: Links back to Topic 16

The trajectory of a projectile: Links back to Topic 12

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Scheme of Learning Overview

17 Projectiles (continued)

Prerequisite Knowledge

From Cambridge Textbook

Student Book 1, Chapter 15	You should be able to find the magnitude and direction of a vector.	1 Find the magnitude and direction of the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.
Student Book 1, Chapter 17	You should be able to use constant acceleration formulae in one dimension.	2 A particle accelerates uniformly from $3\,\mathrm{m\,s^{-1}}$ to $7\,\mathrm{m\,s^{-1}}$ while covering a distance of $60\,\mathrm{m}$ in a straight line. Work out the acceleration.
Chapter 16	You should be able to use constant acceleration formulae in two dimensions.	3 A particle initially has velocity $(3\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ and accelerates at $(\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-2}}$. Find its velocity after 3 seconds.
Chapter 8	You should know the sine double angle identity.	4 Express $\sin x \cos x$ in terms of $\sin 2x$.
Chapter 8	You should know the definition of $\sec x$.	5 Express $\frac{1}{\cos^2 x}$ in terms of $\sec x$.
Chapter 8	You should be able to solve equations involving $\tan x$ and $\sec^2 x$.	6 Solve the equation $2\sec^2 x + \tan x - 5 = 0$ for $0^\circ < x < 180^\circ$.

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Scheme of Learning Overview

18 Forces in context

Key Objectives

- Resolve forces in a given direction in order to calculate the resultant force.
- Use a model for friction.
- Determine the acceleration of a particle moving on an inclined plane.

Lesson Breakdown

1. Resolving forces
2. Coefficient of friction
3. Motion on a slope

AQA specification reference: R2, R4-R6

Rich Tasks

Make it stop!: https://undergroundmathematics.org/vector-geometry/make-it-stop

A frictional story: https://undergroundmathematics.org/vector-geometry/frictional-story

Resolving forces down a plane: https://www.geogebra.org/m/eqX3tdU7

Inclined plane with friction, two masses, and a pulley:

https://www.geogebra.org/m/kZ4H4C7v

Misconceptions

A correct, clear force diagram will help students to understand the situation they are modelling. Labelling the direction of motion enables them to make good decisions about how to apply Newton's second law or the constant acceleration equations. Students should appreciate the difference between mass and weight and be able to add the correct force due to gravity to their diagram. If they are struggling to resolve forces, they may need more practice (Topic 18 of Student Book 1). When dealing with friction, students should recognise the physical implications of the fact that $F = \mu R$ is the maximum or limiting value of friction.

Vocabulary

Components
Coefficient of friction
Limiting equilibrium
Normal reaction
Resolve
Resultant force

Building Links

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Scheme of Learning Overview

18 Forces in context (continued)

Prerequisite Knowledge
From Cambridge Textbook

Student Book 1, Chapter 18	You should be able to add vectors and find magnitudes.	1 Three horizontal forces act on a particle. In newtons, the forces are: $\mathbf{F_1} = 2\mathbf{i}$, $\mathbf{F_2} = 3\mathbf{j}$ and $\mathbf{F_3} = \mathbf{i} - 2\mathbf{j}$. Calculate the magnitude of the resultant force and its angle from the direction $\mathbf{i}$.
Student Book 1, Chapter 17	You should be able to solve problems involving motion with constant acceleration.	2 A force of $5\,\mathrm{N}$ acts upon a particle with mass $2\,\mathrm{kg}$. If the particle is initially at rest, after how many seconds will its displacement be equal to $5\,\mathrm{m}$?

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Scheme of Learning Overview

19 Moments

Key Objectives

- Find the turning effect of a force.
- Work with uniform rods and laminas.
- Understand and use rotational equilibrium.

Lesson Breakdown

1. The turning effect of a force
2. Equilibrium

AQA specification reference: P1, S1

Rich Tasks

Inside outside: http://nrich.maths.org/4767

Balance point: http://nrich.maths.org/4768

Moments: https://www.geogebra.org/m/Qnb2Tzcc

Misconceptions

Students should be encouraged to draw a diagram showing all forces that are present, not only the ones mentioned in a question, the correct dimensions and the position of the centre of mass where appropriate. In order to calculate the moments, they need to think carefully about the direction in which each force operates. For forces in equilibrium, students should practise choosing the most convenient point about which to calculate moments such that they can ignore unknown forces. Work it out 19.1 and Exercise 19B will help them to make decisions about their strategy so as to set up the equations that will help them to solve more challenging problems.

Vocabulary

Centre of mass
Equilibrium
Lamina
Moment
Resultant moment
Rod
Uniform

Building Links

Equilibrium: Links back to Topic 18

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Scheme of Learning Overview

19 Moments (continued)

Prerequisite Knowledge

From Cambridge Textbook

Student Book 1, Chapter 18	You should be able to recognise types of force acting on a particle.	1 A particle is pulled across a smooth horizontal table by a string that is parallel to the table. Draw a diagram and label all the forces acting on the particle.
Student Book 1, Chapter 18	You should understand when a particle is in equilibrium.	2 Three forces act on a particle as shown.  The particle is in equilibrium. Find the magnitude of F.

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Scheme of Learning Overview

20 Conditional probability

Key Objectives

- Use set notation to describe probabilities.
- Work with conditional probabilities in the context of Venn diagrams, two-way tables and tree diagrams.
- Use a formula for conditional probability.

Lesson Breakdown

1. Set notation and Venn diagrams
2. Two-way tables
3. Tree diagrams

AQA specification reference: M2, M3

Rich Tasks

Tree Diagrams, 2-Way Tables and Venn Diagrams: http://nrich.maths.org/9861

Conditional Probability Is Important for All Students!: http://nrich.maths.org/9646

Venn diagrams: http://www.examsolutions.net/tutorials/exam-questions-venn-diagrams/

Misconceptions

Students often find this topic conceptually difficult to begin with and may have trouble extracting information from questions in order to apply it to their Venn diagram. When they try to solve a problem without drawing a diagram, their intuition may prove to be misleading. Learning the conditional probability formula will help, as will confidence in using not only Venn diagrams but other tools such as two-way tables and tree diagrams to illustrate a problem. Work it out 20.1 is an example of a question which is greatly helped by a diagram of some description, and many of the problems in Exercise 20D lend themselves to careful choice of strategy. Students may not notice when their calculations lead to answers which are meaningless, e.g. probability > 1 or < 0, or values that are inconsistent with information given in the question. It is a good idea to encourage them to check their results to see if they make sense.

Vocabulary

Complement
Intersection
Mutually exclusive
Set notation
Union
Venn diagram

Building Links

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Scheme of Learning Overview

20 Conditional probability (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to use tree diagrams to solve problems.	1 A mother has two children, who are not twins. What is the probability that they are both boys or both girls?
Student Book 1, Chapter 1	You should be able to use set notation.	2 Write out this set. $\{\text{prime numbers}\} \cap \{\text{even numbers}\}$
Student Book 1, Chapter 21	You should understand the basic laws of probability including the terms mutually exclusive and independent.	3 In a family having a car is mutually exclusive of having a motorbike. The probability of having a car is $\frac{1}{2}$. The probability of having a motorbike is $\frac{1}{3}$. What is the probability of having neither a car nor a motorbike?
Student Book 1, Chapter 21	You should understand probability distributions, including the binomial distribution.	4 What is the probability of getting 4 heads when 6 fair coins are tossed?

Scheme of Learning Overview

21 The normal distribution

Key Objectives

- Calculate probabilities for a normally distributed random variable.
- Relate any normal distribution to the standard normal distribution.
- Calculate the value of the variable with a given cumulative probability.
- Find mean and standard deviation from information about probabilities.
- Use the normal distribution as a model.
- Use the normal distribution as an approximation to the binomial distribution.

Lesson Breakdown

1. Introduction to normal probabilities
2. Inverse normal distribution
3. Finding unknown μ or σ
4. Modelling with the normal distribution

AQA specification reference: N2, N3

Rich Tasks

Into the Normal Distribution: http://nrich.maths.org/6314

Over-Booking (Normal approximation to binomial): http://nrich.maths.org/4932

Normal Probability Distributions

http://www.intmath.com/counting-probability/14-normal-probability-distribution.php

Normal Distribution

https://www.mathsisfun.com/data/standard-normal-distribution.html

Misconceptions

Two common misconceptions are shown in Work it out 21.1: using P($X > x$) instead of P($X < x$) and using probability instead of z. Notation is important in avoiding such errors and in understanding clearly the difference between X and Z. Students will also find it helpful to sketch a graph and to shade the area corresponding to the probability they need. Confidence with the calculator is an advantage, though writing down probability values from their calculator without showing any working can make students prone to errors. Real life examples help to provide motivation for mastering what can be a tricky subject for many students.

Vocabulary

Mean
Inverse normal distribution
Normal distribution
Standard deviation
Variance
Z-score

Building Links

Introduction to normal probabilities: Links back to Topic 20

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Scheme of Learning Overview

21 The normal distribution (continued)

Prerequisite Knowledge

From Cambridge Textbook

GCSE	You should be able to solve simultaneous equations.	1 Solve these simultaneous equations. $x + 4.72y = 7.32$ $x - 1.28y = 0.435$
Student Book 1, Chapter 20	You should be able to interpret histograms.	2 What is the frequency density of a group of 60 people with masses strictly between 40 kg and 50 kg?
Student Book 1, Chapter 21	You should be able to use the binomial distribution.	3 The probability of rolling a six on a biased dice is $\frac{1}{5}$. The dice is rolled four times. Find the probability of getting exactly 1 six.
Chapter 20	You should be able to work with tree diagrams.	4 There is a 20% chance of it raining. If it rains there is a 60% chance I am late. If it does not rain there is a 25% chance I am late. What is the probability that I am late?
Chapter 20	You should be able to calculate conditional probabilities.	5 Two dice are rolled – one red and one blue. If the total score is 5, what is the probability that the score on the red dice is 3?

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Scheme of Learning Overview

22 Further hypothesis testing

Key Objectives

- Treat the sample mean as a random variable and see how it is distributed.
- Test whether the mean of a normally distributed population is different from a predicted value.
- Test whether a set of bivariate data provides evidence for significant correlation.

Lesson Breakdown

1. Distribution of the sample mean
2. Hypothesis tests for a mean
3. Hypothesis tests for correlation coefficients

AQA specification reference: O1, O3

Rich Tasks

The Sampling Distribution of the Sample Mean http://www.jbstatistics.com/the-sampling-distribution-of-the-sample-mean/

An Introduction to Hypothesis Testing

http://www.jbstatistics.com/an-introduction-to-hypothesis-testing/

Z Tests for one mean – the *p*-value

http://www.jbstatistics.com/z-tests-for-one-mean-the-p-value/

Misconceptions

If students need more practice at writing null and alternative hypotheses for their tests, Exercise 22B Q. 1 may be useful. Work it out 22.1 highlights some common errors in stating hypotheses, finding the standard error for a sampling distribution and calculating *p*-values for one- and two-tailed tests. Exercise 22B Q.3 reinforces these ideas and students may find it useful to sketch a graph to help them identify the critical region(s) and arrive at the correct outcome. Writing conclusions accurately is a crucial part of conducting hypothesis tests. In particular, students should ensure that the statements they make are not too categorical (e.g. 'There is evidence to support the manufacturer's claim that …' rather than 'The manufacturer's claim is correct').

Vocabulary

Correlation coefficient
Critical region
Critical value
Null hypothesis
Population mean
Random variable
Sample mean
Significance level

Building Links

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Scheme of Learning Overview

22 Further hypothesis testing (continued)

Prerequisite Knowledge

From Cambridge Textbook

Student Book 1, Chapter 20	You should be able to interpret correlation coefficients.	1 Information on height, mass, waist size and average time spent exercising per week was recorded from a random sample of adult males. Match the values of the product moment correlation coefficient with each of the sets of variables: A height and mass B height and time spent exercising C waist measurement and time spent exercising. 1 $r = -0.82$ 2 $r = 0.13$ 3 $r = 0.71$
Student Book 1, Chapter 22	You should be able to conduct hypothesis tests using the binomial distribution.	2 A dice is rolled ten times and four sixes are obtained. It is claimed that the dice is biased in favour of getting a six. Test this claim at the 10% level.
Chapter 21	You should be able to conduct calculations using the normal distribution.	3 $X \sim N(175, 10^2)$. Find: a $P(X < 190)$ b $P(150 < X < 185)$ c a such that $P(X > a) = 0.01$.

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